Department of Fundamentos del Análisis Económico.
University of Alicante
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STATISTICS AND INTRODUCTION TO ECONOMETRICS
Problem Set 6

1. The following table contains the ACT scores and the GPA (grade point average) for 8 college students. Grade point average is based on a four-point scale and has been rounded to one digit after the decimal.

GPA	ACT
2.8	21
3.4	24
3.0	26
3.5	27
3.6	29
3.0	25
2.7	25
3.7	30

Using this data:

(a) Estimate the relationship between GPA and ACT using OLS; that is, obtain the intercept and slope estimates in the equation

$$GPA = \beta_0 + \beta_1 ACT + u$$

(b) Does the intercept have a useful interpretation here? Explain.

(c) How much higher is the GPA predicted to be, if the ACT score is increased by 6 points?

(d) Compute the fitted values and residuals for each observation and verify that the residuals (approximately) sum to zero.

(e) What is the predicted value of GPA when $ACT = 22$?

(f) How much of the variation in GPA for these 8 students is explained by ACT? Explain.

2. Consider the following linear consumption function that has been estimated using a sample of annual income ($renta$) and annual consumption ($cons$) (both in dollars) for a sample with 100 American families:

$$\begin{aligned} \widehat{cons} &= -124.84 + 0.853\, renta \\ n &= 100, \qquad R^2 = 0.692 \end{aligned}$$

(a) Interpret the intercept in this equation and comment on its sign and magnitude.

(b) Interpret the slope in this equation.

(c) What is predicted consumption when family income is 25000 dollars?

(d) How much of the variation in consumption is explained by income?

(e) Suppose that using the same sample consumption is measured in thousands of dollars. Compute the new estimated parameters of the model and R^2. If we also measure income in thousands of dollars, what would be the new estimated parameters of the model and R^2?

3. Using a sample of 935 individuals for which the monthly wage in dollars (*wage*) and the result of an IQ test (*IQ*) are observed, the following OLS estimates are obtained:

$$\begin{aligned} \widehat{wage} &= 117 + 8.3IQ && (1)\\ \widehat{wage} &= -2628 + 778.5\log(IQ) && (2)\\ \widehat{\log(wage)} &= 5.9 + 0.0088IQ && (3)\\ \widehat{\log(wage)} &= 2.94 + 0.83\log(IQ) && (4) \end{aligned}$$

(a) Which one of these models assumes that an increase of one point in the IQ test IQ implies a constant change in dollars of the mean of *wage*? Using that model, compute the change in the expected wage given an increase of 10 points in the IQ test.

(b) Which one of these models assumes that an increase of one point in the IQ test IQ has always the same percentage effect on *wage*? Using that model, compute the percentage change in the predicted wage given an increase of 10 points in the IQ test.

(c) Compare the results obtained in the previous parts for an individual whose monthly wage coincides with the mean monthly wage of the sample (958 dollars).

(d) Which one of the models assumes that an increase of 1% in IQ has always, on average, the same percentage effect on *wage*? Using that model, compute the estimated percentage change in wage given an increase of 10 percent in the IQ. Compute the wage increase of an individual that has the mean monthly wage and the mean IQ test result (i.e. a wage of 958 dollars and IQ of 101) given an increase of 10% in the result of the IQ test. Compare this result with the one obtained in part b) for the same individual.

(e) Compute the estimated slopes for each model if *wage* is measured in hundreds of dollars.

4. Consider the following model relating the infant birth weight (*weight*) and the cigarettes the mother smoked during pregnancy (*cigs*)

$$weight = \beta_0 + \beta_1 cigs + u$$

Using a sample of 1388 births, the following results have been obtained

$$\begin{aligned} \widehat{weight} &= \underset{(16.23)}{3395.5} - \underset{(2.56)}{14.57}cigs \\ n &= 1388, \quad R^2 = 0.0227 \end{aligned}$$

where *weight* is measured in grams and *cigs* is the average number of daily cigarettes.

(a) Interpret the slope in this equation.

(b) What is the predicted birth weight when $cigs = 0$? What about when $cigs = 20$ (one pack per day)? Comment on the difference.

(c) Does this simple regression necessarily capture a causal relationship between the child's birth weight and the mother's smoking habits? Explain

(d) In order to predict a weight of 3 kilos and a half, *cigs* should be equal to which value? Explain.

5. Consider the following model relating the time spent sleeping per week and the time spent in paid work.

$$sleep = \beta_0 + \beta_1 totwrk + u$$

where *sleep* is minutes spent sleeping at night per week and *totwrk* is total minutes worked during the week. Using the data in file SLEEP75 from Wooldridge:

(a) Estimate the model and report your results in equation form along with the number of observations and R^2.

(b) Interpret the intercept and the slope in the estimated equation.

(c) How much of the variation in time spent sleeping is explained by time spent working?

(d) If *totwrk* increases by 2 hours, by how much is *sleep* estimated to fall?

6. For the population of firms in the chemical industry, let *rd* denote annual expenditures on research and development (R+D), and let sales denote annual sales (both are in millions of dollars).

   (a) Write down a model that implies a constant elasticity between *rd* and *sales*. Which parameter is the elasticity?

   (b) Now estimate the model using the data in file RDCHEM from Wooldridge. Write out the estimated equation in the usual form.

   (c) What is the estimated elasticity of the R+D expenditure with respect to sales? Explain in words what this elasticity means.

   (d) How would the estimated coefficients change if sales are measured in thousands of dollars? Explain.

7. The file CEOSAL2 from Wooldridge contains information on chief executive officers for U.S. corporations. The variable *salary* is annual compensation, in thousands of dolllars, and *ceoten* is prior number of years as company CEO.

   (a) Write down a model for which one more year as company CEO always implies the same percentage increase in wage.

   (b) Estimate the model in part **a)** and show the estimated parameters, the standard errors, the number of observations and the R^2 in the usual form.

   (c) Interpret the slope in this equation.