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2D Green's tensor for the analysis of dielectric structures with translational geometries

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Summary: In this work we present simulations of dielectric structures based on the volume integral method. The formalism is based on the solution of Helmholtz equation using the Green's tensor technique. By using this technique, the second order differential equation (Helmholtz equation) is transformed into an implicit integral equation. In this work the two dimensional Green tensor will be derived in order to study dielectric structures with translational symmetry. The method is firstly compared with other methods existing in the literature for homogeneous circular dielectric cylinders in order to test the validity of the method proposed. Then numerical simulations of other structures such as dielectric cylinders with elliptical base are performed.

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Keywords: Scattering, Green's tensor, Maxwell equations

1. Introduction

Electromagnetic scattering [1] is a complex process that depends mainly on two parameters: the dielectric properties of the surface (dielectric permittivity, conductivity, etc.) and its geometric structure. There are no analytical solutions for materials with arbitrary shapes; the equations that govern this process can only be solved for some very specific cases; but there are many numerical techniques to simulate a dielectric non-inhomogeneous material [2] such as the finite element method, the finite difference method, the method of lines, the method of moments, etc. In our case, we have chosen a volume integral method based on the use of a Green's tensor [3]. This method has been largely applied, for instance in the study of the scattering of particles of different size and form. A modified version of the method is the so-called discrete dipole method, which has successfully applied to the scattering of particles of different sizes [4]. In this work we obtain the two dimensional Green tensor in order to analyze structures with translational symmetry. The method is firstly compared with other methods existing in the literature for homogeneous circular dielectric cylinders in order to test the validity of the method proposed. Then numerical simulations of other structures such as dielectric cylinders with elliptical base are performed.

2. Theory

2.1. General equations

Let's consider a dielectric medium with a spatially varying dielectric permittivity, $\varepsilon(\boldsymbol{r})$. If this medium is illuminated with an incident monochromatic field $\boldsymbol{E}^0(\boldsymbol{r})$, the total electric field is a solution of the equation:

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) = k_0^2 \varepsilon(\boldsymbol{r}) \boldsymbol{E}(\boldsymbol{r}), \quad (1)$$

where k_0 is the wavenumber. Introducing the so called dielectric contrast:

$$\Delta\varepsilon(\boldsymbol{r}) = \varepsilon(\boldsymbol{r}) - \varepsilon_B, \quad (2)$$

ε_B is the average dilectric permittivity of the medium. Equation (1) can be expressed now as:

$$\nabla \times \nabla \times \boldsymbol{E}(\boldsymbol{r}) - k_0^2 \varepsilon_B \boldsymbol{E}(\boldsymbol{r}) = k_0^2 \Delta\varepsilon(\boldsymbol{r}) \boldsymbol{E}(\boldsymbol{r}), \quad (3)$$

where the incident field $\boldsymbol{E}^0(\boldsymbol{r})$, must be a solution of the corresponding homogeneous equation. The Green tensor associated with the infinite medium $\boldsymbol{G}^B(\boldsymbol{r}, \boldsymbol{r}')$ is a tensor of order 2, which is defined as the solution of the wave equation for a point source. In terms of the Green tensor the solution is:

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}^0(\boldsymbol{r}) + \int_\Omega d\boldsymbol{r}' \, \boldsymbol{G}^B(\boldsymbol{r}, \boldsymbol{r}') k_0^2 \Delta\varepsilon(\boldsymbol{r}) \boldsymbol{E}(\boldsymbol{r}'), \quad (4)$$

where Ω denotes all the volume where scattering can occurs.

2.1. Discretization of the equations

To solve equation (4), first we are going to define a mesh with N volume elements. Each point of the mesh i = 1,2, ..., N is centered on the position $\boldsymbol{r}_i$ and has a volume V_i (in two-dimensional systems this volume is refers to the area). The electric field is discretized as $\boldsymbol{E}_i = \boldsymbol{E}(\boldsymbol{r}_i)$, the dielectric contrast $\Delta\varepsilon_i = \Delta\varepsilon(\boldsymbol{r}_i)$ and the Green tensor $\boldsymbol{G}^B_{ij} = \boldsymbol{G}^B(\boldsymbol{r}_i, \boldsymbol{r}_j)$. With all these elements, the equation (4) becomes:

$$\boldsymbol{E}_i = \boldsymbol{E}^0{}_i \sum_{\substack{j=1 \\ j\neq i}}^{N} \boldsymbol{G}^B{}_{ij} k_0^2 \Delta\varepsilon_j \boldsymbol{E}_j V_j - \boldsymbol{L}\frac{\Delta\varepsilon_i}{\varepsilon_B}\boldsymbol{E}_i + \boldsymbol{M}_i k_0^2 \Delta\varepsilon_i \boldsymbol{E}_i, \tag{5}$$

where

$$\boldsymbol{M}_i = \boldsymbol{E}^0(\boldsymbol{r}) + \lim_{\delta V\to 0} \int_{V_i-\delta V} d\boldsymbol{r}'\, \boldsymbol{G}^B(\boldsymbol{r}_i, \boldsymbol{r}'), \tag{6}$$

2.3. 2-dimensional Green tensor

For a 2-dimensional the Green's tensor can be calculated from a scalar function $g^B_{2D}(\boldsymbol{r}, \boldsymbol{r}')$ as:

$$\boldsymbol{G}^B(\boldsymbol{r}, \boldsymbol{r}') = \left(1 + \frac{\nabla\nabla}{k_B^2}\right) g^B_{2D}(\boldsymbol{r}, \boldsymbol{r}') \tag{7}$$

where $g^B_{2D}(\boldsymbol{r}, \boldsymbol{r}')$ is:

$$g^B_{2D}(\boldsymbol{r}, \boldsymbol{r}') = \frac{i}{4} H_0(k_\rho \rho) \exp(i k_z z) \tag{8}$$

3. Results and discussion

In this part, the technique based on Green's tensor will be compared to other formalisms such as the Fresnel-Kirchhoff method [5] and analytical solution of Maxwell equations for a circular dielectric cylinder [1]. We will suppose that the cylinder is illuminated with a plane wave linearly polarized in the direction of the axes of revolution of the cylinder, with a wavelength of 633 nm. The scattered field is evaluated at plane located 20 mm far away from the cylinder. The radius of the cylinder was of 20 mm, whereas the refractive index was assumed 1.5.

In figure 1 the intensity of the field is represented as function of the horizontal distance to the center of the diffraction pattern. Good agreement between the three methods is observed.

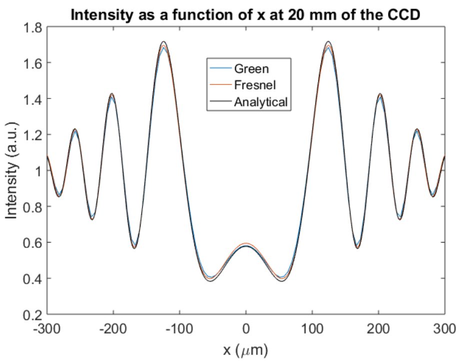

Fig. 1. Intensity as a function of the distance to the center of the diffraction pattern for a circular cylinder

Once validated the Green's tensor technique, the scattering of any structure with translational symmetry can be analyzed. Figure 2 shows the total field for an elliptical dielectric cylinder. The semi-major axis of the ellipse is 30 µm and the semi-minor axis 10 µm. The refractive index is assumed 1.5.

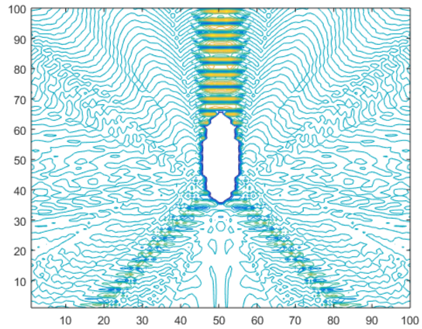

Fig. 2. Intensity of the electric field for an elliptical cylinder

4. Conclusions

In this work, the formalism based on the Green's tensor technique to solve Helmholtz equation is discussed for 2D geometries. The comparison of the method with the Fresnel-Kirchhoff method and the analytical solution of Maxwell equations for a dielectric circular cylinder demonstrate that there is good agreement with the results obtained in the three cases. This permits ananlysing other structures such as a dielectric cylinder with an elliptical base.

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