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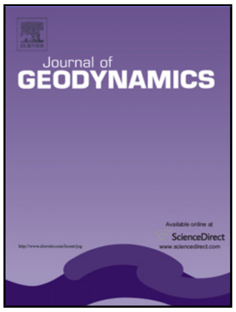

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Testing a New Free Core Nutation empirical model

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Abstract

The Free Core Nutation (FCN) is a free mode of the Earth's rotation caused by the different material characteristics of the Earth's core and mantle. This causes the rotational axes of those layers to slightly diverge from each other, resulting in a wobble of the Earth's rotation axis comparable to nutations.

In this paper we focus on estimating empirical FCN models using the observed nutations derived from the VLBI sessions between 1993 and 2013. Assuming a fixed value for the oscillation period, the time-variable amplitudes and phases are estimated by means of multiple sliding window analyses. The effects of using different a priori Earth Rotation Parameters (ERP) in the derivation of models are also addressed. The optimal choice of the fundamental parameters of the model, namely the window width and step-size of its shift, is searched by performing a thorough experimental analysis using real data.

The former analyses lead to the derivation of a model with a temporal resolution higher than the one used in the models currently available, with a sliding window reduced to 400 days and a day by day shift. It is shown that this new model increases the accuracy of the modeling of the observed Earth's rotation. Besides, empirical models determined from USNO Finals as a priori ERP present a slightly lower Weighted Root Mean Square (WRMS) of residuals than IERS 08 C04 along the whole period of VLBI observations, according to our computations. The model is also validated through comparisons with other recognized models. The level of agreement among them is satisfactory. Let us remark that our estimates give rise to the lowest residuals and seem to reproduce the FCN signal in more detail.

Keywords
Earth, Reference systems, Nutations, Empirical methods

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1. Introduction

The Earth's Free Core Nutation (FCN), also known as the Nearly Diurnal Free Wobble (NDFW) because of its origin and period in the terrestrial frame, is a free-oscillation mode of the Earth that appears in any model consisting of a solid mantle and a fluid core. It is a retrograde motion due to the misalignment between the rotation axes of the mantle and the core (Toomre, 1974; Smith, 1977; Wahr, 1981). Kinematically, it resides in the "nutation frequency band", along with all astronomical forced nutations and sharing with them the same frequency characteristics. As far as the rotation of the mantle (or "Earth rotation") is concerned, it has a long retrograde period of about 430 days (with average amplitude of about 100 μas) relative to the inertial frame, or a period slightly shorter than 1 day in the retrograde-diurnal band, relative to the rotating terrestrial frame. FCN is a free mode of Earth rotation hence it shows variable excitation and damping.

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Some of the astronomical nutations have periods sufficiently close to the FCN that they feel a resonance effect which magnifies their amplitudes, the most notable example being the retrograde annual nutation. In fact, that was how FCN was first identified in nutation time series obtained by VLBI (Herring et al., 1986; Gwinn et al., 1986). Similarly, gravitational effects of FCN have also been detected in surface gravity records from superconducting gravimeter (Defraigne et al., 1995; Richter, 1988; Florsch and Hinderer, 2000) and long-period seismometers (Cummins and Wahr, 1993). The period estimates range between 425-435 days, rather than the predicted 460 days based on the hydrostatic theory for the Earth, implying a core more elliptical than a core in hydrostatic equilibrium would be. In a recent study Krásná et al. (2013) performed the first simultaneous adjustment of solid tides and nutation data to estimate the FCN period, which was determined as 431.18 ± 0.10 sidereal days (-430.00 solar days), close to the value of 430.21 solar days estimated by Lambert (2007).

It is well known that the current IAU 2006/2000A precession/nutation theory needs to be complemented with a model providing the FCN, since otherwise the observation-theory residuals of the Celestial Intermediate Pole (CIP) coordinates would be noticeably large (approaching about 400 *µas as maximum deviations*), thus damaging the accuracy of predictions. The FCN signal exhibits complex patterns of variation affecting its amplitude, period and phase, whose geophysical causes are not fully understood yet, although recent investigations on the effects of geophysical excitations are producing interesting insight. For instance, Vondrak and Ron (2014) used numerical integration of the broad band Brzezinski's equations (1994) to compute the effects of modeled geophysical excitations, whereas Chao and Hsieh (2015) formulated the equations of motion in terms of the temporal convolution of a free-rotational mode of resonance with an excitation function, showing that the resulting dynamical behavior has similar features than the observed signal. Nevertheless, today it is still necessary to use empirical models to describe the inferred FCN signal and to predict it to a limited extent, and the IERS Conventions (2010) (Petit and Luzum, 2010) recommends the use of the model of Lambert (2007), which is regularly updated.

Let us recall that the IAU 2000 nutation theory was fitted with the help of a previous, different FCN model (Herring et al., 2002) with the oscillation amplitude determined as a piecewise linear function of time, defined so that it has linear variations between ‘‘nodes’’ at selected times. Shirai and Fukushima (2000b,a) used a more complex model including an exponential decay due to the dissipation in the core-mantle boundary and the possibility of some sudden jumps. Ferrandiz et al. (2002) developed the first model based on performing the fit to the observed nutations by a Sliding Window approach (with window lengths of either one or two FCN periods), following the ideas already applied successfully by Schuh et al. (2000) in their analyses of Polar Motion (PM). Few years later Lambert (2007) developed a new Sliding Window solution that provides one reference value of the amplitude for each year, which can then be used for interpolation.

The scientific community demands the availability of FCN models with increasingly high accuracy, necessary to fit and predict the observed nutation offsets and to improve the knowledge of the geophysical excitation mechanisms that cause its amplitude, period, and phase variations. In this research we have developed a new empirical FCN model with higher temporal resolution by fitting

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the amplitude parameters directly to the Very Long Baseline Interferometry (VLBI) solution (1993-2013) calculated with the GeoForschungsZentrum (GFZ) version (Nilsson et al., 2015) of the Vienna VLBI Software (VieVS) (Böhm et al., 2012). A comparison with other recently determined empirical FCN models: Malkin (2013), Krásná et al. (2013) and Lambert and Dehant (2007) was included by means of the weight root mean square (WRMS) of the residuals during the entire period of VLBI data. Moreover, we assessed the sensitivity of our empirical FCN model with respect to different a priori Earth Orientation Parameter series, namely IERS 08 C04 and USNO finals.

1. FCN modeling in VLBI data

The orientation of the Earth's rotation axis in the inertial space is customary given by the *X, Y* coordinates of the celestial pole. Using distant quasar sources as reference frame, VLBI is at present the unique technique in Space Geodesy that is capable of accurately observing the 3-D Earth Orientation Parameters (EOP) in space. In the VLBI observation of the variation of the Earth's rotation axis, the empirical convention is such that the bulk of the broad-band signal is considered as polar motion; only the signal residing in the narrow retrograde-diurnal band is considered as nutations.

In this nutation band, the VLBI nutation time series contains the FCN constituent plus a great number of astronomical nutation components, many of which have much larger or comparable amplitudes than the FCN can realistically have. The astronomical nutation models have to be complemented with an empirical term corresponding to the FCN. IERS Conventions (2010) (Petit and Luzum, 2010) recommends an FCN empirical model of Lambert (2007). However, other alternative models are available today (Krásná et al., 2013; Malkin, 2013). In the time domain, the FCN signal is "buried"; it is thus simpler to conceptualize it in the frequency domain. The nutations are purely sinusoidal at periods that are precisely known from celestial mechanics. However, their amplitude and phase are not precisely known, subject to either observational errors or modeling uncertainties. This lack of knowledge is especially severe for components that are close to the FCN period due to the near-resonance magnifying effects mentioned above. As far as FCN itself is concerned, it is thus difficult to completely model and remove the nutation terms, which is necessary in order to reveal the true properties and behavior of FCN. Therefore, one must first carefully model the astronomical nutations to the extent of present knowledge.

The study of Earth Rotation by space geodetic techniques involves determining the transformation matrix between the terrestrial and celestial reference system what is commonly parameterized using the EOP. There are five EOP available and we have focused our discussion on two angles with respect to the celestial frame. The celestial pole coordinates (*X; Y*) are currently derived from the adopted theory of precession-nutation IAU2000A (Mathews et al, 2002), and the corrections (*dX; dY*) or offsets of the celestial pole are the part of the observed CIP location unexplained by the said theory, and are currently provided by VLBI in the form of time series. These offsets are affected by periodic components attributable to the retrograde NDFW of the rotation axis.

The procedure leading to the VLBI time series that we utilized is described as follows. We took all available VLBI data from 1993 to 2013 and discarded all the data before 1993 as a consequence of the poor quality and temporal resolution of VLBI sessions in the eighties, as suggested by Malkin (2013) and Chao and Hsieh (2015) in their FCN studies. Positions and velocities of all stations as

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well as source coordinates were estimated by imposing no-net-translation and no-net-rotation conditions with respect to ITRF2008 (Altamimi et al., 2011) and ICRF2 (Fey et al., 2015). The five EOP were estimated. After single-session adjustment, we discarded the VLBI sessions with a posteriori sigma of unit weight larger than 3.

1. Empirical FCN models

FCN models can be characterized from a weighted least squares fit of these equations (Lambert, 2007):

$$X_{FCN} = A_C \cos\left(\sigma_{FCN} t\right) - A_S \sin\left(\sigma_{FCN} t\right) + X_0$$

$$Y_{FCN} = A_S \cos\left(\sigma_{FCN} t\right) + A_C \sin\left(\sigma_{FCN} t\right) + Y_0$$

where $\sigma_{FCN} = 2\pi/P$ is the frequency of FCN in the Celestial Reference Frame (CRF), A is the amplitude, t is the time relative to J2000.0, P is the period, and X_o and Y_o are constant offsets. These offsets accumulate the low-frequency part of the signal. Therefore, the contribution of the FCN to the CIP offsets (CPO) can be computed by using eq. 1 without taking into account the shift terms X_o and Y_o.

FCN amplitudes and phases are not fix over time, but exhibit a noticeable variation. Causes of this variability need to be identified and predicted with high accuracy, which is an indispensable requirement to fulfill the stringent targets of the Global Geodetic Observing System (GGOS) of the International Association of Geodesy (IAG): *1 mm* position and *0.1 mm/year* in velocity on global scales for the ITRF (Plag and Pearlman, 2009) that are crucial for many practical geophysical applications. That is why a large quantity of empirical FCN models were estimated and tested with different sliding window lengths N_L (from 8 months to 7 years) displaced by different time spans N_D (from 1 day up to 12 months) with the purpose of finding optimum parameters that provide the lowest residuals. We adopted a constant period of -431.18 sidereal days (-430.00 solar days) since the phase and period would be correlated in the fitting. This value was determined recently by Krásná et al. (2013) from VLBI and gravity data, with an accuracy of more than twice the one estimated by Mathews et al. (2002), -430.21 solar days. Upon considering a fixed period, the possibility to estimate the amplitudes from window lengths N_L lesser than the FCN period had to be checked and examined. Therefore, models with high temporal resolution were calculated by applying a conventional adjustment method to fit harmonic constituents independently of whether N_L is smaller or larger than the FCN period. The advantage of the models using a small window size is the ability to recognize and relate the geophysical changes to FCN amplitude and phase variations.

Two different approaches were used to estimate the CPO by VLBI. The estimates were computed starting from different a priori Earth rotation parameters (ERP) series (taking the PM and UT1-UTC values from IERS 08 C04 or USNO finals), and the CIP coordinates from the IAU 2006/2000A precession/nutation theory. Before estimating the FCN models, the CPO differences between the results using IERS 08 C04 and USNO finals as a priori ERP were compared by means of the rms of them, finding a scatter of about 36.9 µas and 31.9 µas in *dX* and *dY* respectively (Fig. 1). All the FCN models were derived from the analysis of the differences between the celestial pole coordinates

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determined from VLBI observations and the corresponding coordinates resulting from the IAU 2006/2000A theory.

1. Results

1.1. Sliding Window: Size and Displacement

A large variety of FCN models based on different a priori ERP (IERS 08 C04 and USNO Finals) and fitted with a constant period of -430.00 solar days were determined using different N_L and N_D. One of the FCN models which were calculated with a sliding window of 2 FCN periods and half-yearly displaced using a priori ERP from IERS 08 C04 is shown in the Fig. 2a and 2b. It seems clearly visible that a low frequency part of the signal still remains in the CPO with approximately the same magnitude and opposite signs for *dX* and *dY* respectively. This apparent small curvature could be removed by using the shift terms *Xo* and *Yo* that appear in the equation 1. Different studies applied several methods to delete these kinds of signals, e.g. Capitaine et al. (2009) fitted a parabola plus a *18.6*-year periodic term to the residuals of the IAU 2006 model with respect to the VLBI measurements.

Theoretically, if all signals (linear and nonlinear) could be perfectly modeled by the IAU 2006/2000A precession/nutation model, the remaining residuals would be close to zero once the FCN effect was deleted. However, this does not happen. Fig. 2c and 2d show the residuals after the free core nutation has been removed with the aforementioned model. These residuals ($[\Delta X, \Delta Y]$ = CPO-FCN) gather the unmodelled, mismodelled or stochastics contributions to nutations caused by the limitations of the theory and the models. To have an idea of the level of uncertainty, the standard deviations (std) for every experiment by both IERS 08 C04 and USNO finals as a priori ERP were estimated and are shown in Fig. 3a and 3b, respectively. It is important to mark that the range of the sliding window length varies from 0.4 to 4 cycles for the whole period and that the windows are displaced from 1 day up to 1 year. The lowest scatterings, with std of about 145 µas, are located on the left side for each approach (tiny N_L), on the contrary the largest values of about 155 µas are on the right, when using large N_L. This test gives an idea of the importance of the parameters N_L and N_D, and justifies the finer resolution. Notice that generally the smallest errors correspond to using a priori ERP from USNO finals, whilst IERS 08 C04 series causes a slight deterioration at the modeling. If the early years (1984-1993) are included in the analysis, the errors raise around 8%, in agreement with similar behavior reported in (Malkin, 2013; Chao and Hsieh, 2015).

Let us remark that the lower rms are obtained for the lower values of the window length N_L, which are noticeably smaller than the FCN period indeed. That may be envisaged as a capability of the model of detecting more details of the signal, but the situation is not so simple. It is known that the first IERS nutation model, the IERS 1996 derived by Herring and included in the IERS Conventions (1996) (McCarthy, 1996) reached a remarkable accuracy but was far from be based in a whole 18.6-years span of data, the period of the main nutation term. However, the use of too short time intervals entails problems and effects the reliability and accuracy of the fitting, The amplitude coefficients can be poorly determined and hence it can be argued to which extent a model really stands for a physical signal.

Therefore, further analyses are needed to decide on the parameters N_L and N_D. In the following we search a compromise to obtain a kind of optimal model balancing the minimization of the root mean

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square (rms) of the fit, and of the coefficient errors, A_c and A_s. As can be seen in Fig. 4a and 4b those errors have opposite patterns, i.e. the shorter the sliding window is, the smaller the rms is, but the larger the coefficient errors are and vice versa. To tackle that, the rms and the error coefficients of the different empirical FCN models estimated have been normalized with respect to their minimum and maximum values and finally summed between them. Different error weightings are conceivable, but we did not find a clear advantage in other alternative choices. Thus, the minimum values of the combined error indicate the best FCN models according to this option. As shown in Fig. 4c and following those criteria, it is easy to conclude that the parameter N_L shows the greatest gradient with optimal values between 0.7 and 1.6 FCN periods. On the contrary and as suggested by Fig. 3, the displacement (N_D) between the subsequent fit is less significant since the residuals hardly change with it. Having that in mind and to reduce the high computational cost of the experiments, the range of variation of parameter N_L was reduced to 300 - 561 days for the final analyses (with N_L varying 1 by 1) and we only focused in three different displacements N_D, namely 1 day, 15 days and 1 month. Fig. 4d shows the optimum sliding window length for the aforementioned cases computed from different a priori ERP. In all these estimates the optimum value was located in a narrow neighborhood of 400 days. We performed additional behavior tests for longer displacements, e.g. 3 and 5 months. It is noteworthy to mention that for those experiments the optimum length was reached in 429 and 436 days respectively. It indicates that, the larger the parameter N_D is, the larger the optimum parameter N_L is driven to be.

1. Comparison with other models and discussion

Once the optimal value of parameter N_L was set in 400 days, a particular FCN model was computed with that fixed length and moving the window by one-day steps. In this section, we focused on the comparisons with other FCN models (Krásná et al., 2013; Lambert 2007; Malkin 2013) for mutual validation. Krásná et al. (2013) recently determined a FCN model using groups of 4 years (3.4 FCN periods) measurement data displaced one year. It means that data from 1984.0 to 1988.0 were involved in the first run and thus the estimates are valid for 1986.0. The second global solution included data from 1985.0 till 1989.0 and so on. She used a period of -430.00 solar days. Lambert (2007) estimated a FCN model (recommended in the IERS Conventions (2010)) using a period of -430.21 solar days, a sliding window size of two years and displaced by one year. The latter FCN model, developed by Malkin (2013), is computed by using the running reference interval of 431 days with a shift of one day between sequential reference intervals. All the FCN series estimated with the aforementioned models and the model parameters A_c and A_s that appear in equation 1 are depicted in Fig. 5. All of these series are reasonably near to each other along the whole period except at certain intervals close to the years 1996 or 2002. Table 1 summarizes the statistics analysis between the reported models. In order to make the Krásná's model comparable with the other, the calculations with this model are made only using data up to 2009. We used the whole span for the rest of comparisons.

It can be seen in Table 1 that the Ac and A_S coefficients of all the models show a satisfactory level of agreement, below 30 µas. The highest consistency occurs between the model in this paper and Malkin's (2013) with a scatter of about 13 µas. The larger discrepancies (almost twice) of ours with respect to Lambert's and Krásná's models could be attributed to different groups of data used in the adjustment or to differences in the fundamental parameters. The last two columns of Table 1 show the wrms difference of the residuals between our FCN estimates and the other models in the time domain. The results show that the best level of consistency, close to 19 µas, corresponds to Malkin's

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and our model. However, this wrms value increases up to 30 µas with Krásná's and Lambert's models. Identical pattern can be seen by using different a priori EOP series (IERS 08 C04 and USNO finals).

In addition to the previous mutual validation, other error analyses have been performed (Table 2). We display the STD of the residuals CPO-FCN after each FCN model is removed from the CPO. The smallest residuals, of about 146 µas, correspond to our model computed with a sliding window length of 400 days, along with the USNO Finals as a priori ERP. Models of Lambert (2007) and Krásná et al. (2013) achieve the less accurate results, although the increase is of few µas. On the other hand, the USNO Finals series always reaches better accuracy than IERS 08 C04 for all the models with a slight improvement of around 3 µas.

Next, supplemental VLBI estimates were computed using different a priori ERP (IERS 08 C04 or USNO Finals) including a sort of nutations (CPO) given in the USNO finals and IERS 08 C04 instead of the IAU 2006/2000A precession/nutation model plus FCN model. With a view to find the maximum consistency and agreement, these new calculations, which should be free of FCN, were compared with remaining residuals of the different empirical FCN models by means of the WRMS difference. The finest results (minimum WRMS of about 70 µas) can be found in our own model and Malkin (2013), by using USNO finals. Again, the FCN models estimated from IERS 08 C04 as a priori ERP achieves the less accurate results with WRMS of about 15 µas larger than USNO finals.

Once the usefulness of choosing a sliding window length of 400 days has been tested in terms of accuracy, the amplitude and phase variations of each model are estimated and compared. Fig. 6 shows that all the FCN models exhibit similar variation patterns in their amplitudes and phases in spite of their different scenarios, but the variations associated to Lambert's and Krásná's model are smoother, as expected since they are computed with running two-year and 4-year windows respectively and with one-year shifts. As it can be seen, depending on the settings (N_L and N_D), the amplitude and phase variations can be determined with variable accuracy and approximation. In general, the shorter the length of the sliding window is, the rougher the curves appear and vice versa. The higher level of details likely corresponds to a better sensitivity of the Malkin's and the new model with respect to the geophysical changes. Of course, it is not possible to ensure that all of the inferred FCN signal corresponds actually to FCN since physical models capable of reproducing the observed behavior with clarity and enough accuracy are not available yet.

Let us note the that FCN seems to follow an exponential decay law to a large extent from the early years up to 2000. After 2000, this behavior is reversed about until 2013. To further investigate the occurrence of long-period or decadal variations, we show in Fig. 7 the periodograms of amplitudes and phases for each FCN model used in the previous comparisons. There is a high level of agreement among all of them, Krásná et al. (2013) showing a slight shift of the lower frequency peaks perhaps due to the use of larger parameters N_L and N_D. Regarding the amplitude, the maximum spectral power concentrates near the 5 years period. Its origin is unclear, but it may not be an artifact but have actual physical origin; let us recall that Vondrak and Ron (2014) found error

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decreases when changing the initial values of their numerical integrations (from 1989.0 to 2013.5) at epochs 1994.3, 1999.3, 2003.8 and 2007.8. That behavior could be associated to strong ENSO events or certain geomagnetic jerk events as pointed out by Malkin (2014) and Shirai et al. (2005) among other candidates. Further investigation is required to elucidate the issue. As for the phase, we remark the appearance of a small peak close to the 6.6-years period, which seems due to the beating between the FCN and the annual retrograde (as for Chandler wobble) signals. The 5-years peak is also present, with similar power than the 6.6 and 4 years ones.

We think that the material presented in this section supports the validity of our new FCN model, whose competing accuracy and higher resolution were shown in section 4.

1. Conclusions

Previous studies of FCN mostly focused on the FCN's resonance effect on neighboring astronomical nutations found in VLBI and gravity data. Such indirect evidences are well suited for estimating FCN's natural period, but unable to provide information about the behavior of FCN over time. Direct evidences for the latter begin to emerge as VLBI data quality improves. In this study, we estimated different empirical FCN models using the residual VLBI nutation time series (CPO) from 1993.0 to 2013.0 obtained after the astronomical nutations were accounted for and then removed according to the IAU 2006/2000A precession/nutation model. Phase and amplitude variations were examined. Furthermore, different EOP (IERS 08 C04 and USNO Finals) were analyzed with a view to know which series achieves the optimal FCN model. Finally, a comparison with Malkin (2013), Krásna et al. (2013) and Lambert (2007) model was done. In all the estimates the period was fixed to the recent value determined by Krásná et al. (2013).

A large number of FCN models were estimated and assessed. The methodology and the choice of the sliding window length vary for each simulation. Least-squares estimates of the coefficients of the FCN models were done at each interval of sliding window with different displacements between the subsequent fit. As already anticipated by Lambert (2007) in a test performed with synthetic data, the choice of a proper sliding window size (N_L) is a relevant matter, since inappropriate values could cause accuracy variations of about 15 μas. Less significant is the displacement between the subsequent fit (N_D) in which the residuals hardly change. We have performed a thorough analysis of the errors depending on parameter setting concluding that the use of FCN models determined from a sliding window size of 400 days with a minimal displacement between the subsequent fit (one-day step) improves the accuracy of the modeling; achieving always the best statistics with slight improvements with respect to Malkin’s, Lambert’s and Krásná’s et al. models of about 1 μas, 2 μas and 5 μas, respectively. The highest agreement occurs with Malkin (2013). In addition, the usage of IERS 04 C08 as a priori ERP series gives slightly larger residuals. This conclusion was already addressed by Malkin (2010) between IERS 08 C4 and NEOS (U.S. National Earth Orientation Service). Therefore the USNO Finals series provides the smaller residuals when the empirical FCN models are assessed. The combination of IAU 2006/2000A with the proposed FCN model can fit the VLBI observations of X, Y with a WRMS of about 70 μas, which is about a factor of two larger than the GGOS accuracy goal (1 mm). However, this value increases by 15 μas when the IERS 08 C04 series is used as a priori ERP. All of these comparisons provide a first validation of the new model, which seems to be more sensitive to geophysical changes, e.g. sparse sudden events.

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The FCN is essentially the only signal that remains above the noise level caused by the unmodelled geophysical signals and the inaccuracies of the theory and models. It clearly has a time-variable amplitude during the studied period, as can be demonstrated by a simple wavelet time-frequency spectrum. A sliding window least-squares fit of the FCN amplitude yields similar results: To first order, the amplitude hovers around 0.3 mas prior to 1990, then it drops to about half of that value. Dynamically the FCN is a free-oscillation mode in the Earth's rotation and hence a decaying harmonic function of time if left unexcited. Thus the time-variable amplitude is a clear evidence that the FCN has been subjected to episodic, if not continual excitations by some geophysical processes. The situation is analogous to the continual geophysical excitation of the Chandler wobble, another free oscillation in the Earth's rotation, resulting in its time-variable amplitude that has been long observed and studied since around 1900. While the excitation of the Chandler wobble can be traced to meteorological origin (Furuya et al., 1996; Gross, 2000), long-term variation of the Chandler wobble amplitude has not been explained. Similarly, there has been no clear evidence as to the excitation source(s) of FCN. In spite of this, the new tested empirical FCN model of high temporal resolution can help to enhance the understanding about the amplitude and phase variations, which in turn is related to the geophysical excitations. In summary, wider window lengths produce smoother amplitude and phase variations and vice versa.

An ultimate pursuit in front of us is to identify the geophysical phenomena that excited the FCN in the way that is observed. One possible candidate is again meteorological disturbances in the form of broad-band variations of atmospheric, oceanic, and hydrological angular momentum. Furthermore, on excitation-domain study, one can confidently presume that some major FCN excitation occurred around 1990. One can further speculate the possibility of certain large and deep seismic events, some episodic jerky disturbances in the core, or even major solar mass-ejection events exerting magnetic torques on the Earth.

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		Std: Ac, As (μas)		WRMS (μas)	
Comparison		**IERS 08 C04**	**USNO Finals**	**IERS 08 C04**	**USNO Finals**
This study	**Malkin**	13.46	12.84	18.76	19.06
	Lambert	25.22	24.87	31.55	31.13
	Krásná	28.43	27.40	28.65	28.71
Malkin	**Lambert**	23.63		34.06	33.91
	Krásná	27.63		31.45	32.18
Lambert	**Krásná**	18.31		20.13	20.09

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Fig. 1 *CPO differences using USNO finals and IERS 08 C04 as a priori ERP values.*

Fig. 2 Upper: Empirical FCN model plus the shift terms X_o and Y_o estimated from a N_L = 2 FCN cycles (periods) and N_D = 6 months. Lower: Residuals of the IAU 2006/2000A precession/nutation model w.r.t. VLBI time series after removing that FCN model. A priori ERP from IERS 08 C04.

	Std: CPO-FCN (μas)		WRMS (μas)	
	IERS 08 C04	**USNO Finals**	**IERS 08 C04**	**USNO Finals**
This study	149.71	146.71	87.01	72.49
Malkin	150.44	147.40	85.79	71.17
Lambert	151.74	148.65	90.95	77.06
Krásná	155.38	152.53	88.28	80.38

Fig. 3 Standard deviation of the residuals after each determined FCN model was removed from the CPO obtained from different a priori ERP. Left: IERS 08 C04. Right: USNO Finals. Units: µas.

Fig. 4 Upper: Rms (left) and error coefficients of the fits (right) of FCN to CPO obtained from IERS 08 C04 as a priori ERP. Lower: Sum of Normalized errors: rms and error of model coefficients (left), and the optimum sliding window size (right) estimated from different a priori ERP and different N_D (plotted with different colors).

Fig. 5 Upper: Coefficients Ac (left) and As (right) of different empirical FCN models, during the interval 1993-2013. Lower: Different FCN series used in this study (see text). Notice that Krásná's model only has available coefficients up to 2009. A priori ERP: USNO finals.

Fig. 6 FCN amplitudes (left) and phase variations (right) estimated from different models. A priori ERP: USNO finals.

Fig. 7 Periodogram realized to the FCN amplitude (left) and phase variations (right) using different models.

Table 1 Standard deviation of the Cosine and Sine coefficients and the wrms difference of the residuals between our FCN model and the other models estimated from different a priori ERP: IERS 08 C04 and USNO Finals.

Table 2 Standard deviation of the residuals CPO-FCN after each FCN model is removed from the CPO. WRMS differences between the remaining residuals once FCN was removed using different models w.r.t. VLBI estimates computed from different a priori ERP including the CPO. A priori ERP from IERS 08 C04 and USNO Finals.

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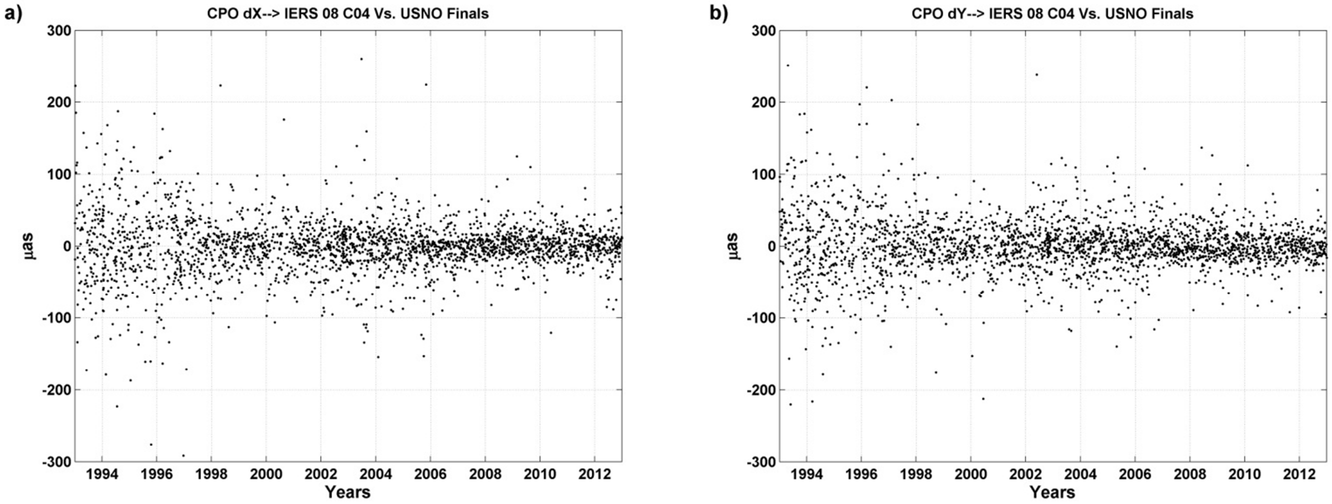

Figure 1: CPO differences using USNO finals and IERS 08 C04 as a priori ERP values.

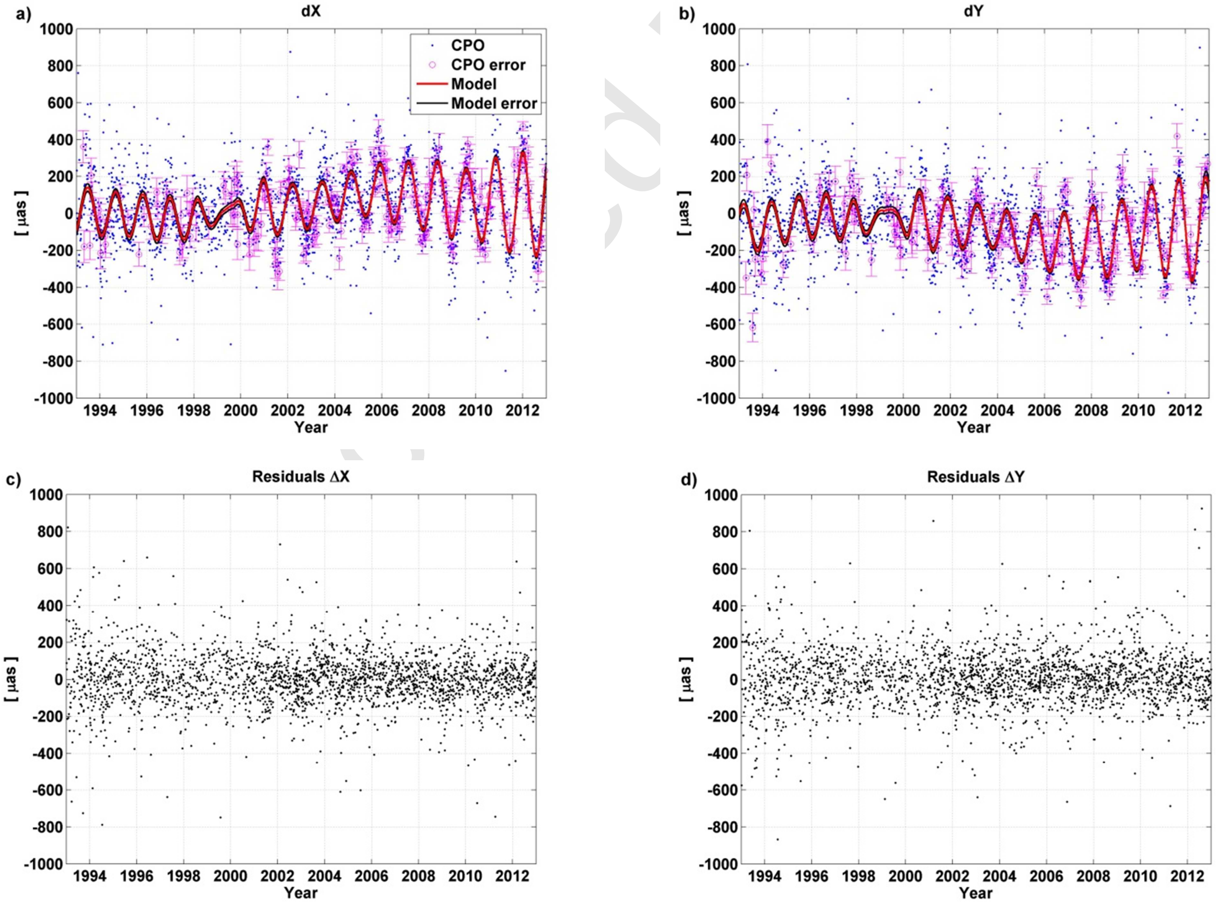

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Figure 2: Upper: Empirical FCN model plus the shift terms X_o and Y_o estimated from a $N_L = 2$ FCN cycles (periods) and $N_D = 6$ months. Lower: Residuals of the IAU 2006/2000A precession/nutation model w.r.t. VLBI time series after removing that FCN model. A priori ERP from IERS 08 C04.

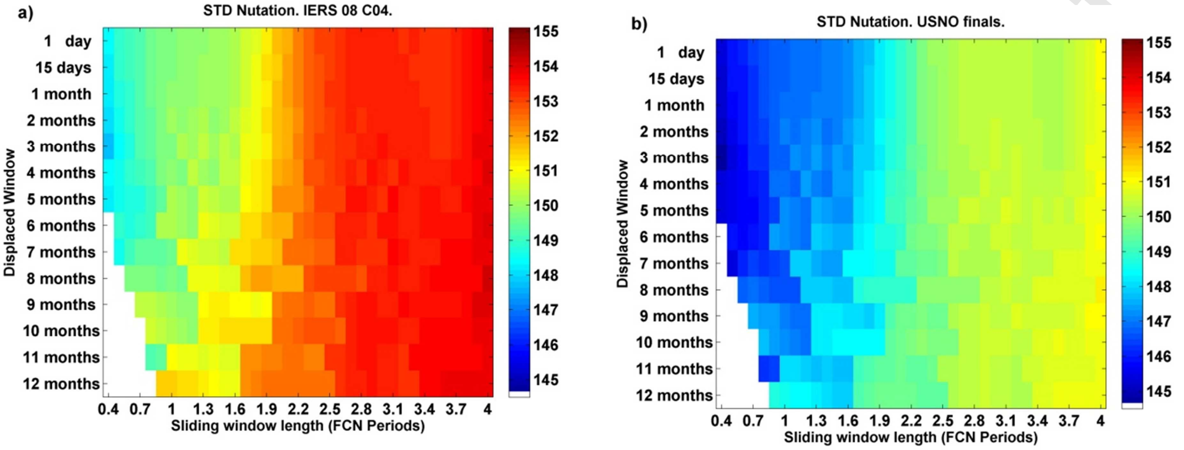

Figure 3. Standard deviation of the residuals after each determined FCN model was removed from the CPO obtained from different a priori ERP. Left: IERS 08 C04. Right: USNO Finals. Units: µas.

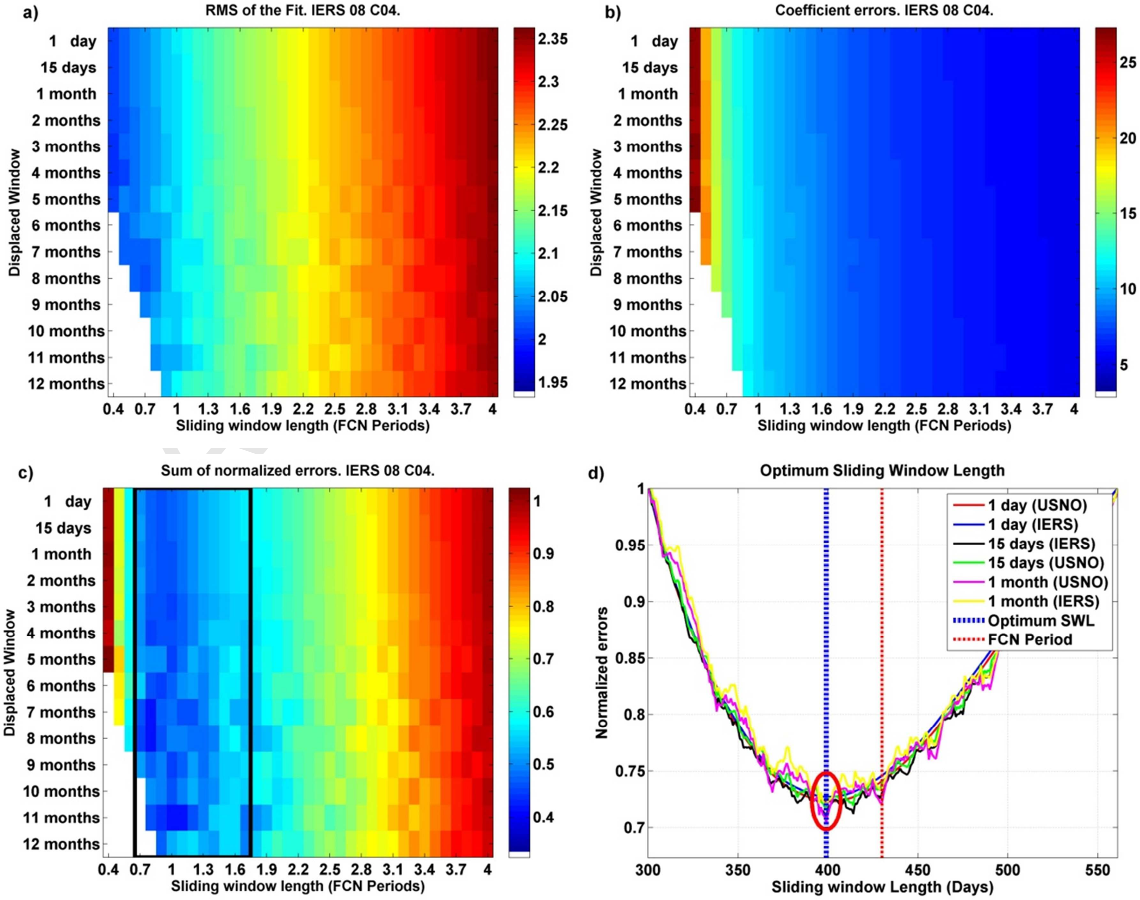

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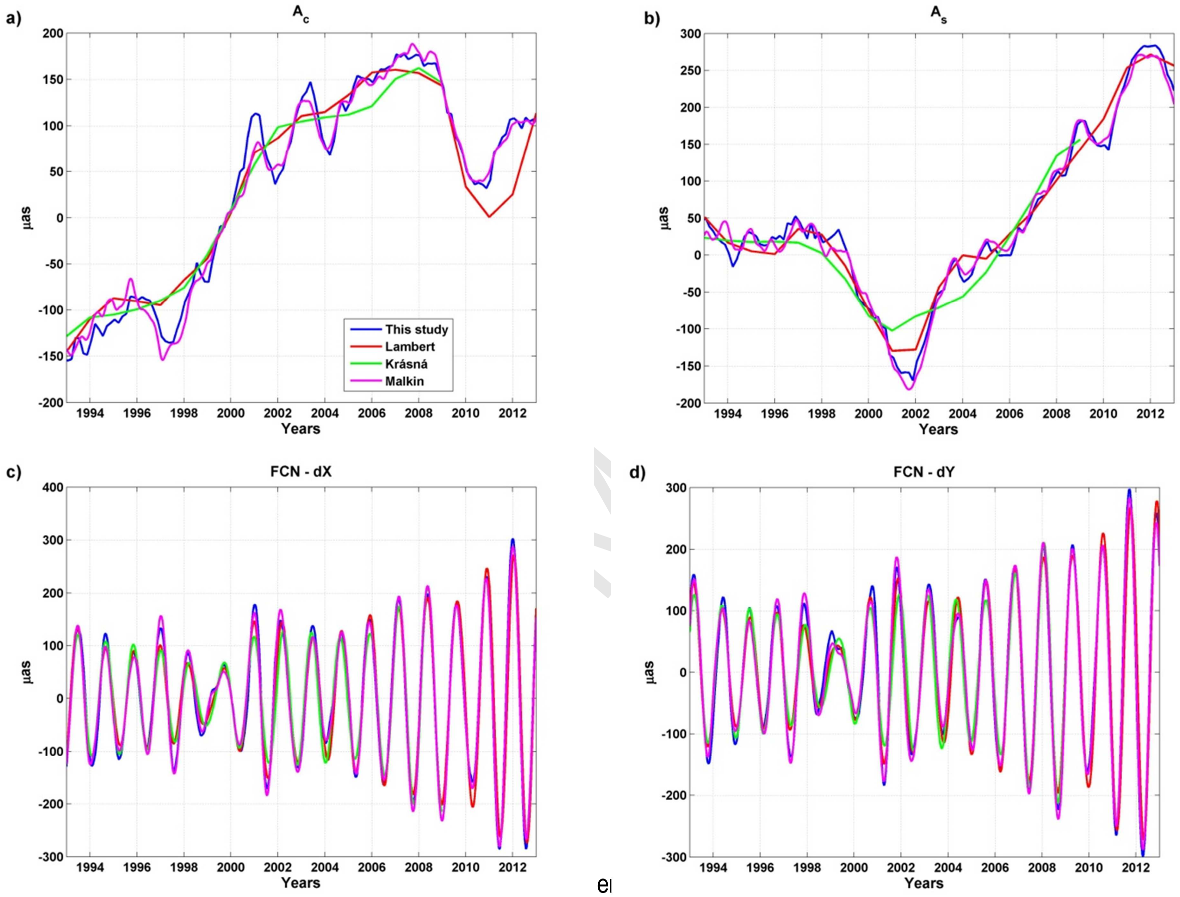
Figure 4. Upper: Rms (left) and error coefficients of the fits (right) of FCN to CPO obtained from IERS 08 C04 as a priori ERP. Lower: Sum of Normalized errors: rms and error of model coefficients (left), and the optimum sliding window size (right) estimated from different a priori ERP and different N_D (plotted with different colors).



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Lower: Different FCN series used in this study (see text). Notice that Krásna's model only has available coefficients up to 2009. A priori ERP: USNO finals.