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All-Pay Auction Equilibria in Contests*

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Abstract

We analyze (non-deterministic) contests with anonymous contest success functions. There is no restriction on the number of contestants or on their valuations for the prize. We provide intuitive and easily verifiable conditions for the existence of an equilibrium with properties similar to the one of the (deterministic) all-pay auction. Since these conditions are fulfilled for a wide array of situations, the predictions of this equilibrium are very robust to the specific details of the contest. An application of this result contributes to fill a gap in the analysis of the popular Tullock rent-seeking game because it characterizes properties of an equilibrium for increasing returns to scale larger than two, for any number of contestants and in contests with or without a common value.

Keywords: (non-) deterministic contest, all-pay auction, contest success functions.
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1. Introduction

In a contest agents exert irreversible effort to increase their probability of winning a prize. Contests have been used to analyze a variety of situations including lobbying, rent-seeking and rent-defending contests, litigation, political campaigns, conflict, patent races, arms races, sports events or R&D competition. Moreover, recent papers (like e.g. Alesina and Spolaore (2006), Baron and Diermeier (2006), Konrad (2000a, 2000b) or Polborn and Klumpp (2006)) have embedded contests in larger political economy models in order to capture the effect of conflict on other variables of interest.

Particularly when a contest model is embedded in a larger game, it is desirable that equilibrium payoffs do not change too much as the primitives of the contest change. Otherwise, the predictions of the larger model might not be robust to changes in the primitives of the contest subgame. The present paper determines a class of contests with fairly different primitives that admits essentially the same equilibrium. Equilibrium predictions within this class of contests can, thus, be considered robust to the specification of the contest. Moreover, the class includes two prominent models of contests.

The crucial element in the specification of a contest is the so-called contest success function (CSF), which associates to each vector of contestants' effort levels a lottery specifying for each agent a probability of getting the prize. In the literature there are two prominent ways to model contests.

First, there is the all-pay auction, in which the player exerting the highest effort wins the prize with probability one. Such a contest is therefore called deterministic (or perfectly discriminating). It has been analyzed by Hillman and Riley (1989), Baye *et al.* (1993, 1996) or Che and Gale (1998), among others.

For later reference we summarize the results of Hillman and Riley (1989) and Baye *et al.* (1996) as follows. Denote the valuation of bidder B_i for the prize by V_i and suppose that $V_1 \geq V_2 \geq \cdots \geq V_n$. There exists a Nash equilibrium in mixed-strategies to the all-pay auction. In this equilibrium, bidder B_1 randomizes uniformly on $[0, V_2]$, while bidder B_2 abstains with probability $1 - V_2/V_1$ and adopts the same mixed-strategy as B_1, given that he enters the contest. All other contestants abstain with probability one. Expected equilibrium payoffs are $E\Pi_1 = V_1 - V_2$ and $E\Pi_j = 0$ for all B_j with $j > 1$. The expected revenue is $ER = V_2(V_1 + V_2)/(2V_1)$.[1]

In the sequel we will use the term *all-pay auction equilibrium* to indicate an equilibrium in which the expected equilibrium bids, payoffs and revenues (but not necessarily the distributions of bids) are as in the (deterministic) all-pay auction (see Definition 3.1). Note that these equilibria have important implications for the participation in the aforementioned applications of contests. It is sufficient to deal with two contestants, because further players prefer to abstain.

[1] In many instances this equilibrium is unique, see Remark 3.2 for more details.

Second, a very prominent class of contest games is the so-called Tullock's Rent-Seeking Game. Given a vector of efforts b and R, a positive parameter measuring returns to scale from effort, in Tullock's specification the probability that bidder B_i wins the contest is given by

$$\Psi_i^T(b) = \frac{b_i^R}{\sum_{j=1}^n b_j^R} \; . \tag{1.1}$$

Note that if $R = 0$, that is, the so-called contest success function is completely insensitive to effort, the extreme case of a (fair) lottery is obtained. The opposite case of extreme sensitivity ($R \to \infty$) in which only efforts matter yields the (deterministic) all-pay auction. Hence, we might think of R as specifying how much the extreme requirement of the deterministic all-pay auction is relaxed through chance in the assignment of the prize.

Tullock's Rent-Seeking Game has been analyzed by Tullock (1980), Pérez-Castrillo and Verdier (1992), Baye *et al.* (1994) and Skaperdas (1996), among others. Equilibria in this game are well understood when R is relatively small, because then there exist pure strategy equilibria. However, this is not so for larger R. For $2 < R < \infty$, we are only aware of one study (Baye *et al.* (1994)), which restricts to two contestants with equal valuations. For this large range of parameter values, the widely applied Tullock's Rent-Seeking Game offers, hence, no prediction concerning rent-seeking outlays when there are more than two contestants or when valuations differ. Moreover, it is not known what properties of the deterministic all-pay auction extend to the non-deterministic Tullock's Rent-Seeking Game.

For tractability reasons applications of Tullock's Rent-Seeking Game suppose very often that $R = 1$. This case yields very different results from the deterministic all-pay auction. For instance, equilibrium payoffs are, in general, different. As a consequence, more than two contestants might have an incentive to participate actively in the contest. Thus, it is no longer sufficient to deal with two contestants. Further differences between Tullock's Rent-Seeking Game and deterministic all-pay auction exist and the reader may find discussions in Nitzan (1994), Che and Gale (2000) and Fang (2002).

The purpose of the present paper is, hence, twofold:

1. We analyze to what extent the equilibrium predictions of the deterministic all-pay auction are robust to different amounts of randomness in the assignment process for the price. This randomness might be introduced following (1.1) with R finite, but it is worth to point out at this point that we do not limit our analysis to logit formulations of the CSF.

2. We contribute to close the gap in the analysis of Tullock's Rent-Seeking Game, because our main result applies for $2 < R < \infty$, for any number of contestants, and for any valuations for the political prize the contestants might have.

Our main result specifies conditions on the CSF that are sufficient for an all-pay auction equilibrium to exist. The main conditions are three. Anonymity is used to construct an equilibrium for general situations building on an equilibrium of the symmetric two bidder contest. While the deterministic all-pay auction is anonymous, the other two conditions relax the requirement that the highest bidder wins the contest for sure. Sufficient Discrimination (SD) says that the contest has to be deterministic enough. Sufficient Monotonicity (SM) requires that increasing one's bid should yield a sufficiently high win probability. We show then that these conditions are fulfilled under a variety of very different CSFs, including Tullock's Rent-Seeking Game.

Contests have been reviewed, for example, in Nitzan (1994) and Konrad (2007). Usually, papers on contests specify a particular CSF and analyze equilibrium. Consequently, there are few papers dealing with a general class of CSFs and we are not aware of any carrying out an analysis at our level of generality.[2] The present paper is most related to Che and Gale (2000), Alcalde and Dahm (2007) and Baye *et al.* (1994). Che and Gale analyze a family of linear difference-form contests with two bidders that is characterized by a non-negative parameter. Similarly to Tullock's Rent-Reeking Game, the scalar specifies how deterministic the contest is. As a result, the family contains the polar cases of the (fair) lottery and the (deterministic) all-pay auction. Che and Gale analyze mixed-strategy equilibria and show the convergence of the equilibrium to that of the all-pay auction as the difference-form contest approaches the all-pay auction. In contrast, the present paper specifies conditions under which a non-deterministic contest (e.g. Tullock's Rent-Seeking Game with $2 < R < \infty$) admits all-pay auction equilibria.[3] Alcalde and Dahm define the Serial Contest (for a formal definition see Subsection 4.2) which is a different family of contests that also includes the two polar cases depending on a scalar. They show that the Serial Contest admits all-pay auction equilibria when the contest is deterministic enough. The present paper obtains this result as a special case. However, contrary to the present paper, their proof relies on the homogeneity of degree zero of the CSF. Both papers follow Baye *et al.* (1994) by using an auxiliary contest with a finite bidding space to in order to analyze mixed-strategy equilibria in the original contest.

This paper is organized as follows. The next section introduces the class of contests analyzed in the present paper and defines an auxiliary contest with a finite grid on the bidding space. Section 3 establishes our main result which we apply in Section 4 to specific contests. The last section offers some concluding remarks.

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[2] For example, Szidarovszky and Okuguchi (1997) focus on logit formulations of the CSF with twice differentiable, strictly increasing, and concave effectivity functions. Malueg and Yates (2006) study homogenous CSFs.

[3] In principle, the class of contests analyzed in the present paper includes Che and Gale's contest. However, since their contest does not fulfill condition (SD) when the scalar is finite our main result does not apply.

2. Contests

2.1. Preliminaries

There are $n > 1$ players wishing to participate in a contest. The set of contestants or bidders is denoted by $\mathcal{B} = \{B_1, \ldots, B_i, \ldots, B_n\}$. Each contestant has a valuation for the object, denoted by V_i, and submits a bid $b_i \in \mathbb{R}_+$. Outlays are irreversible. Bidders are risk-neutral, and they bid simultaneously. The valuations are common knowledge and without loss of generality ordered such that $V_1 \geq V_2 \geq \ldots \geq V_n > 0$.

It is assumed that the contest administrator commits to determine the winner through a contest success function. This function associates, to each vector of bids $b = (b_1, \ldots, b_n)$, a lottery specifying for each agent a probability of getting the object.

Definition 2.1. *[CSF] A contest success function is a mapping*

$$\Psi : \mathbb{R}_+^n \to \Delta^n$$

such that for each $b \in \mathbb{R}_+^n$, $\Psi(b)$ is in the $n-1$ dimensional simplex, i.e. $\Psi(b)$ is such that, for each i, $\Psi_i(b) \geq 0$, and $\sum_{i=1}^n \Psi_i(b) = 1$.

Throughout this paper we assume that contest success functions satisfy the following Incentive Property.

Definition 2.2. *[IP] We say that CSF Ψ satisfies the Incentive Property if, for each bidder B_i, and other agents' bids $b_{-i} \in \mathbb{R}_+^{n-1} \setminus \{0\}$,*

$$\Psi_i(b_i, b_{-i}) \geq \Psi_i\left(b_i^{'}, b_{-i}\right) \text{ whenever } b_i \geq b_i^{'}; \tag{2.1}$$

$$\Psi_j(b_i, b_{-i}) \leq \Psi_j\left(b_i^{'}, b_{-i}\right) \text{ for all } j \neq i, \text{ whenever } b_i \geq b_i^{'}; \text{ and} \tag{2.2}$$

$$\Psi_i(b_i, b_{-i}) > 0 \text{ only if } b_i > 0. \tag{2.3}$$

Let us observe that [**IP**] is a natural condition that is satisfied by all the (homogeneous) CSFs studied in the literature. In particular, (2.1) specifies a weak monotonicity property, of each bidder's winning probability, in her own bid; (2.2) establishes that when some bidder's winning probability increases (resp. decreases), then the winning probability of any other bidder decreases (resp. increases); and (2.3) says that no bidder has a positive winning probability unless her bid is positive (or all bidders bid zero).

Given the contest success function Ψ, agents' expected utility from participating in the contest, when the vector of bids is b, is

$$E\Pi_i(b) = \Psi_i(b)V_i - b_i, \qquad \forall B_i \in \mathcal{B}. \tag{2.4}$$

We denote a mixed-strategy for player B_i by μ_i and indicate the associated strategy profile by μ.

2.2. A Class of Contest Success Functions

We describe now properties of the class of contest success functions analyzed in this paper. The first axiom is Anonymity, a property establishing that each agent's probability is independent of her label and depends only on the vector of bids.

(A) Anonymity: For any permutation function π of $\mathcal{B}$ (i.e., a bijection $\pi : \mathcal{B} \to \mathcal{B}$) we have $\Psi(\pi(b)) = \pi(\Psi(b))$ for all b.

Note that this axiom also implies that all bidders submitting identical bids must obtain equal probabilities of winning. Specifically, for the degenerated bid vector (all contestants bid zero), Anonymity and the definition of a CSF imply that the CSF assigns win probability $1/n$ to all contestants, as e.g. in Baye *et al.* (1994).

The present paper makes use of continuity properties of contestants' payoff functions by applying results of Dasgupta and Maskin (1986) to contests. Therefore, a natural requirement is continuity of the CSF. However, the most commonly used CSFs, like the perfectly discriminating all-pay auction or Tullock's Rent-Reeking Game, are not continuous everywhere.[4]

To avoid excluding these CSFs, we allow for a weaker form of continuity of the CSF. Given a vector of bids $b = (b_i, b_{-i}) \in \mathbb{R}^n_+$, define the highest bid of a contestant other than B_i as $b^i_{\max}$, i.e.

$$b^i_{\max} = \max_{j \neq i} \; b_j.$$

The following property assures that the set of discontinuities of the CSF is 'small' and that it 'pays' to increase outlays slightly at these points.

(DS) Discontinuity Set: Given $B_i \in \mathcal{B}$, if Ψ_i is discontinuous at b, then:

(a) $b_i = b^i_{\max}$; and

(b) $\Psi_i\left(b'_i, b_{-i}\right) = 1$ for all $b'_i > b_i$.

For instance, in the popular Tullock's Rent-Seeking Game the CSF is continuous everywhere except at the degenerate bid vector, when all contestants bid zero. Thus, $b^i_{\max} = 0$. The second part of (DS) requires in such a case simply that in order to obtain a strictly positive win probability contestant B_j, distinct of B_i, has to participate actively in the contest. In other words, zero outlays by a contestant imply that this player has no chance to win the contest. But note that (DS) is general enough to accommodate the perfectly discriminating all-pay auction in which the CSF is continuous everywhere except when two or more contestants tie for the highest bid.

In the following we focus on contests assigning win probabilities through an anonymous CSF fulfilling (DS).

[4] In fact, it is easy to see that homogenity of degree zero of the CSF implies that if $\Psi_i(b)$ is continuous at the degenerated bid vector (all contestants bid zero), then $\Psi_i(b)$ must be constant. This fact was pointed out by Corchón (2000).

2.3. The Continuous and the Finite Contest

In this paper we follow the approach in Baye *et al.* (1994) by relating the original contest with continuous strategy space to another one in which there is a finite grid on the bidding space. Note that the latter is realistic when there is a smallest monetary unit, like in experimental settings. Given some $G \in \mathbb{N}_+$, the contest is *finite* with grid G and smallest monetary unit $1/G$ if the strategy space is discrete such that only bids that coincide with the grid $\{m, m+1/G, m+2/G, \ldots, m+(G-1)/G, m+1\}$ for all $m \in \mathbb{N}_+$ are feasible. We refer to this game as the finite contest and indicate an arbitrary element of the grid by x/G, where $x \in \mathbb{N}_+$.

As a starting point for our analysis we follow Baye *et al.* (1994) and apply results of Dasgupta and Maskin (1986) to our model. Consider a contest with n bidders and common value. Let $\bar{\mu}^G = (\bar{\mu}_1^G, .., \bar{\mu}_n^G)$ denote an equilibrium to the contest with finite grid G. The next lemma establishes existence of a symmetric mixed-strategy equilibrium to both the continuous and finite contest and relates these equilibria.[5]

Lemma 2.3. *Consider a contest with common value V and contest success function satisfying (A) and (DS). This contest possesses a symmetric mixed-strategy Nash equilibrium, both when the strategy space is finite and when it is continuous. Moreover, the profile $\bar{\mu} = lim_{G\to\infty}\bar{\mu}^G$ exists and constitutes a mixed-strategy Nash equilibrium to the continuous contest.*

Proof. Note that the existence of a common value and (A) imply that both the finite and the continuous contest are symmetric games. With this, the existence of a symmetric equilibrium for the contest with finite grid G follows from Lemma 6 in Dasgupta and Maskin (1986). We show that the conditions of their Theorem 6 are also satisfied. This theorem guarantees the existence of a symmetric mixed-strategy equilibrium when the strategy space is continuous. In addition, the proof of Dasgupta and Maskin's Theorem 6 shows that the limiting equilibrium of a finite approximation to the strategy space as the grid size goes to zero is indeed an equilibrium to the continuous game. The application of their theorem requires some conditions to be fulfilled. First, the sum of payoffs must be upper semi-continuous. Since $\sum_{i=1}^n E\Pi_i(b) = V - \sum_{i=1}^n b_i$ is continuous, it is upper semi-continuous, too. Second, $E\Pi_i(b)$ must be bounded, which holds as $-V \leq E\Pi_i(b) \leq V$ for $b_i \in [0, V]$ and $i = 1, 2, \ldots, n$. This completes the proof when the CSF is continuous. For discontinuous CSFs fulfilling (DS) two further properties must be fulfilled. Third, one must be able to express a set of points that includes the discontinuities as a function relating the strategies of pairs of contestants. Given (DS) the identity function can be used to define this set. Fourth, a so-called

[5] From Dasgupta and Maskin's results it also follows that the equilibrium strategy has no atom at zero effort, when this is a point of discontinuity of the CSF.

property α must hold. Let $k \geq 1$ denote the cardinality of the bid $b^i_{\max}$ in b_{-i}. Property α is fulfilled, since

$$\lim \inf_{b_i \longmapsto^+ b^i_{\max}} E\Pi_i\left(b_i, b_{-i}\right) = V - b^i_{\max} > \frac{V}{k+1} - b^i_{\max} \geq E\Pi_i\left(b^i_{\max}, b_{-i}\right),$$

holds. Thus, Theorem 6 in Dasgupta and Maskin (1986) can be applied. ■

3. The Main Result

As explained in the Introduction, in this section we give conditions for the existence of an equilibrium to non-deterministic contests that has properties of the one of the deterministic all-pay auction. We define first what we mean by an all-pay auction equilibrium.

Definition 3.1. *Let $V_1 \geq V_2 \geq \ldots \geq V_n$. In an all-pay auction equilibrium μ^* the expected bid of contestant B_1 is $E\left(\mu_1^*\right) = V_2/2$ and the one of contestant B_2 is $E\left(\mu_2^*\right) = \left(V_2\right)^2 / \left(2V_1\right)$. All other contestants abstain from the contest (by bidding zero). Contestant B_1's expected equilibrium payoff is $E\Pi_1\left(\mu^*\right) = V_1 - V_2$, while for all other contestants $E\Pi_{i\neq 1}\left(\mu^*\right) = 0$. The expected revenue is $ER\left(\mu^*\right) = V_2(V_1 + V_2)/(2V_1)$.*

Remark 3.2. *In the perfectly discriminating all-pay auction there is a unique equilibrium if $V_2 > V_3$. When there is a multiplicity of equilibria, in no equilibrium there is a contestant whose expected payoff exceeds the one specified in the statement. Moreover, the only case in which there is no revenue equivalence among equilibria is when more than one contestant have the second highest valuation which is strictly lower than the highest one. See Baye* et al. *(1996) for more details.*

It turns out that we can guarantee the existence of an all-pay auction equilibrium when besides properties (A) and (DS) the contest success function is sufficiently discriminating and monotonic in two active player contests.[6] The following two conditions will be used in the finite game. Remember that in the discrete setting bidding $(x+1)/G$ represents a marginal increase of the bid x/G. For simplicity of exposition, let $\bar{x}$ denote the integer such that $\bar{x} \leq GV_1 < \bar{x} + 1$.

The first condition specifies a minimum win probability that outbidding the opponent by the minimum amount must yield.

[6] These additional properties are related to the case in which at most two agents' bids are positive. Since we deal with anonymous CSFs, there is no loss of generality by assuming that those agents are bidders B_1 and B_2. Thus, we abuse notation and denote by (b_1, b_2) a vector in $\mathbb{R}^n$ such that $b_i = 0$ for all $i \neq 1, 2$.

(SD) Sufficient Discrimination: For each $x \in \{0, 1, ..., \bar{x}\}$,

$$\Psi_1\left(\frac{x+1}{G}, \frac{x}{G}\right) \geq \frac{x+2}{2(x+1)}. \tag{3.1}$$

Note that the right hand side, RHS from now on, is a strictly decreasing function with values in $[1, 1/2)$. Thus, the CSF must be sufficiently discriminating in favor of the higher bidder. Hence, (SD) specifies a lower bound on how much the extreme case of the (deterministic) all-pay auction, in which the higher bidder wins the contest for sure, can be relaxed.

The next condition requires that a marginal increase of the bid x/G to $(x+1)/G$ yields a sufficiently higher win probability than before.

(SM) Sufficient Monotonicity: $\forall x \in \{0, 1, ..., \bar{x}\}$ and $\forall j \in \{x+1, ..., \bar{x}\}$

$$\frac{\Psi_1\left(\frac{x+1}{G}, \frac{j}{G}\right) - \Psi_1\left(\frac{x}{G}, \frac{j}{G}\right)}{\Psi_1\left(\frac{x}{G}, \frac{j}{G}\right)} \geq \Psi_1\left(\frac{x+1}{G}, \frac{x}{G}\right) - \Psi_2\left(\frac{x+1}{G}, \frac{x}{G}\right). \tag{3.2}$$

Note, again that (SM) is fulfilled in the extreme case of the (deterministic) all-pay auction.

We are now in a position to present our main result.

Theorem 3.3. *Let $V_1 \geq V_2 \geq \cdots \geq V_n$. Suppose the contest success function satisfies (A), (DS), (SD) and (SM). Then the contest possesses an all-pay auction equilibrium.*

To prove this result we use several lemmata. For a sketch of the proof consider the following example with particularly simple equilibrium strategies.

Example 3.4. *Consider Tullock's Rent-Seeking Game with $R = 2$. In the case of a common value, the first order conditions characterize maximizers of expected utility for each agent.*[7] *Both contestants bid half of their common valuation. Lemma 2.3 established the existence of a symmetric equilibrium to common value contests for more general situations.*

Building on this symmetric equilibrium we construct an equilibrium to two-player contests without common value when, say, $V_1 \geq V_2$. Note that in the symmetric equilibrium the rent is completely dissipated and contestants obtain zero payoffs. Lemma 3.5 shows that the same is true under (SD) and (SM) in a wide class of symmetric equilibria. Notice also that the increase in B_1's valuation w.r.t. the symmetric situation does not change the problem of contestant B_2. Hence, given $b_1^* = V_2/2$, her best reply is still to

[7] See e.g. Pérez-Castrillo and Verdier (1992) or Nti (1999) for a formal analysis of this maximization problem.

bid $b_2^* = V_2/2$, on one hand, or, on the other, to bid zero. So she is also willing to mix between the two. If she mixes with the right frequency, then the maximization problem of contestant B_1 admits the same solution as in the symmetric game. Consequently, the following is an equilibrium to the asymmetric contest. Contestant B_1 bids the optimal strategy of the symmetric game $b_1^* = V_2/2$ and contestant B_2 abstains with probability $(1 - V_2/V_1)$ and bids $b_2^* = V_2/2$ whenever she participates. Lemma 3.6 establishes such a result for any symmetric equilibrium in which the rent is completely dissipated.
The last step is to observe that further contestants with lower valuations than V_2 cannot do better than B_2. Given the specified bids of the first two contestants they prefer to abstain from the contest. Thus, the described strategies constitute an all-pay auction equilibrium in mixed-strategies to the n-player Tullock's Rent-Seeking Game with $R = 2$ and asymmetric valuations.

The next lemma, and its proof, follows some of the reasoning in Baye *et al.* (1994) and generalizes it to a broader class of contests.

Lemma 3.5. *Consider a 2-bidder contest with finite grid G and common value V in which the contest success function satisfies (A), (SD) and (SM). In any symmetric Nash equilibrium $\bar{\mu}^G = (\bar{\mu}_1^G, \bar{\mu}_2^G)$ it is true that for $i = 1, 2$:*

$$\text{(1) } 0 \leq E\Pi_i(\bar{\mu}^G) \leq \frac{1}{G} \text{ and (2) } E(\bar{\mu}_i^G) = \frac{V}{2} - E\Pi_i(\bar{\mu}^G).$$

Proof. First of all, let us introduce some additional notation. Given G, and agent B_i's strategy μ_i^G, μ_{ik}^G denotes the probability that agent B_i assigns to bidding k/G. Moreover, it is easy to see that, at equilibrium, $\bar{\mu}_{ik}^G = 0$ for any agent B_i and bid such that $k > \bar{x}$. To prove Lemma 3.5, we will concentrate on agent B_1. A similar reasoning applies to agent B_2.

(1) (a) For the lower bound: The expected payoff from bidding x/G when the opponent follows the equilibrium strategy $\bar{\mu}_2^G$ is

$$E\Pi_1\left(\frac{x}{G}, \bar{\mu}_2^G\right) = V\sum_{j=0}^{\bar{x}} \bar{\mu}_{2j}^G \Psi_1\left(\frac{x}{G}, \frac{j}{G}\right) - \frac{x}{G}. \tag{3.3}$$

Choosing $x = 0$, contestant B_1 can secure herself $E\Pi_1\left(0, \bar{\mu}_2^G\right) \geq 0$. Thus, $E\Pi_1(\bar{\mu}^G) \geq 0$.
(b) For the upper bound, since $\bar{\mu}^G$ is an equilibrium, agent B_1 must react optimally to B_2's strategy. Thus, for all x/G: (i) $E\Pi_1\left(x/G, \bar{\mu}_2^G\right) \leq E\Pi_1\left(\bar{\mu}^G\right)$, (ii) $E\Pi_1\left(x/G, \bar{\mu}_2^G\right) = E\Pi_1\left(\bar{\mu}^G\right)$ if $\bar{\mu}_{1x}^G > 0$ and (iii) $\bar{\mu}_{1x}^G = 0$ if $E\Pi_1\left(x/G, \bar{\mu}_2^G\right) < E\Pi_1\left(\bar{\mu}^G\right)$. Using (3.3), condition (i) can be rewritten as

$$V\sum_{j=0}^{\bar{x}} \bar{\mu}_{2j}^G \Psi_1\left(\frac{x}{G}, \frac{j}{G}\right) \leq E\Pi_1\left(\bar{\mu}^G\right) + \frac{x}{G}. \tag{3.4}$$

Let $x/G \geq 0$ be the lowest bid that is part of the symmetric mixed-strategy equilibrium.[8] By (ii) condition (3.4) holds with equality. Using (A), we have

$$\frac{1}{2}\bar{\mu}^G_{2x} + \sum_{j=x+1}^{\bar{x}} \bar{\mu}^G_{2j}\Psi_1\left(\frac{x}{G}, \frac{j}{G}\right) = \frac{1}{V}\left[E\Pi_1\left(\bar{\mu}^G\right) + \frac{x}{G}\right]. \tag{3.5}$$

For $x+1$ condition (3.4) becomes

$$\Psi_1\left(\frac{x+1}{G}, \frac{x}{G}\right)\bar{\mu}^G_{2x} + \frac{1}{2}\bar{\mu}^G_{2(x+1)} + \sum_{j=x+2}^{\bar{x}} \bar{\mu}^G_{2j}\Psi_1\left(\frac{x}{G}, \frac{j}{G}\right) \leq \frac{E\Pi_1\left(\bar{\mu}^G\right)}{V} + \frac{x+1}{VG}. \tag{3.6}$$

Computing $\bar{\mu}^G_{2x}$ from equation (3.5) and substitution in inequality (3.6) yields

$$\begin{aligned}
&\left(\tfrac{1}{2} - 2\Psi_1\left(\tfrac{x+1}{G}, \tfrac{x}{G}\right)\Psi_1\left(\tfrac{x}{G}, \tfrac{x+1}{G}\right)\right)\bar{\mu}^G_{2(x+1)} + \\
&+ \left(\Psi_1\left(\tfrac{x+1}{G}, \tfrac{x+2}{G}\right) - 2\Psi_1\left(\tfrac{x+1}{G}, \tfrac{x}{G}\right)\Psi_1\left(\tfrac{x}{G}, \tfrac{x+2}{G}\right)\right)\bar{\mu}^G_{2(x+2)} + \\
&+ \ldots + \left(\Psi_1\left(\tfrac{x+1}{G}, \tfrac{\bar{x}}{G}\right) - 2\Psi_1\left(\tfrac{x+1}{G}, \tfrac{x}{G}\right)\Psi_1\left(\tfrac{x}{G}, \tfrac{\bar{x}}{G}\right)\right)\bar{\mu}^G_{2\bar{x}} \leq \\
&\leq \tfrac{1}{V}\left[E\Pi_1\left(\bar{\mu}^G\right) + \tfrac{x+1}{G} - 2\left(E\Pi_1\left(\bar{\mu}^G\right) + \tfrac{x}{G}\right)\Psi_1\left(\tfrac{x+1}{G}, \tfrac{x}{G}\right)\right] = \\
&= \tfrac{1}{V}\left[\tfrac{1}{G}\left(x + 1 - 2x\Psi_1\left(\tfrac{x+1}{G}, \tfrac{x}{G}\right)\right) - E\Pi_1\left(\bar{\mu}^G\right)\left(2\Psi_1\left(\tfrac{x+1}{G}, \tfrac{x}{G}\right) - 1\right)\right].
\end{aligned} \tag{3.7}$$

Note that (SM) implies that every term on the left hand side, LHS from now on, of condition (3.7) is non-negative. Suppose, by way of contradiction, that $E\Pi_1\left(\bar{\mu}^G\right) > 1/G$. The RHS of condition (3.7) is strictly smaller than

$$\frac{1}{VG}\left(x + 1 - 2x\Psi_1\left(\frac{x+1}{G}, \frac{x}{G}\right) - 2\Psi_1\left(\frac{x+1}{G}, \frac{x}{G}\right) + 1\right).$$

Under (SD) this expression is smaller than zero, a contradiction.
(2) We have that in a symmetric equilibrium $E\Pi_i(\bar{\mu}^G) = V\Pr\{B_i\ wins\} - E(\bar{\mu}^G_i)$. Summing up for both agents gives $2E\Pi_1\left(\bar{\mu}^G\right) = V[\Pr\{B_1\ wins\} + \Pr\{B_2\ wins\}] - 2E(\bar{\mu}^G_1)$ and rearranging yields the statement. ■

Lemma 3.6. *Let $\mathcal{C}^S$ be a (continuous) 2-bidder contest with common value $\tilde{V}$. Let $\mathcal{C}^A$ be the same contest with asymmetric valuations $V_1 \geq V_2 = \tilde{V}$. If $\mu^* = (\mu^*_1, \mu^*_2)$ is a symmetric (possibly mixed) Nash equilibrium strategy profile to $\mathcal{C}^S$ in which the rent is completely dissipated (in expectation), then the following strategy profile $\upsilon^* = (\upsilon^*_1, \upsilon^*_2)$ constitutes a Nash equilibrium to $\mathcal{C}^A$:*

[8] I.e., $\bar{\mu}^G_{1x} > 0$, and $\bar{\mu}^G_{1j} = 0$ for all $j < x$.

- *Contestant B_1 bids $\upsilon_1^* = \mu_1^*$ and*
- *contestant B_2's strategy υ_2^* is such that she abstains from the contest with probability $(1 - V_2/V_1)$ and bids μ_2^* whenever she participates.*

Proof. Note first that in $\mathcal{C}^S$ the complete dissipation of rents implies that $\tilde{V} = E(\mu_1^*) + E(\mu_2^*)$. Since the equilibrium is symmetric, we have $E(\mu_i^*) = \tilde{V}/2,\ i \in \{1, 2\}$. The symmetry of the game assures that on average each player wins half of the times and, thus, in $\mathcal{C}^S$ we have $E\Pi_i(\mu^*) = 0,\ i \in \{1, 2\}$.

To see that in $\mathcal{C}^A$ contestant B_2 has no profitable deviation from υ_2^*, note that, since $\upsilon_1^* = \mu_1^*$ and $V_2 = \tilde{V}$ is the same in $\mathcal{C}^S$ and $\mathcal{C}^A$, any pure strategy in $\mathcal{C}^A$ yields the same as in $\mathcal{C}^S$ and B_2 obtains $E\Pi_2(\upsilon^*) = 0$. She is, hence, willing to abstain with probability $(1 - V_2/V_1)$.

For B_1 note that in $\mathcal{C}^S$, given the mixed-strategy μ^* by B_2, all pure strategies b_1 in the support of μ^* maximize

$$E\Pi_1(b_1, \mu^*) = \tilde{V} E[\Pr\{B_1 \text{ wins}|b_1, \Psi, \mu^*\}] - b_1, \tag{3.8}$$

where $E[\Pr\{B_1 \text{ wins}|b_1, \Psi, \mu^*\}]$ is B_1's expected win probability from the pure strategy b_1 when the CSF is Ψ and B_2 mixes according to the equilibrium strategy μ^*. Note that, although we do not know whether μ^* is a continuous, discrete, or partially continuous and discrete distribution, the following must be true. For any constant A, any b_1 which is a maximizer of (3.8) is also a maximizer of

$$A + \tilde{V} E[\Pr\{B_1 \text{ wins}|b_1, \Psi_1, \mu^*\}] - b_1. \tag{3.9}$$

The proof is completed by noticing that (3.9) with $A = (1 - V_2/V_1)V_1$ is the payoff of the pure strategy b_1 in $\mathcal{C}^A$, with $\upsilon_2^* = \mu^*$ conditional on entry.[9] ■

We are now in a position to prove Theorem 3.3.

Proof of Theorem 3.3. Suppose $V_1 \geq V_2 \geq \cdots \geq V_n$. and that the contest success function satisfies (A), (DS), (SD) and (SM). We show the existence of an all-pay auction equilibrium by construction.

Suppose there were two contestants with common value V_2. Lemma 2.3 and successively Lemma 3.5 can be applied. This establishes the existence of a symmetric mixed-strategy equilibrium in which the rent (in expectation) is completely dissipated because both contestants bid (in expectation) $V_2/2$. Application of Lemma 3.6 allows to conclude that in any two-bidder contest without common value, say, $V_1 \geq V_2$ there exists equilibrium strategies for B_1 and B_2 with the properties specified in Definition 3.1. Assume there are further bidders with valuations lower or equal to V_2. These contestants

[9] This is because $\tilde{V} = V_2$, and V_1 cancels.

B_j with $j > 2$ cannot do better than bidding zero and obtain expected payoffs of zero. To see this take any pure strategy b'. Given μ_1^*, contestant B_2 obtains $E\Pi_2(\mu_1^*, b') \leq 0$ in the two contestants game. By (A), we have that $E\Pi_i(\mu_1^*, \mu_2^*, b') = E\Pi_2(\mu_1^*, b', \mu_2^*)$; and by Condition (2.2), $E\Pi_2(\mu_1^*, b', \mu_2^*) \leq E\Pi_2(\mu_1^*, b')$. The expected bids imply the expressions for expected equilibrium payoffs and revenue in the statement of Theorem 3.3. ■

4. Applications

In this section we apply Theorem 3.3 to specific contests, mainly by checking conditions (SD) and (SM). This shows the practical applicability of these conditions. We start by verifying existing results for the deterministic all-pay auction and the Serial Contest. We turn then to the derivation of new results. Of particular interest is here Tullock's Rent-Seeking Game. Although the hypothesis of Theorem 3.3 does not include homogeneity of the CSF, we focus on this class of CSFs, because of its relevance for applications.[10]

4.1. The Deterministic All-Pay Auction

It is instructive to start with the deterministic all-pay auction and derive an equilibrium to this game without using existing results. It is straightforward to see that the deterministic all-pay auction satisfies the hypothesis of Theorem 3.3. Now Theorem 3.3 says that this game has an all-pay auction equilibrium. This is indeed the case; for this is a well established result (Hillman and Riley (1989) and Baye *et al.* (1996)). Note that this equilibrium is unique when $V_2 > V_3$ (see Remark 3.2). Thus, we conclude:

Proposition 4.1. *The deterministic all-pay auction has an all-pay auction equilibrium.*

4.2. The Serial Contest

One way to relax the extreme requirement of the deterministic all-pay auction that the highest bidder wins the contest with probability one, is through the Serial CSF (Alcalde and Dahm (2007)). Without loss of generality suppose that the vector of bids is ordered such that $b_1 \geq b_2 \geq ... \geq b_n$.[11] Given a scalar $R > 0$, the serial CSF assigns

$$\Psi_i^S(b) = \sum_{j=i}^{n} \frac{b_j^R - b_{j+1}^R}{j \cdot b_1^R} \text{ for all } B_i \in \mathcal{B}, \tag{4.1}$$

[10] One interpretation of homogeneity of degree zero is that it does not matter whether lobbying expenditures are measured in dollars or in euros. See also the further discussion in Malueg and Yates (2006).

[11] If necessary relabel the set of bidders.

with $b_{n+1} = 0$.

Using homogeneity, condition (SD) becomes

$$\frac{x+2}{2(x+1)} \leq 1 - \frac{1}{2}\left(\frac{x}{x+1}\right)^R \Leftrightarrow \left(\frac{x}{x+1}\right)^{R-1} \leq 1, \tag{4.2}$$

which holds for all $R \geq 1$. On the other hand, (SM) can be written as

$$1 - 2\left(\frac{x}{x+1}\right)^R + \left[\left(\frac{x}{x+1}\right)^R\right]^2 \geq 0, \tag{4.3}$$

which is true for all $R \geq 0$. Summarizing, we have the following.

Proposition 4.2. *For $R \geq 1$, the Serial Contest has an all-pay auction equilibrium.*

4.3. Tullock's Rent-Seeking Game

Tullock's CSF is defined as in equation (1.1). Again, using homogeneity, conditions (SD) and (SM) simplify. The former becomes

$$\frac{x+2}{2(x+1)} \leq \frac{(x+1)^R}{(x+1)^R + x^R} \Leftrightarrow (x+2)x^R \leq x(x+1)^R, \tag{4.4}$$

which is fulfilled for $x = 0$. For $x > 0$, (following Baye *et al.* (1994), p. 379) we obtain

$$\frac{x+2}{(x+1)} \leq \left(\frac{x+1}{x}\right)^{R-1} \Leftrightarrow 1 + \frac{1}{x+1} \leq \left(1 + \frac{1}{x}\right)\left(1 + \frac{1}{x}\right)^{R-2}.$$

This holds for $R \geq 2$. The latter condition (SM) can be written as

$$(x^R + j^R)((x+1)^R + x^R) \geq ((x+1)^R + j^R)2x^R, \tag{4.5}$$

which is true for all $R \geq 0$. We have proved the following result.

Proposition 4.3. *For $R \geq 2$, Tullock's Rent-Seeking Game has an all-pay auction equilibrium.*

Although the explicit derivation of the equilibrium mixed-strategies is beyond the scope of the present paper, we conclude this subsection computing four examples of the symmetric two-bidder Tullock's Rent-Seeking Game with a finite strategy space. We represent the cases of R equal to 2, 3, 5 and ∞ with a grid of $G = 11$ in Figure 4.1. The computations suggest that, as the returns to scale increase, the bulk of probability mass shifts to the right and some mass is attached to low bids. As R increases further, μ^* becomes more and more uniformly distributed, which is the optimal bidding strategy in the all-pay auction.[12]

[12] Due to the finiteness, contestants obtain very low but strictly positive expected profits (smaller than 0.06). Moreover, the expected bid – even of the discrete all-pay auction – is strictly lower than

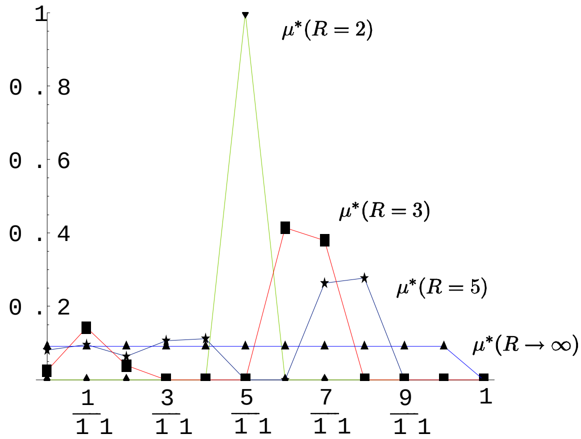

Figure 4.1: $Q = 11$ and $R \in \{2, 3, 5, \infty\}$

4.4. Combining Tullock's and the Serial Contest

Consider a contest administrator who wants to design a contest that has properties of both Tullock's Rent-Seeking Game and the Serial Contest. In the γ-TS contest win probabilities are assigned following

$$\Psi_i^{TS}(b) = \gamma \Psi_i^T(b) + (1-\gamma)\Psi_i^S(b), \tag{4.6}$$

with $\gamma \in [0, 1]$ and for all $B_i \in \mathcal{B}$.

Using homogeneity, condition (SD) becomes

$$\frac{x+2}{2(x+1)} \leq \gamma \frac{(x+1)^R}{(x+1)^R + x^R} + (1-\gamma)\left[1 - \frac{1}{2}\left(\frac{x}{x+1}\right)^R\right],$$

which we know holds for $R \geq 2$ because then (4.4) and (4.2) hold. Condition (SM) can be written as

$$\frac{1}{2}\frac{\gamma\frac{(x+1)^R}{(x+1)^R+j^R} + (1-\gamma)\frac{1}{2}\left(\frac{x+1}{j}\right)^R}{\gamma\frac{x^R}{x^R+j^R} + (1-\gamma)\frac{1}{2}\left(\frac{x}{j}\right)^R} \geq \gamma\frac{(x+1)^R}{(x+1)^R+x^R} + (1-\gamma)\left[1 - \frac{1}{2}\left(\frac{x}{x+1}\right)^R\right].$$

Notice that only the LHS depends on j. Furthermore, this expression can be shown to be increasing in j. Thus, it suffices to verify (SM) for $j = x + 1$. In this case (SM)

0.5 (but larger than 0.44). Baye *et al.* (1994) have shown that in the two player case the symmetric equilibrium of the discrete all-pay auction converges to the unique equilibrium of the continuous strategy space all-pay auction.

becomes

$$\frac{1}{4} \geq \gamma^2 \frac{(x+1)^R}{(x+1)^R + x^R} \frac{x^R}{x^R + (x+1)^R} + (1-\gamma)^2 \frac{1}{2}\left(\frac{x}{x+1}\right)^R \left[1 - \frac{1}{2}\left(\frac{x}{x+1}\right)^R\right]$$
$$+\gamma(1-\gamma)\left\{\left[1 - \frac{1}{2}\left(\frac{x}{x+1}\right)^R\right] \frac{x^R}{x^R + (x+1)^R} + \frac{1}{2}\left(\frac{x}{x+1}\right)^R \frac{(x+1)^R}{(x+1)^R + x^R}\right\}.$$

Given that (4.5) and (4.3) hold it is enough to show that

$$\frac{1}{2} \geq \left[1 - \frac{1}{2}\left(\frac{x}{x+1}\right)^R\right] \frac{x^R}{x^R + (x+1)^R} + \frac{1}{2}\left(\frac{x}{x+1}\right)^R \frac{(x+1)^R}{(x+1)^R + x^R},$$

which is true for all $R \geq 0$. We obtain, hence, the following.

Proposition 4.4. *For $R \geq 2$ and for all $\gamma \in [0, 1]$, the γ-TS contest has an all-pay auction equilibrium.*

Notice that this result may be interpreted as saying that the all-pay auction equilibrium is very robust. Given that the same equilibrium exists for any combination of the two contests –provided that the contest is deterministic enough ($R \geq 2$)– the model builder does not really have to decide which model is more realistic.

4.5. The Serial Contest with Spillover Effects

Note that instead of the power function $f_i(b) = b_i^R$ any homogenous production function for lotteries might be combined with the basic functional form of either Tullock's logit structure or the serial formulation in order to generate another homogeneous CSF.

Consider, for example, the case in which effort represents advertising. Malueg and Yates (2006) introduce a CES production function in order to capture such a setting. Here a contestant's success depends on her private effort b_i (her own advertising). But there might be also a public aspect or spillover effect of effort (e.g. increased consumer awareness of the product generated through rivals' advertisements). The following is a variation of the production function introduced in Malueg and Yates

$$f_i(b) = \left(ab_i^R + cb_i^T \sum_{k \neq i} b_k^{R-T}\right)^{1/S}, \text{ for all } B_i \in \mathcal{B}, \tag{4.7}$$

where $a \geq c \geq 0$, $R > T \geq 0$ and $S > 0$. Note that if $c = 0$, then (4.7) reduces to the classical power function. Notice also that for $T = 0$, we obtain the exact expression used by Malueg and Yates. In this case a contestant who exerts no effort might still have a positive probability of winning. When $T > 0$, a positive win probability requires non-zero effort.

With this we can define e.g. the Serial Contest with spillover effects in which win probabilities are assigned following

$$\Psi_i^{SS}(b) = \sum_{j=i}^{n} \frac{f_j(b) - f_{j+1}(b)}{j \cdot f_1(b)} \text{ for all } B_i \in \mathcal{B}, \tag{4.8}$$

where $f_i(b)$ is defined as in (4.7) and $f_{n+1}(b) = 0$.

Consider the following simple example in which $R = 3$ and $S = T = a = c = 1$. We obtain

$$f_i(b) = b_i \left(b_i^2 + \sum_{k \neq i} b_k^2 \right), i = 1, ..., n.$$

By homogeneity, condition (SD) requires

$$\frac{x+2}{2(x+1)} \leq 1 - \frac{x^3 + x(x+1)^2}{2((x+1)^3 + x^2(x+1))} \Leftrightarrow \frac{x^3 + x(x+1)^2}{2((x+1)^3 + x^2(x+1))} \leq \frac{x}{2(x+1)}.$$

Straightforward manipulation shows that this is true. For (SM), we have that

$$\begin{aligned}
\Psi_1\left(\frac{x+1}{G}, \frac{j}{G}\right) &= \frac{(x+1)^3 + (x+1)j^2}{2(j^3 + (x+1)^2 j)}; \\
\Psi_1\left(\frac{x}{G}, \frac{j}{G}\right) &= \frac{x^3 + xj^2}{2(j^3 + x^2 j)}; \text{ and} \\
\Psi_1\left(\frac{x+1}{G}, \frac{x}{G}\right) &= 1 - \frac{x^3 + x(x+1)^2}{2((x+1)^3 + x^2(x+1))}.
\end{aligned}$$

Thus, it must hold that

$$\frac{(x+1)^3 + (x+1)j^2}{2(j^3 + (x+1)^2 j)} \geq 2\left(1 - \frac{x^3 + x(x+1)^2}{2((x+1)^3 + x^2(x+1))}\right) \frac{x^3 + xj^2}{2(j^3 + x^2 j)},$$

which is true if $x = 0$. For $x > 0$, it is required that

$$\frac{\left((x+1)^3 + (x+1)j^2\right)\left(j^3 + x^2 j\right)}{\left(j^3 + (x+1)^2 j\right)\left(x^3 + xj^2\right)} \geq \frac{2((x+1)^3 + x^2(x+1)) - \left(x^3 + x(x+1)^2\right)}{((x+1)^3 + x^2(x+1))}.$$

Expanding terms yields the condition

$$j^5\left(2x + 2x^2 + 1\right) + j^3\left(4x + 8x^2 + 8x^3 + 4x^4 + 1\right) + j\left(x^2 + 4x^3 + 7x^4 + 6x^5 + 2x^6\right) \geq 0.$$

We have, hence, shown the following.

Proposition 4.5. *In the example in which $R = 3$ and $S = T = a = c = 1$, the Serial Contest with spillover effects has an all-pay auction equilibrium.*

5. Discussion

The present paper has offered a robustness analysis of the predictions of the deterministic all-pay auction. In this auction the highest bidder always wins with probability one. We have analyzed non-deterministic contests which respond to different degrees to the highest bid when assigning the prize. This setting includes –but is not limited to– the popular Tullock's Rent-Seeking Game. Our model is quite general because we did not suppose the existence of a common value and we did not restrict the number of contestants. Not surprisingly, our main result can be interpreted as saying that if the contest is 'not too far away' from the polar case of the all-pay auction, it admits essentially the same equilibrium. This is an important result as it implies that conclusions of models that embed an all-pay auction in a larger model are robust to changes in the contest structure.

However, it is somewhat surprising that the contest can be 'quite far away' from the polar case and that there are different mathematical formulations through which one might depart from the deterministic case. These conclusions follow from the application of our main result to specific contests. A by-product of our analysis here is to provide an equilibrium to Tullock's Rent-Seeking Game for increasing returns to scale larger than two, for any number of contestants, and for any valuations for the political prize the contestants might have. But future research concerning this contest in the general setting is still needed in order to determine an equilibrium for $R \in (n/(n-1), 2)$, to derive the explicit equilibrium strategies and to determine the complete set of equilibria.[13]

The question of robustness of the predictions of the deterministic case is important because the polar case has important properties –some of which are known to be not fulfilled when the contest is non-deterministic enough, say, in Tullock's Rent-Seeking Game with $R = 1$ (see Che and Gale (2000) or Fang (2002)). This refers to properties concerning incentives for more than two agents to participate in the contest, rent dissipation, exclusion principle (Baye et al. (1993)), and the preemption effect (Che and Gale (2000)). Our analysis implies that these properties are fulfilled in a wide range of non-deterministic contests.

[13] Note that our analysis implies that there are multiple equilibria. Lemma 2.3 implies that there is a symmetric equilibrium for, say, three contestants with common value. However, Theorem 3.3 establishes an asymmetric all-pay auction equilibrium. It seems, therefore, reasonable that results similar to the ones in Baye *et al.* (1996) can be derived.

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