EX 1

ARA group

1. Find the eigenvalues and eigenspaces of the following matrices and determine if they are diagonalizable:

   (a)
   $$\begin{pmatrix} 4 & 5 \\ -1 & -2 \end{pmatrix}$$

   (b)
   $$\begin{pmatrix} 1 & 0 \\ 1 & -2 \end{pmatrix}$$

2. Let A be invertible. Show that λ is an eigenvalue of T if and only if $\lambda \neq 0$ and λ^{-1} is an eigenvalue of A^{-1}.

3. Find the minimal polynomials of
   $$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$
   $$B = \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix}$$
   $$C = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

4. Find an $n \times n$ matrix with fundamental polynomial x^2.

5. Prove that A is invertible if and only if its fundamental polynomial has a nonzero constant term.

6. Let $A \in \mathbb{R}^{n \times n}$ have n distinct eigenvalues. Show that A is diagonalizable.

7. Find the possible eigenvalues of a matrix A such that $A^2 = A$.

8. Show that a 2×2 real symmetric matrix is diagonalizable..