Gaussian noise elimination in colour images by vector-connected filters

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Abstract

This paper deals with the use of vector-connected filters for eliminating Gaussian noise in colour images. This class of morphological filters suppresses noise but preserves the contours of the objects. We impose a total order between pixels for morphological processing. Once the HSI space has been adapted, we employ it in the lexicographical order. As such, all of the morphological operations are vectorial. After having defined the vectorial geodesic operators, they are then employed to eliminate Gaussian noise.

1. Introduction

In image processing, one of the most important tasks is the improving of the visual presentation or the reduction of noise. With regard to colour images, this can be done in many different ways. In [12], a directional vectorial filter is defined, which minimises the angle between vectors to eliminate the noisy pixel. In [1], the so-called vectorial median filters are an extension of the scalar median filter. In such a case, the ordering of the vectors is done with an Euclidean distance value. In [2], a combination of morphological alternate filters are used with RGB to reduce the noise.

In this paper, we show the utility of using the vectorial geodesic reconstruction algorithms for eliminating Gaussian noise from the colour images. Section 2 presents the colour space chosen for mathematical processing. In Section 3, we extend the geodesic operations to colour images. In Section 4, we apply the vector-connected filters for eliminating the Gaussian noise in colour images. Finally, our conclusions are outlined in the final section.

2. Colour mathematical morphology

Mathematical morphology is a non-linear image processing approach which is based on the application of lattice theory to spatial structures [10]. The definition of morphological operators needs a totally ordered complete lattice structure [9]. The application of mathematical morphology to colour images is difficult, due to the vectorial nature of the colour data (RGB, CMY, HSI, YIQ…). Many works have been carried out on the application of mathematical morphology to colour images [3,4,5,6,7]. The most commonly adopted approach is based on the use of a lexicographical order which imposes a total order on the vectors. This way, we avoid the false colours in an individual filtering of signals. Let $\mathbf{x}=(x_1, x_2,\ldots,x_n)$ and $\mathbf{y}=(y_1, y_2,\ldots, y_n)$ be two arbitrary vectors ($\mathbf{x},\mathbf{y} \in Z^n$). An example of lexicographical order o_{lex}, will be:

$$x < y \quad if \quad \begin{cases} x_1 < y_1 \quad or \\ x_1 = y_1 \quad and \quad x_2 < y_2 \quad or \\ x_1 = y_1 \quad and \quad x_2 = y_2 \ldots \quad and \ldots x_n < y_n \end{cases} \tag{1}$$

On the other hand, it is important to define the colour space in which operations are to be made. The intuitive systems (HSI, HLS, HSV,…) are widely used in image processing as they represent the information in a similar way to the human brain. We use HSI colour space. The preference or disposition of the components of the HSI in the lexicographical ordering depends on the application and the properties of the image. Ordering with luminance (intensity) in the first position is the best way of preserving the contours of the objects in the image I→H→S (lattice influenced by intensity). In situations in which the objects of interest are highly coloured or in which only objects of a specific colour are of interest, the operations with hue in the first position are the best H→I→S (lattice influenced by hue).

However, the HSI colour space has some significant drawbacks that prevent the correct formation of the lattice with HSI, such as:

- Instability of saturation in the highest and lowest levels of intensity.
- A lack of internal order in the hue component.
- Hue of no value in zero saturation.

The use of HSI colour morphology requires that the above-mentioned inconveniences be solved. In the following section, we solve such drawbacks.

2.1. The adapting of the HSI colour space to form complete lattices

There are a great number of variations for the transformation of RGB to HSI. Some instability arises, however, in saturation for small variations of RGB values. To avoid this, we must linearly reduce the saturation as the intensity or luminance increases or decreases. This way, we change from a cylindrical geometrical

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representation to a double-cone shape. The saturation map is now more amenable to morphological treatment.

A problem arises with the hue in trying to form a complete lattice with HSI. Hue is angle-valued, if $\mathbf{f}(p)=(f_I(p), f_H(p), f_S(p))$ are the colour values of the pixel p from a colour image in the HSI colour space, then $f_H(p)\in[0,2\pi)$. In addition, one cannot order hues from their lowest to their highest values. To order hues, Hanbury [3] and Peters [7] use a hue-valued structuring function. Hues are ordered according to the absolute value of a distance function between the image hue and a reference hue (infimum):

$$d(f_H(p), H_{ref}) = \begin{cases} \left|f_H(p) - H_{ref}\right| & if \quad \left|f_H(p) - H_{ref}\right| \le \pi \\ 2\pi - \left|f_H(p) - H_{ref}\right| & if \quad \left|f_H(xp) - H_{ref}\right| > \pi \end{cases} \tag{2}$$

With this definition, the value of the distance will fluctuate between 0 and 180 (360º/2). In a range [0,1] the distance fluctuate between 0 and 0.5. In this case, when the hue has no value, it is represented with a value of 1.

Finally, from the definition of the complete lattice, we must ensure that all of the elements in the lattice can be ordered and compared. There is a problem with the hue when the saturation is zero. This means that the hue signal cannot be used to order pixels in such cases. We can define a saturation threshold S_{Th} which determines which pixels have a significant hue and which have not. In the lattice, if there is a pixel p with $f_S(p)<S_{Th}$ the lexicographical order I→H→S must be changed to I→S, H→I→S must be changed to I→S, etc.

3. Vector-connected filters

Morphological filters by reconstruction have the property of suppressing details, preserving the contours of the remaining objects [8,11]. The use of these filters in colour images requires an ordering relationship among the pixels of the image. For the vectorial morphological processing the lexicographical ordering previously defined o_{lex} will be used. As such, the infimum ($\wedge_v$) and supremum ($\vee_v$) will be vectorial operators, and they will select pixels according to their order o_{lex} in the HSI colour space [5].

Once the orders have been defined, the morphological operators of reconstruction for colour images can be generated and applied. A elementary geodesic operation is the geodesic dilation. Let **g** denote a marker colour image and **f** a mask colour image (if $o_{lex}(\mathbf{g})\le o_{lex}(\mathbf{f})$, then $\mathbf{g} \wedge_v \mathbf{f} = \mathbf{g}$). The vectorial geodesic dilation of size 1 of the marker image **g** with respect to the mask **f** can be defined as:

$$\delta_{v\mathbf{f}}^{(1)}(\mathbf{g}) = \delta_v^{(1)}(\mathbf{g}) \wedge_v \mathbf{f} \tag{3}$$

where it is very important that the infimum operation is done with the same lexicographical ordering as the vectorial dilation. This way, there is not false colours in the result.

The vectorial geodesic dilation of size n of a marker colour image **g** with respect to a mask colour image **f** is obtained by performing n successive geodesic dilations of **g** with respect to **f**:

$$\delta_{v\mathbf{f}}^{(n)}(\mathbf{g}) = \delta_{v\mathbf{f}}^{(1)}\left[\delta_{v\mathbf{f}}^{(n\text{-}1)}(\mathbf{g})\right] \tag{4}$$

with $\delta_{v\mathbf{f}}^{(0)}(\mathbf{g}) = \mathbf{f}$

Geodesic transformations of bounded images always converge after a finite number of iterations [11]. The propagation of the marker image is impeded by the mask image. Morphological reconstruction of a mask image is based on this principle.

The vectorial reconstruction by dilation of a mask colour image **f** from a marker colour image **g**, (both with the same dominion and $o_{lex}(\mathbf{g}) \le o_{lex}(\mathbf{f})$) can be defined as:

$$R_{v\mathbf{f}}(\mathbf{g}) = \delta_{v\mathbf{f}}^{(n)}(\mathbf{g}) \tag{5}$$

where n is such that $\delta_{v\mathbf{f}}^{(n)}(\mathbf{g}) = \delta_{v\mathbf{f}}^{(n+1)}(\mathbf{g})$.

The vectorial reconstruction is an algebraic opening (increasing, anti-extensive and idempotent) only if all the operations between pixels respect the total order, in our case, the lexicographical order.

4. Application: Gaussian noise elimination

Vector-connected filters will be used to eliminate Gaussian noise from colour images. Chromatic Gaussian noise, in contrast to impulsive noise, changes the entire definition of the image. Its effect is most obvious in the homogenous regions, which become spotty. With vector-connected filters we can reduce the Gaussian noise by merging the flat zones in the image. As such, the noisy image is simplified and the contours of the objects are preserved.

The image used in our study is a colour image of Parrots, which has been contaminated by a Gaussian noise with a variance of 20. In Figure 1, the original image and the noisy one are presented.

We apply a vectorial opening by reconstruction, VOR:

$$\gamma_{v\mathbf{f}}^{(n)} = \delta_{v\mathbf{f}}^{(n)}(\varepsilon_v{}^{(s)}(\mathbf{f})) \tag{6}$$

to the noisy image. All of the operations are made with a lexicographical order o_{lex}: I→H→S, (H_{ref}=0º). The vectorial erosion of the opening by reconstruction is made with increasing sizes of the structuring element 's', from 5x5 to 11x11. In Figure 2, the results of the connected filter are observed: i.e., in accordance with the increase of the structuring element, the noise is reduced, but the image is darkened (anti-extensive operation).

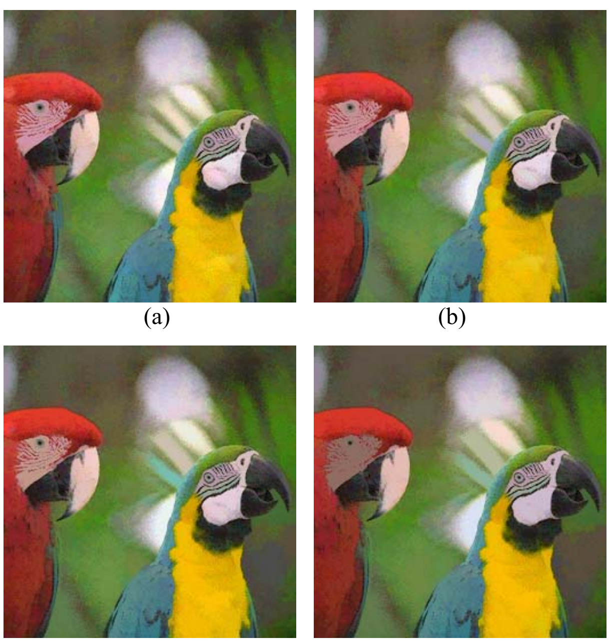
We now apply a vectorial closing by reconstruction VCR:

$$\phi_{v_{\mathbf{f}}}^{(n)} = \varepsilon_{v_{\mathbf{f}}}^{(n)}(\delta_v^{(s)}(\mathbf{f})) \tag{7}$$

to the original image and we obtain the results which are presented in Figure 3. In this case, the image is clarified as the size of the structuring element 's' of the dilation is increased. VCR is an extensive operation.

(a) (b)

Figure 1. Parrots Image. (a) original image, (b) noisy image.

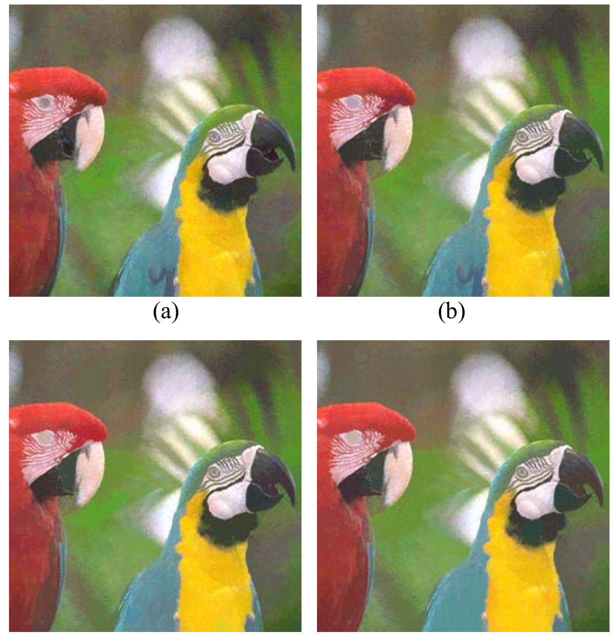


(a) (b)

(c) (d)

Figure 2. Vectorial openings by reconstruction (VOR) from: 5x5-eroded image (a), 7x7-eroded image (b), 9x9-eroded image (c), 11x11-eroded image (d).

In Figure 4, the effect of reducing flat zones within the noisy image is shown. In particular, the upper left corner of the images is detailed. Figure 4.a shows the original noisy section. In Figures 4.b and 4.c, we can observe the progressive reduction of flat zones as the size of the structuring element increases in the operation of erosion of the vectorial opening by reconstruction.



(a) (b)

(c) (d)

Figure 3. Vectorial closings by reconstruction (VCR) from: 5x5-dilated image (a), 7x7-dilated image (b), 9x9-dilated image (c), 11x11-dilated image (d).

(a) (b) (c)

Figure 4. Detail of simplification of the image. (a) noisy section. (b) VOR-7x7. (c) VOR-11x11.

Finally, in order to attenuate the dark and light effects of the previous filters, we calculate the mean signal of both filters. The mean images are shown in Figure 5.

We use the normalised mean squared error (NMSE) to assess the performance of the different means of the filters. The NMSE test for colour images is calculated from the RGB tri-stimulus values [2]. In addition to the NMSE test, three subjective criteria can be used: visual attenuation of noise, contour preservation and detail preservation.

The evaluation of the quality with subjective criteria is divided into four categories: i.e., excellent (4), good (3), regular (2) and bad (1). The performance of the filters on the noise-polluted Parrots image is illustrated in Table 1. The value of the NMSE test for the noisy one is 0.0952.

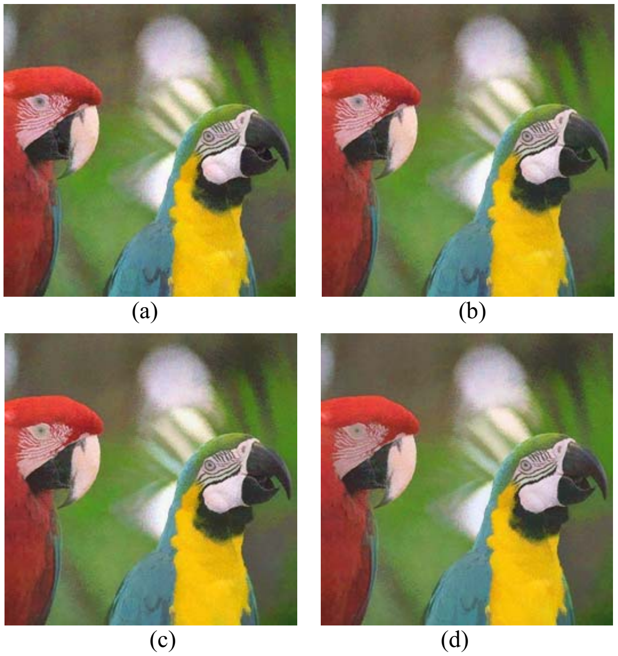


Figure 5. Final results. (a) Mean VOR-VCR5x5, (b) Mean VOR-VCR 7x7, (c) Mean VOR-VCR 9x9, (d) Mean VOR-VCR 11x11.

Table 1. Colour NSME Test

SE	NMSE	Visual atten. of noise	Contour preservation	Detail preservation
5X5	0.0775	3	4	3
7X7	0.0778	3	3	3
9X9	0.0787	4	3	3
11x11	0.0809	4	3	2

The most optimal filter, according to the NSME test, is the one that employs a structuring element of either 5x5 or 7x7 in size in the vectorial morphological operations. Regarding the subjective evaluation of the quality of the filter, the visual attenuation of the noise has a proportionally inverse relationship with the conservation of contours and details. The best visual attenuation of noise is the one obtained from filtering the image with a large structuring element, while the conservation of structures is achieved with smaller structuring elements.

5. Conclusions

In this paper, we have presented an algorithm for eliminating Gaussian noise in colour images. This algorithm includes the colour geodesic reconstruction operations as a novelty. The geodesic method for reducing noise has been shown to be efficient for a Gaussian noise of a variance of 20. In the experiments carried out, better results where obtained for lower variances, specifically 10 and 15.

6. References

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