Paraconsistent Multivalued Logic and Coincidentia Oppositorum: Evaluation with Complex Numbers

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Abstract

Paraconsistent logic admits that the contradiction can be true. Let p be the truth values and P be a proposition. In paraconsistent logic the truth values of contradiction is \( v(P \land \neg P) = p(1 - p) = 1 \Rightarrow p - p^2 - 1 = 0 \)

\( \Rightarrow p^2 - p + 1 = 0 \). This equation has no real roots but admits complex roots \( p = e^{\pm \frac{i\pi}{3}} \). This is the result which leads to develop a multivalued logic to complex truth values. The sum of truth values being isomorphic to the vector of the plane, it is natural to relate the function V to the metric of the vector space \( R^2 \). We will adopt as valuations the norms of vectors. The main objective of this paper is to establish a theory of truth-value evaluation for paraconsistent logics with the goal of using in analyzing ideological, mythical, religious and mystic belief systems.

Keywords: belief systems, circle of truth, coincidentia oppositorum, contradiction, complex number, denier, logic coordinations, paraconsistency, propositions, truth values


1. Introduction

"Philosophy is a collection of big mistakes, but mistakes so seemingly close to an aspect of truth, that they require serious consideration as premises, at least until their consequences and revelations become temporarily exhausted." (Florencio Asenjo, 1985)

“If ... we construe [this statement] as [a statement] about a sun which is actually all it is able to be, then we see clearly that this sun is not at all like the sensible sun. For while the sensible sun is in the East, it is not in any other part of the sky where it is able to be. [Moreover, none of the following statements are true of the sensible sun:] “It is maximal and minimal, alike, so that it is not able to be either greater or lesser”; “It is everywhere and anywhere, so that it is not able to be elsewhere than it is”; “It is all things, so that it is not able to be anything other than it is”—and so on.” (Nicholas of Cusa)

What we need initially is an answer to the question “What is a Contradiction?” That can only be had with a definition of the term. The English term itself derives from the Latin verb contradictio (contradicere), “I speak against” (“to speak against”). But the initial definition of “contradiction” comes to us from Aristotle. In the Greek the term Aristotle used was antiphasis. That term is composed of two Greek words. The term anti is a preposition. In this use, it means “against.” The second term, phasis, comes from the verb phēmi, which means “to say, speak or tell.” It connotes the act of expressing opinion, thought or belief; and, thus, of having an opinion, thought or belief. The term phasis itself means a “saying, speech, sentence, affirmation or assertion.” A fair etymological definition of the term antiphasis, then, is that it means a “saying, speech, sentence, affirmation or assertion against.” So Latin and Greek provide the same basic meaning. But both leave us with the question against what? And we shall answer that in due course. For now, however, we need to look at Aristotle’s own definition of the term.

In the current paradigm of consciousness, duality is perceived be a binary state of mutual exclusion. One sees this notion reflected in human thought and language where something must be “either X or Y”, but not “both X and Y”. A new paradigm of consciousness is required that no longer operates in a “dualistic” notion of “either/or”, but one that conveys a “holistic” notion of “both/and”. We currently view duality as a disjunctive rather than a conjunctive aspect of being. The difference between this dualistic and holistic paradigm of consciousness can be symbolically expressed in the language of logic. Current dualistic paradigm of consciousness \( X \neq Y; X \cup Y \) (something is either X or Y); a disjunctive exclusion. Emerging holistic paradigm of consciousness \( X = Y; X \cap Y \) (something is both X and Y); a conjunctive inclusion. This conceptual paradigm of viewing the world in such an exclusionary and disjunctive dualistic state has been programmed into us by an outdated Cartesian philosophical worldview and a Newtonian scientific view.
of the universe. This modern paradigm of dualistic thought has been prevalent ever since René Descartes proclaimed. That tradition clings to the principle of non-contradiction, but understood as rejection of contradiction hereinafter abbreviated as RC. Well, is paraconsistent any approach that rejects this same RC, ie admit that certain contradictions can be true (not necessarily all, of course). In particular, today is paraconsistent a treatment of problems such as a philosophy of religion that accommodates certain antinomian assertions and in doing so, offered as underlying logic to build theory, not classical logic is a logic of Aristotelian stamp, but one of the denominated precisely paraconsistent logics.

It seems to us that the philosophy of paraconsistency can propose a concept of modern rationality which will enable us to restore and gradually elaborate in never ending self-criticism "the vision of the whole" as a co-evolutionary unity of mankind and Nature. To the basics of this modern rationality would belong of the non-exclusive relation between analytical and dialectical thinking, their developmental unity. The desirable unifying can be conceived of in various ways. It follows from this paper that we are skeptical about the proposal to unify analytical and dialectical thinking through a kind of reduction of the latter to the first by applying the idea of paraconsistency. It would mean to reduce the whole to a part. What we propose is to conceive analytical thinking as a part of and a derivative from something more complex and more fundamental.

The main objective of the authors is to establish a theory of truth-value evaluation for paraconsistent logics, unlike others who are in the literature (Asenjo, 1966; Avron, 2005; Belnap, 1977; Bueno, 1999; Carnielli, Coniglio and Lof D'ottaviano, 2002; Dunn, 1976; Tanaka et al, 2013), with the goal of using that paraconsistent logic in analyzing ideological, mythical, religious and mystic belief systems (Nescolarde-Selva and Usó-Doménech, 2013, Usó-Doménech and Lloret-Climent, 2014; Usó-Doménech and Nescolarde-Selva, 2012).

2. Historical Perspective

Numerous paraconsistent logical calculi have been constructed which allow the formula $P \land \neg P$ to be true (derivable) under some special conditions and thus tolerate $P \land \neg P$ without becoming trivial. To provide some grounds for this theory let us take a look at Aristotle's theory of contrariety from the point of view of modern dialectic. This is detailed account of what Aristotle calls Antiphasis is to be found in Metaphysics Book 4. Aristotle's examples of the four kinds of opposites are: double and half, bad and good, blindness and sight and he sits and he does not sit. Aristotle was deeply interested in investigating the modes of opposition and their ontological relevance in the early, middle and late period of his philosophizing. He ascribed to the opposites an important role in almost all fields of reality, in Nature, in society as well as in thought, but disagreed with that ontological overestimation of the role of opposites, which he found in many preceding Greek thinkers. The second is his misinterpretation of Heraclitus in the sense of Protagoras' relativism hereby not only the sophistic relativism, but also the Heraclitian anticipations of dialectical ontology.

Aristotle is right in insisting that the denial of this principle would lead to a kind of total trivialization of human thinking and people would become prisoners of a helpless tenet "which prevents a thing from being made definite by thought". Now let us compare three following allegedly synonymous formulations. Aristotle took all three as stating the same principle and in different places mutually argues the truth of each of them from the presupposed evidence of each of them.

1) "Contradictory propositions are not true simultaneously". This statement is, as already mentioned, acceptable and respected on the new ontology.

2) "Contradictories cannot be predicated at the same time". This statement would be unacceptable if interpreted in the following way: (in the European tradition translated as "contradictio") is for Aristotle sometimes the conjunction of two sentences (or statements, propositions) of which one affirms what the other denies; sometimes either part of this conjunction; sometimes the negation of any given subject, property, relation, action etc. (e.g. man - not-man, changing - unchanging).

3) "Contraries cannot at the same time belong to the same subject" if taken, as Aristotle did, as a general principle valid for all entities.

These opposites are, for Heraclitus, to be taken in unity, as constituting in their opposition and unity something identical. If sometimes in the dialectical tradition Heraclitus' position was characterized as claiming not only the unity, but even the identity of opposites, never was the Leibnizian identity meant, allowing us to replace one of the identical expressions and/or concepts by the other mutually and thus to remove completely the opposition.

A closely related, but more general, acceptance that figures in most literary manuals defines paradox as an apparent contradiction which, upon examination, actually reveals a hidden, startling truth. It is important to stress, however, that the apparent contradiction of paradox occurs only in the surface meaning of the opposing statements, each of which is found to be true in some sense or to a certain degree. Paradox therefore uses the language of apparent nonsense to express startling «truths» that exceed the bounds of logic and propositional discourse. A third meaning of paradox is technically called «antinomy»: an insoluble contradiction in which asserting the truth of a particular proposition necessarily entails asserting that proposition's falsity (Quine, 1966). The prototypical antinomy is the Paradox of the Liar, which asserts, in effect: "This statement is false". Clearly, if that statement if false, it is also true, in the same sense and to the same degree. For our purposes here, it is important to bear in mind that what renders antinomies insoluble is their wholly internal reference and their self-contained quality. Beyond their utterly fixed terms, there remains no logical or semantic space, and no other level of abstraction, which permit an assertion of even partial truth or falsity. One of the chief sources of paradox literature in the West is Plato's Parmenides. Through a barrage of paradoxical utterances, the dialogue not only treats of such eminently philosophical questions as “unity and diversity”, “likeness and unlikeness” and “being and non-being”. It also provides a model of Plato's rhetorical art, including a practical model for the training of novice orators.
Parmenides says to the young Socrates: There is an art which is called by the vulgar «idle talking», and which is often imagined to be useless; in that art you must train yourself, now that you are young, or truth will elude your grasp (Plato, 1973). Parmenides goes on to demonstrate that, simply put; this art consists in arguing opposite sides of a question. “Truth” is thus shown to lie, not so much between, as beyond extremes, each of which is in some way deficient, at once partially true and partially false. Further, truth is also shown to prove elusive and paradoxical, as set forth in the dialogue’s startling “conclusion” about what the truth seems to be: [Parmenides]: Let this much be said; and further let us affirm what seems to be the truth, that, whether [the] one is or is not, [the] one and the others [pluralitY] in relation to themselves and one another, all of them, in every way, are and are not, appear to be and appear not to be. [Socrates]: Most true. (Plato, 1973).

In his Of Learned Ignorance (De docta ignorantia), written in 1440, Nicholas of Cusa adopted an equally paradoxical approach to questions of truth, albeit within a Christian intellectual framework. A clear echo of Socrates’ knowing only that he knows nothing, and the Pauline distinction between worldly and godly wisdom, the title of Cusa’s first chapter reads: “How Knowledge is Ignorance” (Cusanus, 1986). Yet such ignorance becomes increasingly «learned», hence increasingly unknowing, through reflective contemplation of the created order, which unfolds in time as a perplexing admixture of unity and plurality, likeness and unlikeness, being and non-being—a coincidentia oppositorum, or an alternately conflicting and harmonious blend of contraries. A source of paradox literature is Erasmus’ The Praise of Folly. Modeled after the classical type of “paradoxical encomium”, Erasmus’ work seems often to conclude that many species of “folly” are wisdom, and that folly is at once good and bad, laudable and despicable. Yet the viciously circular text resists anything akin to univocal interpretation, owing primarily to its status as an exaltation of folly pronounced by Folly herself. Indeed, the character’s declamation presents an extreme case of self-reference, self-praise, self-love (philautia) and a lack of self-knowledge, which casts even its seemingly «truthful» utterances in doubt.

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The paradoxical nature of the coincidentia oppositorum receives primary attention in much contemporary exegetical literature in the West. It is often glossed by interpreters as an intentional confound to conventional logic, which is based on the law of the excluded middle, an
theory of objects (Peña, 1985) will admit as non illogical the principle of characterization attributable to Meinong:

**Principle of Characterization:** *An entity who comes presented as being in one way or another, has effectively a determination to be in one form or another; but with the precision of which does not necessarily follow, that an entity that has the property to be in one form or another, that entity is in one form or another*

When this principle is complied is produced named rule of characterization. If we have a system with the principle but not the rule of characterization, the system wills a weak Meinongian system. Instead a system with the principle and with a restricted version of the rule will be a strong Meinongian system. What cannot be is a system that having both and unrestricted both the full version of the principle as that of the rule; this system would be deliquescent, in it each sentence (syntactically well formed) would be both a thesis (asserted). But it is possible to have a treatment with both, rule and principle in its full and unrestricted versions. But it does not follow that do not fit but both treatments containing restricted or qualified versions. E.g., be a version according to which, if having the property A implies have the property B, then each entity will have one another. This may seem a simple tautology, but it is not demonstrable in the systems of first-order logic. Moreover, the treatment Meinongian weakly we are surmising could contain some much nuanced version of the rule of characterization, a version that would prevent the emergence of contradictions. Within a treatment so, arguably, if God is instituted or presented as having such and such characteristics, then He has effectively such features; but there will not continue necessarily having the features in question (although it may follow that in many cases).

Then the sentence \( P = "X \text{ has the property of being in one form or another and has the property of not being in one form or another}" \) is no real contradiction in the classical sense and does not move in terms of truth and falsity absolute. That phrase appears only real contradiction if we add the rule of characterization. This last rule makes adopt a Meinongian weak approach.

Some of the adherents of this trend in contemporary logic investigate explicitly also its philosophical presuppositions and implications (Bueno, 2010; Carnielli and Marcos, 2001). Among other problems, the question of the relationship between the idea of paraconsistency and the traditional and/or contemporary forms of dialectical thinking is being examined. It seems to us that in the philosophy of paraconsistency a differentiation can be observed today. One of the tendencies, represented by Arruda (Arruda et al, 1980), da Costa (da Costa and Wolf, 1980), Quesada (1989) while assessing highly important philosophical implications of the logic of paraconsistency, insists upon the view that paraconsistency is closely linked with the theory of logical calculi. The philosophizing logicians of this tendency give, as a rule, only modest hypothetical accounts of the relationship between paraconsistency and dialectic. The other tendency, represented by G. Priest (Priest, 1989, 1995, 1998; Priest, Routley, and Norman, 1989), dares to defend vehemently more radical and ambitious assumptions about the philosophical and scientific implications of paraconsistent logic, concerning not only the relation to dialectic, but also the conception of rationality in general. Let us have a
closer critical look at some main claims of the philosophy of paraconsistency from a special point of view, namely, from the point of view of secular (ontopraxeological) dialectic which aims at elaborating a theory of modern rationality taking inspiration from Hegel's critique of Kant and Marx's critique of Hegel. Needless to say, no simple reception of any philosophy of the past is able to cope with our contemporary problems of rationality. References to Kant and Hegel remain mostly mere decoration. Priest's use of the calculus-oriented notion of inconsistency in his interpretation of the so-called Kant/Hegel thesis about the inherently inconsistent nature of human reason seems to us to be misleading. Supposing we accept Kant's argumentation in his "Transcendental Dialectics" as a justification of the statement that our thinking is in its very nature (apparently, but necessarily) inconsistent: then Hegel's critique of Kant's antinomies should be taken as an attempt at a new consistency which corrects the antinomic dialectic of (apparently, but necessary) inconsistency of human reason in a section of its usage. The dictum of a unitary "Kant/Hegel thesis" hides this difference.

Following Priest, we will say that a logical system is paraconsistent, if and only if its relation of logical consequence is not "explosive", i.e., iff it is not the case that for every formula P and Q, P and not-P entails Q; and we will say a system is dialectical, iff it is paraconsistent and yields (or "endorses") true contradictions, named "dialetheias". A paraconsistent system enables to model theories which in spite of being (classically) inconsistent are not trivial, while a dialectical system goes further, since it permits dialetheias, namely contradictions as true propositions. Still following Priest, semantics of dialectical systems provide truth-value gluts (its worlds or set-ups are overdetermined); however, truth-value gaps (opened by worlds or set-ups which are underdetermined) are considered by Priest to be irrelevant or even improper for dialectical systems. Besides that, sometimes the distinction is drawn between weak and strong paraconsistency, the latter considered as equivalent with dialectics. A reader of recent literature in this field may have an impression that dialectics as strong paraconsistency is more a question of ontology than of logic itself, namely that it states the existence of "inconsistent facts" (in our actual world) which should verify dialetheias. One more introductory remark has to be put here: in recent literature of paraconsistency there are no quite unanimous, among paraconsistent logicians generally accepted distinction between paraconsistent and dialectical logical systems. But it remains an open question whether; semantically paradoxes express any "inconsistent facts".

2.1. Coincidentia oppositorum in Mysticism: The Early Kabbalah Case

The Kabbalists use the term, achdut hashvaah, to denote that Ein-sof, the Infinite God, is a “unity of opposites,” (Figure 1).

Or as Scholem (1974, p. 88) at one point translates the term a “complete indistinguishability of opposites,” one that reconciles within itself even those aspects of the cosmos that are opposed to or contradict one another. Sefer Yetzirah, an early (3rd to 6th century) work which was of singular significance for the later development of Jewish mysticism, had said of the Sefirot (the ten archetypal values through which divinity is said to constitute the world) “their end is imbedded in their beginning and their beginning in their end.” (Kaplan, 1997, p. 57). According to Yetzirah, the Sefirot are comprised of five pairs of opposites: “A depth of beginning, a depth of end. A depth of good, a depth of evil. A depth of above, a depth of below. A depth of east, a depth of west. A depth of north, a depth of south. The 13th century Kabbalist Azriel of Gerona was perhaps the first Kabbalist to clearly articulate the doctrine of coincidentia oppositorum. For Azriel “Ein Sof …is absolutely undifferentiated in a complete and changeless unity…He is the essence of all that is concealed and revealed.” (Azriel. 1966). According to Azriel, Ein-sof unifies within itself being and nothingness, “for the Being is in the Nought after the manner of the Nought, and the Nought is in the Being after the manner [according to the modality] of the Being... the Nought is the Being and Being is the Nought. (Scholem, 1987, p. 423). For Azriel, Ein-sof is also "the principle in which everything hidden and visible meet, and as such it is the common root of both faith and unbelief." (Scholem, 1987, pp. 441-442). Azriel further held that the very essence of the Sefirot, the value archetypes through with Ein-sof is manifest in a finite world, involves the union of opposites, and that this unity provides the energy for the cosmos. The nature of sefirot is the synthesis of everything and its opposite. For if they did not possess the power of synthesis, there would be no energy in anything. For that which is light is not dark and that which is darkness is not-light. Further, the coincidence of opposites is also a property of the human psyche; “we should liken their (the Sefirot) nature to the will of the soul, for it is the synthesis of all the desires and thoughts stemming from it. Even though they may be multifarious, their source is one, either in thesis or antithesis.” (Azriel. 1966, p. 94). Azriel was not the only Kabbalist to put forth a principle of coincidentia oppositorum. The early Kabbalistic Source of Wisdom describes how God's name and being is comprised of thirteen pairs of opposites (derived from the 13 traits of God enumerated in Chronicles), and speaks of a Primordial Ether (Avir Kadmon), as the medium within which such oppositions are formed and ultimately united.
The concept of achdut hashvaah figures prominently in the Lurianic Kabbalah, which became the dominant theosophical and theological force in later Jewish mysticism and Chasidism. Isaac Luria (1534-72) wrote very little, but his chief expositor, Chayyim Vital (1543-60) records: Know that before the emanation of the emanated and the creation of all that was created, the simple Upper Light filled all of reality...but everything was one simple light, equal in one hashvaah, which is called the Light of the Infinite. (Zohar III, 113a. Vol. 5, p. 153). While Vital’s account suggests a unity of opposites in the godhead only prior to creation, a close examination of the Lurianic Kabbalah reveals a series of symbols that are applicable to God, the world and humanity, and which overcome the polar oppositions of ordinary (and traditional metaphysical) thought. Indeed, each of the major Lurianic symbols expresses a coincidence of opposites between ideas that are thought to contradict one another in ordinary thought and discourse. For example, Luria held that the divine principle of the cosmos is both Ein-sof (without end) and Ayin (absolute nothingness), that creation is both a hitpashut (emanation) and a Tzimtzum (concealment), that Ein-sof is both the creator of the world and is itself created and completed through Tikkan ha-Olam, the spiritual, ethical and “world restoring” acts of humanity, and, finally, that the Sefirot are both the originating elements of the cosmos and only fully realized when the cosmos is displaced and shattered (via Shevirat ha-Kelim, the “Breaking of the Vessels”).

A closer examination of two key elements in the Lurianic system, Tzimtzum (concealment/contraction) and Shevirat ha-Kelim (the “Breaking of the Vessels”) can provide further insights into the Lurianic conception of the coincidence of opposites.

In the symbol of Tzimtzum (the withdrawal, concealment and contraction of the infinite that gives rise to the world) there is a coincidence of opposites between the positive acts of creation and revelation and the negative acts of concealment, contraction and withdrawal. For Luria, God does not create the world through a forging or emanation of a new, finite, substance, but rather through a contraction or concealment of the one infinite substance, which prior to such contraction is both “Nothing” and “All.” Like a photographic slide, which reveals the details of its subject by selectively filtering and thus concealing aspects of the projector’s pure white light (which is both “nothing” and “everything”), Ein-sof reveals the detailed structure of the finite world through a selective concealment of its own infinite luminescence. By concealing its absolute unity Ein-sof gives rise to a finite and highly differentiated world. Thus in the symbol of Tzimtzum there is a coincidence of opposites between addition and subtraction, creation and negation, concealment and revelation. In order to comprehend the notion of Tzimtzum, one simultaneously think two thoughts, for example, one thought pertaining to divine concealment and a second pertaining to (this concealment as) creation and revelation.

For Luria, the further realization of Ein-sof is dependent upon a second coincidence of opposites; between creation and destruction, symbolized in the Shevirat ha-Kelim, the “Breaking of the Vessels,” Ein-sof is only fully actualized as itself, when the ten value archetypes which constitute the Sefirot are shattered and are subsequently restored by humankind (Tikkun ha-Olam). While Ein-sof is the source and “creator” (Zohar III, 113a. Vol. 5, p. 153), of all, Ein-sof paradoxically only becomes itself, through a rupture which results in a broken and alienated world in need of humanity’s “restoration” and repair (Tikkun). For Luria, Ein-sof is propelled along its path from “nothing” (Ayin) to “something” (Yesh), through the creative and restorative acts of humankind; for it is only humanity acting in a broken and displaced world, that can undertake the mitzvot, the creative, intellectual, spiritual and ethical acts that fully actualize the values and traits that exist only potentially within God. It is for this reason that the Zohar proclaims “He who ‘keeps’ the precepts of the Law and ‘walks’ in God’s ways... makes’ Him who is above.” (Zohar III, 113a. Vol. 5, p. 153). Thus, just as humanity is dependent for its existence upon Ein-sof, Ein-sof is dependent for its actual being upon humanity. The symbols of Ein-sof, Shevirah (rupture) and Tikkun (Repair) thus express a coincidence of opposites between the presumably opposing views that God is the creator and foundation of humanity and humanity is the creator and foundation of God.

3. Contradiction and Deniers

Let F be a non-countable set whose algebraic structure is at least that of a semi-ring. F is a semi-ring, a ring or a group.

We lay the following definitions and fundamental axioms:

**Axiom 1:** Any proposition P has a truth value p, element of a set E which is a part, not countable and stable for multiplication of the set F.

**Axiom 2:** Any proposition P is endowed with a valuation $v \in [0,1]$ such that $v = V(p)$, $V$ reciprocal application of E on $[0,1]$ subject to the following conditions:

1) $V^{-1}(0) = 0$.
2) $V(p_1, p_2) = V(p_1)V(p_2)$ being $p_1$ and $p_2$ two truth values.

**Axiom 3:** Truth value $p^*$ denotes the negation or contradiction of P denoted as $\neg P$ and $V(p + p^*) = 1$.

Let $P_i$ be n propositions, $i = 1, 2, ..., n$ of $p_i$ and $p_i^*$ be the truth values of their contradictories. Then:

**Definition 1:** A compound proposition (or logical coordination or logical expression) of order n is a proposition whose truth value $c$ is a function $f_n$ of $p_i$ and $p_i^*$:

$$c = f_n(p_1, p_1^*, p_2, p_2^*...p_n, p_n^*)$$

$f_n$ values in F; it determines a truth value if $c \in E$. The condition of existence of a compound proposition defined by $f_n$ is $c \in E$, or what is equivalent, $V(c) \in [0,1]$. 

**Axiom 4:** $f_n$ is a polynomial in which each index 1, 2, ..., $n$ must be at least once and that all coefficients are equal to unity.

**Definition 2:** Let $p + p^* = u$ be. $u$ is a denier of the proposition $P$ if the following three conditions are fulfilled (Usó-Domènech, Nescolarde-Selva and Pérez-Gonzaga, 2014):

a). $u \in E$

b). $V(u) = 1$; $u$ unitary truth value (from axiom 3)

c). $u - p = p^* \in E$ (from axiom 1)

Paraconsistent logic admits that the contradiction can be true. Then $v(P \land \neg P) = p(1 - p) = 1 \Rightarrow p - p^2 - 1 = 0 \Rightarrow p^2 - p + 1 = 0$. This equation has no real roots but admits complex roots $p = e^{\pm \frac{\pi}{3}}$. This is the result which leads to develop a multivalued logic to complex truth values. The sum of truth values being isomorphic to the vector of the plane, it is natural to relate the function $V$ to the metric of the vector space $\mathbb{R}^2$. We will adopt as valuations the norms of vectors. $E$ is the set of complex numbers of modulus less or equal to 1 and the function $V$ is such that $V(p) = |p|^2$ and it has satisfied axiom 2.

Let $P$ be a proposition of fixed truth value $p = |p|e^{i\alpha}$. If $p = 0$, $\alpha$ is indeterminate, we agree to take $\alpha = 0$. According to definition 2, here a denier is a unitary complex number $u = e^{i\omega}$ such that $|u - p| \leq 1$. Putting $\varphi = \theta - \alpha$ (Figure 2) this inequality entails $\cos \varphi \geq \frac{|p|}{2}$, be $|\varphi| \leq \theta$, $\theta = \arccos \frac{|p|}{2}$.

![Figure 2. Circle of truth](image)

In summary

$$\varphi \in [-\theta, \theta]; \quad \cos \theta = \frac{|p|}{2}, \quad \theta \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right]$$

(1)

Deniers $u$ of $P$ form a continuous set: the sector of the circle of truth (trigonometric circle) of angle $2\theta$ whose vector $p$ is collinear to the bisector (Figure 3).

![Figure 3. Location of U: arc NPN; NN' mediatrix of OA](image)

A denier is determined by the angle $\vartheta$. $u(\varphi)$ is a bijective function.

Contradictory proposition $\neg P$ provides, fixed $p$, a continuous set of truth values $p^* = |p^*|e^{i\alpha^*}$ and then:

$$|p^*| \in [1 - |p|, 1]; \varphi = 0 \Leftrightarrow |p^*| = 1 - |p|;$$

(2)

Putting $\omega = \alpha^* = \varphi^*, \varphi = -\theta \Rightarrow \varphi^* = \pi - 2\theta; \varphi = 0 \Rightarrow \varphi^* = 0 \Rightarrow \varphi^* = 2\theta - \pi; \text{ in summary,}$

$$\varphi^* \in [2\theta - \pi, \pi - 2\theta] \Rightarrow \varphi - \varphi^* \in [\theta - \pi, \pi - \theta]$$

(3)

On the other hand we have $\alpha^* - \alpha = \varphi - \varphi^*$.

The truth contradiction $v(P \land \neg P) = V[pp^*] = 1$ requires $|p| = 1$ where $\theta = \pm \frac{\pi}{3}$ and also $|p^*| = 1$ where $\varphi = \pm \frac{\pi}{3}$ and $\varphi^* = \pm \frac{2\pi}{3}, \varphi - \varphi^* = \pm \frac{2\pi}{3} = \alpha^* - \alpha$.

Solutions of $v(P \land \neg P) = 1$ are finally (Figure 4):

$$p = e^{i\alpha}, \alpha \text{ anyone}; \ p^* = e^{i\left(\frac{\alpha \pm \frac{2\pi}{3}}{3}\right)}.$$

![Figure 4.](image)

Then, multivalued logic with complex truth values is paraconsistent.

Another characteristic of paraconsistent logic is that the negation of the negation does not necessarily leads back to the original proposition as Hegel said. If $u^* \neq u, v(\neg \neg P) = V(u^* - p^*) \neq V(p); v(\neg \neg P) \neq v(P)$. 

4. Conditions

4.1. Condition 1

It is written \[ |u_1u_2 - p_1p_2| \leq 1 \] with the above notation:

\[ e^{i(\theta_1 + \phi_2)} - |p_1||p_2| \leq 1 \]

Condition 1 back to

\[ \cos(\phi_1 + \phi_2) \geq \frac{|p_1||p_2|}{2} \]

There is a continuous set of deniers \( u_1(\phi_1) \) and other deniers \( u_2(\phi_2) \) that that satisfy (eg \( \phi_1 + \phi_2 = 0 \)). As well the \( \neg P_1 \lor \neg P_2 \) incompatibility exists, provides \( p_1 \) and \( p_2 \) fixed, on a continuous set of truth values \( u_1u_2 - p_1p_2 \) and we have:

\[ v(-P_1 \lor \neg P_2) = v(- (P_1 \land P_2)) \]

Similarly \( P_1 \lor P_2 \) does exist, provided that:

\[ \cos(\phi_1^* + \phi_2^*) \geq \frac{|p_1||p_2|}{2} \]

satisfied, for example if \( \phi_1 + \phi_2 = 0 \).

Similarly the implication \( P_1 \Rightarrow P_2 \) on condition that:

\[ \cos(\phi_1^* + \phi_2^*) \geq \frac{|p_1||p_2|}{2} \]

4.2. Condition 2

Posing \( \alpha_1 - \alpha_2 = \alpha \), that \( |p_1 + p_2| = |p_1 e^{i\alpha} + p_2|^2 \) it result that \( \forall p_1 \neq 0, \forall p_2 \neq 0, |\alpha| \geq \frac{2\pi}{3} \) we have:

\[ |p_1 + p_2| > 1 \Rightarrow |\alpha| < \frac{2\pi}{3} \]

If \( |p_1 + p_2| > 1 \), which requires non-zero \( p_1 \) and \( p_2 \), complementarity \( P_1 \land P_2 \) does not exist and \( \neg P_1 \land \neg P_2 \) must exist.

**Theorem 1**: Can be found deniers \( u_1 \) and \( u_2 \), such that

\[ |p_1^* + p_2| \leq 1 \]

**Proof**

Just for this inequation is satisfied that \( |\alpha_1^* + \alpha_2^*| \geq \frac{2\pi}{3} \).

After (4) \( \alpha_1^* - \alpha_2^* = (\phi_1 - \phi_1^*) - (\phi_2 - \phi_2^*) + \alpha \).

Let \( 0 \leq \alpha = \frac{2\pi}{3} - \beta \) be. After (3) the maximum value of \( \phi_1 - \phi_1^* \) is \( \pi - \theta_1 > 0 \) and the one of \(- (\phi_2 - \phi_2^*) \) is \( \pi - \theta_2 > 0 \) and therefore

\[ \sup |\alpha_1^* - \alpha_2^*| = 2\pi - (\theta_1 + \theta_2) + \frac{2\pi}{3} - \beta > \frac{5\pi}{3} - \beta \]

because \( \theta_1 + \theta_2 < \pi \).

The result is \( \sup |\alpha_1^* - \alpha_2^*| \geq \pi \) since \( 0 \leq \beta \leq \frac{2\pi}{3} \); or sufficient condition is \( |\alpha_1^* - \alpha_2^*| \geq \frac{2\pi}{3} \).

There is a continuous set of values of \( |\alpha_1^* - \alpha_2^*| \) and therefore of deniers \( u_1(\phi_1) \) and \( u_2(\phi_2) \) which satisfy this condition.

4.3. Condition 3

It is written \( |p_1p_2 + p_1^*p_2^*| \leq 1 \). As condition 2, it is sufficient for it is fulfilled that the angle of non-zero vectors \( p_1p_2 \) and \( p_1^*p_2^* \) is \( \geq \frac{2\pi}{3} \) or else after (4) that:

\[ |\phi_1^* - \phi_1 + \phi_2^* - \phi_2| \geq \frac{2\pi}{3} \]

or in according (3)

\[ \sup |\phi_1^* - \phi_1 + \phi_2^* - \phi_2| = 2\pi - (\theta_1 + \theta_2) > \pi \]

4.4. Condition 4

It is written \( |p_1p_2 + p_1^*p_2^*| \leq 1 \). Studied by the same method it proves to be satisfied if (sufficient condition)

\[ |\phi_1 - \phi_1^* + \phi_2 - \phi_2^*| \geq \frac{2\pi}{3} \]

It is an inequation whose solution is the same of (5). The result is that concordance \( P_1 \Xi P_2 \) and discordance \( P_1 \chi P_2 \) can exist together; then one is the negation of the other by denier \( u_1u_2 \).

5. Propositional Paraconsistent Algebra

Propositional algebra can be built on the set of complex truth values. The main normal binary propositions are the following:

1. **Conjunction**:

\[ v(P_1 \land P_2) = |p_1p_2|^2 = |p_1|^2 |p_2|^2 \]  \hspace{1cm} (8)

2. **Incompatibility**:

\[ v(-P_1 \lor \neg P_2) = |u_1u_2 - p_1p_2|^2 = |p_1||p_2|^2 \]  \hspace{1cm} (9)

\[ v(-P_1 \lor \neg P_2) = v(- (P_1 \land P_2)), \text{ denier } u_1u_2 \]

3. **Disjunction**:

\[ v(P_1 \lor P_2) = |u_1u_2 + p_1u_2 - p_1p_2|^2 \]

\[ = |p_1|e^{i\phi_2} + |p_2|e^{i\phi_1} - |p_1||p_2|^2 \]  \hspace{1cm} (10)

\[ v(P_1 \lor P_2) = v(-(-P_1 \land -P_2)), \text{ denier } u_1u_2 \]
4. Implication:
\[
\begin{align*}
\nu(P \Rightarrow P_2) &= |u_1u_2 - p_1(u_2 - p_2)|^2 \\
&= \left| e^{i(\theta_1 + \theta_2)} - |p_1|\left| e^{i\phi_2} - |p_2|\right|^2 \right|^2 \\
\nu(P_1 \lor P_2) &= \nu(\neg(P_1 \land -P_2)), \text{ denier } u_1u_2
\end{align*}
\]
(11)

5. Concordance:
\[
\begin{align*}
\nu(P_1 \land P_2) &= |u_1u_2 - u_2p_1 - u_1p_2 + 2p_1p_2|^2 \\
&= \left| e^{i(\theta_1 + \theta_2)} + |p_1|e^{i\phi_2} - |p_2|e^{i\phi_1} + 2|p_1||p_2|^2 \right|^2 \\
\nu(P_1 \land P_2) &= \nu(\neg(P_1 \Rightarrow P_2)), \text{ denier } u_1u_2
\end{align*}
\]
(12)

6. Discordance:
\[
\begin{align*}
\nu(P_1 \lor P_2) &= |u_1u_2 + u_1p_2 - 2p_1p_2|^2 \\
&= \left| e^{i(\theta_1 + \theta_2)} - |p_1|e^{i\phi_2} - |p_2|e^{i\phi_1} + 2|p_1||p_2|^2 \right|^2 \\
\nu(P_1 \lor P_2) &= \nu(\neg(P_1 \equiv P_2)), \text{ denier } u_1u_2
\end{align*}
\]
(13)

7. Complementarity:
\[
\nu(P_1 \land P_2) = |p_1 + p_2|^2 = \left| \left| p_1 \right| e^{i(\alpha_1 - \alpha_2)} + \left| p_2 \right|^2 \right|^2
\]
(14)

8. Inverse complementarity:
\[
\nu(\neg(P_1 \land P_2)) = \left| p_1 + u_2 - p_1 - p_2 \right|^2
\]
\[
= \left| e^{i\phi_1} - |p_1|e^{i(\alpha_1 - \alpha_2)} + e^{i\phi_2} - |p_2| \right|^2
\]
(15)

9. Equivalence:
\[
\nu(P_1 \land P_2) = |p_1 + u_2 - p_2|^2 = \left| \left| p_1 \right| e^{i(\alpha_1 - \alpha)} + \left| p_2 \right|^2 \right|^2
\]
(16)

Here intervenes the angle $\alpha_1 - \alpha_2$ of vectors $p_1, p_2$.

Seek what deniers should be chosen so that if $\nu(P_1) = \nu(P_2)$, that is to say, if $|p_1| = |p_2| = |p|, \alpha_1 \neq \alpha_2$, we have: $\nu(P_2 \land P_1) = 1 = \nu(P_1 \lor P_2)$.

We then: $\nu(P_2 \land P_1) = \left| p \left( e^{i\alpha} - 1 \right) + e^{i\phi_2} \right|^2$ where $\alpha = \alpha_1 - \alpha_2$. So that $\nu(P_2 \land P_1) = 1$, the necessary and sufficient condition is:
\[
\sin \left( \frac{\alpha}{2} - \phi_2 \right) = |p| \sin \frac{\alpha}{2}
\]
(17)

Similarly, for $\nu(P_1 \land P_2) = 1$ the necessary and sufficient condition is:
\[
\sin \left( \frac{\alpha}{2} + \phi_1 \right) = |p| \sin \frac{\alpha}{2}
\]
(18)

of which $\phi_1 = \phi_2, \phi_2$ solution of (17).

Figure 5 shows the geometric representation.

5.1. Normal Propositions of Order $n$

1. Conjunction:
\[
\nu(P_1 \land P_2 \land \ldots \land P_n) = \left| p_1p_2 \ldots p_n \right|^2
\]
(19)

2. Incompatibility:
\[
\nu(-P_1 \lor -P_2 \lor \ldots \lor -P_n) = \left| p_1u_2 \ldots p_n - p_1p_2 \ldots p_n \right|^2
\]
(20)

3. Disjunction:
\[
\nu(P_1 \lor P_2 \lor \ldots \lor P_n) = \left| p_1 + p_2 \ldots p_n \right|^2
\]
(21)

4. Complementarity:
\[
\nu(P_1 \land P_2 \land P_3 \land \ldots \land P_n) = \left| p_1 + p_2 + \ldots + p_n \right|^2
\]
(22)

5. Inverse complementarity:
\[
\nu(-P_1 \lor P_2 \lor \ldots \lor -P_n) = \left| u_1 + u_2 + \ldots + u_n \right|^2
\]
(23)

6. Paraconsistent Boolean Logic

It is the Boolean reduction of strong paraconsistent logic; modules of complex truth values there can be only 0 or 1. The circle of truth is there reduced to its center and its circumference. Although Boolean, this logic differs radically from the classical logic: it remains paraconsistent. The contradiction can be true there. We may have verified all the normal binary propositions that the propositional algebra of the paraconsistent Boolean logic contains well beyond the classical logic as a special case.

Since $\nu(P) = 0 \Rightarrow \nu(-P) = 1$, $p^*$ indeterminate and $\nu(P \land -P) = 0$.

Since $\nu(P) = 1 \Rightarrow \theta = \pm \frac{\pi}{3}$. Since $\left| p^* \right|$ must be Boolean $\phi$ can only take two values:
\[
\phi = 0 \Leftrightarrow \nu(-P) = 0 \Rightarrow \nu(P \land -P) = 0
\]
\[
\phi = \pm \frac{\pi}{3} \Leftrightarrow \nu(-P) = 1 \Rightarrow \nu(P \land -P) = 1
\]
(24)
It has always: \( \phi^* = -\phi \Rightarrow \alpha^* - \alpha = 2\phi \).

1. **Conjunction:** The truth table of the conjunction is identical to that of classical logic.

2. **Disjunction:** From (10), it is false if \( v(P_1) = v(P_2) = 0 \) and true if \( v(P_1) = 0 \) and \( v(P_2) = 1 \) or if \( v(P_1) = 1 \) and \( v(P_2) = 0 \).

   Condition 1 is written \( \cos \left( \phi_1^* + \phi_2^* \right) \geq \frac{1}{2} \) \( \phi^* = -\phi \) thus the disjunction exists only if \( \phi_1 + \phi_2 = -\pi, 0, \pi \); we have:

   \[
   v(P_1 \lor P_2) = \begin{cases} \\
   v(P_1 \lor P_2) = 0 \text{ if } \phi_1 = \pm \frac{\pi}{3}, \phi_2 = \pm \frac{\pi}{3} \\
   v(P_1 \lor P_2) = 1 \text{ if } \phi_1 + \phi_2 = \pm \frac{\pi}{3} \\
   \text{or } \phi_1 = \phi_2 = 0 
   \end{cases}
   \]

   Hence the truth table of disjunction (Table 1):

   **Table 1. Truth table of disjunction \( P_1 \lor P_2 \):**

<table>
<thead>
<tr>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_1 \lor P_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0/0*</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

   Not conform evaluation to the classical logic is indicated by *.

   It coincides with that of classical logic, in case \( v(P_1) = v(P_2) = 1 \), is chosen \( \phi_1 \) and \( \phi_2 \) such that \( \phi_1 \phi_2 = 0 \).

3. **Implication:** From (11) is true if \( \forall \phi_2, v(P_1) = 0 \). If \( v(P_1) = v(P_2) = 1 \) the condition of existence is

   \( \cos (\phi_1 - \phi_2) \geq \frac{1}{2} \); such as \( v(P_1 \Rightarrow P_2) = |e^{i\phi_1} - e^{i\phi_2} + 1| \),

   were \( v(P_1 \Rightarrow P_2) = 1 \) in all cases permitted by the condition of existence except where \( \phi_1 = \phi_2 = \pm \frac{\pi}{3} \) for which \( v(P_1 \Rightarrow P_2) = 0 \). If \( v(P_1) = 1 \) and \( v(P_2) = 0 \) were

   \( v(P_1 \Rightarrow P_2) = |e^{i\phi_1} - 1|; v(P_1 \Rightarrow P_2) = 0 \) if

   \( \phi_1 = 0, v(P_1 \Rightarrow P_2) = 1 \) if \( \phi_1 = \pm \frac{\pi}{3} \).

   Hence the truth table of implication (Table 2):

   **Table 2. Truth table of implication \( P_1 \Rightarrow P_2 \):**

<table>
<thead>
<tr>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_1 \Rightarrow P_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0/0*</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0/1*</td>
<td>1</td>
</tr>
</tbody>
</table>

   It coincides with that of classical logic if one rejects the case \( v(P_1) = v(P_2) = 1 \) the choice \( \phi_1 = \phi_2 = \pm \frac{\pi}{3} \) and the case \( v(P_1) = 1 \) and \( v(P_2) = 0 \) the choice \( \phi_1 = \pm \frac{\pi}{3} \).

   These rejections are required to conduct a rigorous deduction in paraconsistent Boolean logic: the fundamental articulation of the deduction is indeed true implication denoted \( \Rightarrow \), that if \( P_1 \) is true requires true \( P_2 \).

4. **Concordance:** From (12) is true if \( v(P_1) = v(P_2) = 0 \).

   If \( v(P_1) = 1 \) and \( v(P_2) = 0 \) then \( v(P_1 \Xi P_2) = \sqrt{|e^{i\phi_1} - 1|^2} \) therefore \( v(P_1 \Xi P_2) = 0 \) if \( v(P_1 \Leftrightarrow P_2) = 1 \) \( \phi_1 = 0 \) and \( v(P_1 \Xi P_2) = 0 \) if \( \phi_1 = \pm \frac{\pi}{3} \); same if \( v(P_1) = 0 \) and \( v(P_2) = 1 \) we have \( v(P_1 \Leftrightarrow P_2) = 0 \) if \( \phi_2 = 0 \) and \( v(P_1 \Leftrightarrow P_2) = 1 \) if \( \phi_2 = \pm \frac{\pi}{3} \). If \( v(P_1) = v(P_2) = 1 \) the concordance does not exist when \( \phi_1 = \pm \frac{\pi}{3}, \phi_2 = \pm \frac{\pi}{3} \), but it is true in all other cases.

   Hence the truth table of concordance (Table 3):

   **Table 3. Truth table of concordance \( P_1 \Leftrightarrow P_2 \):**

<table>
<thead>
<tr>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_1 \Leftrightarrow P_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0/0*</td>
</tr>
<tr>
<td>0</td>
<td>0/1*</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0/1*</td>
</tr>
</tbody>
</table>

   It coincides with that of the equivalence of classical logic if when \( v(P_1) = 1 \) and \( v(P_2) = 0 \) is chosen \( \phi_1 = 0 \) and when \( v(P_1) = 0 \) and \( v(P_2) = 1 \) is chosen \( \phi_2 = 0 \).

   Importantly, to conduct a rigorous reasoning with these choices, the concordance becomes identical to the deductive equivalence.

### 7. Reflections

The argument concerning belief systems may be circumvented if one claims that ordinary belief is not deductively closed. That is, at least, controversial, but an ideal reasoner should aspire to closure. Considering the case of a paraconsistent system being used as a metalinguage to analyze a belief system, it is also the task of paraconsistent logic to define paraconsistent contradictions, that is, contradictions that are so threatening to this belief system that they really compromise rational inference-making within the belief system. This “bad” kind of inconsistency can be quantitative (too many classic contradictions may be a sign that even paraconsistency cannot save the belief system) or qualitative - that is, the classic contradiction in question is so strong (for example, a proof that all statements of the belief system can be proved both true and false) that it is also a paraconsistent contradiction, a contradiction that even a paraconsistent logician cannot accept. This argument implies the idea that the set of paraconsistent contradictions is a subset of the set of classic contradictions and that is indeed a rather intuitive idea. But we cannot think of any conclusive argument against the existence of a paraconsistent contradiction that is not a classic contradiction, so this idea is only a plausible conjecture.

To appreciate the significance for metaphilosophy of the recent developments in paraconsistent logic showing how, within formal systems, contradictory propositions can be held simultaneously without trivialization. The scientist's conception of the search for truth is partly
motivated and partly justified by the ancient rejection of all contradictions. But this rejection is no longer a logical imperative. Indeed, it cannot be endorsed without, at least, severe qualifications that rob it from its argumentative bite. Thus the way will be open to adopt a novel understanding of the search for truth. And we shall present a model that conceives it as the determination of the range of legitimate answers to a given question (without precluding answers that, to an extent, contradict each other).

Rescher’s objection to syncretism in metaphilosophy stems from his belief that because of its readiness to embrace all different answers to a given question, it is bound to hold contradictory answers. But this, according to Rescher, is not rationally acceptable. It is tantamount to destroying the cognitive nature of philosophy, its aspiration to constitute a search for truth: “To accept a plurality of answers is not to have answers at all; an unending openness to a variety of possibilities, a constant yes-and-no leaves us in perpetual ignorance”. (Rescher, 1995. p. 350; cf p. 344.) The variety of metaphilosophical pluralism here outlined rejects this view. And, in order to understand why it does, we must now evaluate the validity of two logical arguments that have traditionally been used to uphold it.

According to the first, the inclusion of contradictory propositions within the same conceptual space is ruled out by the principle of contradiction. The second is a formal argument known since the Middle Ages (ex absurdum sequitur quodlibet), which in contemporary symbolic logic becomes the proof that from the simultaneous assertion of two contradictory sentences everything can be deduced. (Hilbert and Ackermann, 1928) During the 20th century, however, a growing body of formal developments called paraconsistent logic, which in the last decade has critically undermined this view.

Paraconsistent logic was in some sense born of the realization that consistency, in its classical sense, was not a good enough criteria to discriminate between good and bad belief system, exactly because our actual reasoning is, it seems, much more able to cope with inconsistent premises than classical logic. Indeed, it has become a motto in many circles of non-classical logic that classical logic simply is not an accurate model of human rationality.

References

ANNEX A

We will represent in the following table a comparison between three logics: classical (CL), quasi-paraconsistent (QPL) (Usó-Domènech, Nescolarde-Selva and Pérez-Gonzaga, 2014) and strong paraconsistent (SPL).

<table>
<thead>
<tr>
<th>Notation</th>
<th>Name</th>
<th>CL truth values ( p_1, p_2 \in [0,1] )</th>
<th>QPL truth values ( p_1, p_2 \in [0,1] )</th>
<th>SPL truth values ( p_1, p_2 \in [0,1] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 \land R_2 )</td>
<td>Conjunction</td>
<td>( p_1 p_2 )</td>
<td>( p_1 p_2 )</td>
<td>(</td>
</tr>
<tr>
<td>( \neg R_1 \lor \neg R_2 )</td>
<td>Incompatibility</td>
<td>( 1 - p_1 p_2 )</td>
<td>( u_1 u_2 - p_1 p_2 )</td>
<td>( e^{(\phi_1 + \phi_2)} -</td>
</tr>
<tr>
<td>( R_1 \lor R_2 )</td>
<td>Disjunction</td>
<td>( p_1 + p_2 - p_1 p_2 )</td>
<td>( u_2 p_1 + u_1 p_2 - p_1 p_2 )</td>
<td>(</td>
</tr>
<tr>
<td>( R_1 \Rightarrow R_2 )</td>
<td>Implication</td>
<td>( 1 - p_1 + p_1 p_2 )</td>
<td>( u_1 u_2 - p_1 (u_2 - p_2) )</td>
<td>( e^{(\phi_1 + \phi_2)} -</td>
</tr>
<tr>
<td>( R_1 \Leftrightarrow R_2 )</td>
<td>Concordance</td>
<td>( 1 - p_1 - p_2 + 2 p_1 p_2 )</td>
<td>( u_1 u_2 - (u_2 p_1 + u_1 p_2) + 2 p_1 p_2 )</td>
<td>(</td>
</tr>
<tr>
<td>( R_1 \oplus R_2 )</td>
<td>Discordanse</td>
<td>( p_1 + p_2 - 2 p_1 p_2 )</td>
<td>( u_2 p_1 + u_1 p_2 - 2 p_1 p_2 )</td>
<td>(</td>
</tr>
<tr>
<td>( R_1 \vDash R_2 )</td>
<td>Complementarity</td>
<td>( p_1 + p_2 )</td>
<td>( p_1 + p_2 )</td>
<td>(</td>
</tr>
<tr>
<td>( \neg R_1 \vDash \neg R_2 )</td>
<td>Inverse complementarity</td>
<td>( 2 - p_1 - p_2 )</td>
<td>( p_1^* + p_2^* )</td>
<td>( e^{\phi_1} -</td>
</tr>
<tr>
<td>( R_1 \equiv R_2 )</td>
<td>Equivalence</td>
<td>( 1 + p_1 - p_2 )</td>
<td>( p_1 + u_2 - p_2 )</td>
<td>(</td>
</tr>
<tr>
<td>( \neg R_1 \equiv \neg R_2 )</td>
<td>Inverse equivalence</td>
<td>( 1 - p_1 + p_2 )</td>
<td>( u_1 - p_1 + p_2 )</td>
<td>(</td>
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