INITIATING A PROJECT FOR LANGUAGE-AND-LEARNER RESPONSIVENESS IN MATHEMATICS CONTENT TEACHING

Núria Planas, José María Alfonso, Juan Gabriel Rave-Agudelo

Autonomous University of Barcelona

Although our community has come to know that language is an important resource for mathematics teaching and learning, there is a less fine-grained understanding of how developmental work with mathematics teachers can be designed to support content teacher talk that is language-and-learner responsive. In this report, we first discuss our theoretical framework with the tools of naming and lexicalization, interpreted as uses in classroom teacher talk of content-related word names and explanatory sentences with the potential to reduce specific learning difficulties. We then change the emphasis to explore challenges of thinking a version of the framework and tools for work with mathematics teachers in order to inform their decisions as to how and why selected names and/into explanations can be particularly responsive in content teaching.

OUR RESEARCH AND DEVELOPMENTAL PROJECT

The path from research findings and theories about mathematics teaching and language to practical proposals of developmental work with mathematics teachers on content teaching, or even vice versa, is not straightforward. Any formulation of how to traverse this path is problematic because theoretical tools may not be directly applicable or easily understandable in the developmental site. In the domain of mathematics teacher development, the common aspiration of producing research that can impact on the professional learning and knowledge of teachers, and ultimately on content teaching practices in classrooms, is nonetheless essential and remains a driving force (Adler, 2021). As we share this aspiration, we continue to draw on sociocultural approaches to language and mathematics (e.g., Pournara, Adler, Pillay, & Hodgen, 2015; Planas, Morgan, & Schütte, 2021) in our pursuit of content teaching that is language-and-learner responsive, or grounded on the provision of talk in the interaction with learners that is mathematically focused and responds to learning demands and challenges.

In this report, we present two theoretical tools in construction —*naming* and *lexicalization*—, and examine their potential and nuances for use in developmental work on language-and-learner responsive content teaching with mathematics teachers. These tools and the framework they conform were implemented implicitly rather than explicitly in a pilot intervention study with two secondary school teachers on the exploration of mathematical languages at the levels of word names and explanatory sentences for the teaching of algebraic concepts (Planas, 2019, 2021). At its actual stage of conceptualization, the framework and the tools reveal strong theoretical and practical interest. Whilst it is relatively uncomplicated to identify the potential of the

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.). *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 315-322). PME.

theoretical tools for reflection on specialized meaning making in language, it is not clear-cut the process towards specifying how to introduce them to teachers in ways that are not too highly conceptual to be practical for them, and that enhance content mathematics teaching aimed at the reduction of school learners' challenges.

Following this introduction, the report is structured to discuss two questions. In the first section, we discuss: 1) How do the theoretical tools of naming and lexicalization relate to the study of content teacher talk? In the second section, we discuss: 2) How can they be reinterpreted into developmental tools for work towards language-and-learner responsiveness in mathematics content teaching? We finish with some remarks about possibilities of continuing the refinement and expansion of the framework.

THEORETICAL TOOLS FOR THE STUDY OF TEACHER TALK

Our framework for the study of teacher content talk started to unravel backed up by intensive revision of literature on language and mathematics teaching and specifically grounded on Halliday's functional grammar (1985). Without diminishing the importance of nonverbal and paralinguistic tools in language, the intention was to increase the understanding of mathematical meaning making enhanced at the levels of words and sentences in classroom teacher talk. Earlier field research on mathematical meaning making in classroom talk (e.g. Pimm, 1987; Schleppegrell, 2007) already suggested the study of dense noun phrases, being and having verbs, conjunctions with technical meaning or logical connectors, which all fit into our focus on words and sentences. Today, analyses of mathematics teacher talk often privilege the study of conversational patterns and communicational moves and, when mathematical content specificity is also addressed, words and sentences tend to be studied in general terms and subsumed to, instead of interacting with, the broader discourse level. Instances of words and sentences are often illustrated and said to be mathematically and pedagogically relevant but the criteria of relevance are not detailed or focused on.

The interconnected distinction in Halliday (1985) between the linguistic forms in a language and their functions to produce situated meaning expresses the diverse ways by which words, sentences, and discourses in a language system and an interactional situation are lexically elaborated to communicate meaning (Morgan, 2021). Alongside the study of discourses or larger language units over isolated words, words into sentences, and sentences, meaning making crucially develops at granular linguistic levels. In this regard, the experiences of teachers and learners in classroom content teaching and learning are subject to the complexity of using words into/and sentences to communicate some meanings considered as (more) appropriate amongst all those possibly lexicalized —i.e., encoded with precise meaning— in the interaction and the language system. In Halliday (1978, p. 195), a register is precisely, "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings." In the mathematics classroom, the forms used to encode meanings within a mathematical content register may also bring with them less appropriate or unintended meanings. In order to address this complexity, we

examined two tools in language; one at the level of the set of words or lexicon in a language system, the other at the level of further lexical elaboration into sentences:

- *Naming* or giving word names from mathematical content registers
- *Lexicalization* or giving sentences with encoded explanations of mathematical content meaning

In this version of the tools, responsiveness in teacher talk is basically a function of content specificity through the use of content-related names and explanations. In the current more advanced version, responsiveness in teacher talk is a function of content and learning specificity through the use of content-related names and explanations aimed at supporting content learning challenges (see Figure 1). The refinement to strengthen the emphasis on learner responsiveness has therefore led to:

- *Naming* or giving word names from mathematical content registers oriented to reduce content learning challenges
- *Lexicalization* or giving sentences with encoded explanations of mathematical content meaning oriented to reduce content learning challenges

Language-and-learner responsiveness in this way emphasizes the learning goal without losing the focus on curricular content demands. It connects mathematical meaning making in content teaching to mathematical meaning misunderstood or overlooked by learners. If we think of the teaching of fractions, for example, language-and-learner responsive names and explanations would address and challenge field-documented learning misunderstandings such as the common belief that the parts of the continuous whole are equal-shape (Darrough, 2015). Equal-size and (non)equal-shape would be instances of naming within the fraction register, and the equal-size parts of a continuous whole are not always equal-shape would be an instance of lexicalization. If we consider the teaching of angles, in a lesson with dynamic software where secondary school learners keep referring to angles as static bounded regions only (Mitchelmore & White, 2000), the rotation about a point also makes an angle, would be an instance of lexicalization including important names.

Language responsiveness and learner responsiveness are then different phenomena in content teacher talk with special connection between them. Language responsiveness in content teaching exists as soon as the language of mathematics is made explicit and public at the levels of words, sentences, and discourse, although it does not necessarily address the needs or demands of learners in content learning. Learner responsiveness therefore involves language responsiveness, but the converse cannot be argued.

CONTENT REGISTER	*Theoretical refinement*
	Studying language responsiveness
Words Sentences	Giving names from the content register Giving explanations of mathematical content meanings
Form-function	
	Studying language-and-learner responsiveness
Words Sentences	Giving names from the content register aimed at **reducing learning challenges** Giving explanations of mathematical content meanings aimed at **reducing learning challenges**

Figure 1: Successive versions of the theoretical framework.

TOWARDS A DEVELOPMENTAL VERSION OF THE FRAMEWORK

The reinterpretation of research tools for use in developmental practice implies shifts in meaning. This is the case with the reinterpretation in Adler (2021, p. 83) of "naming" —very close in meaning to our first version of naming— as *word use* in the teaching version of the Mathematics Discourse in Instruction frame. We rethink naming and lexicalization in mutually supportive ways, rather than treated separately, as *word names into/and explanatory sentences* (see Figure 2), whose communication in teacher talk can prevent or diminish learning challenges shown to be persistent across school ages, individual learners and classroom settings. Field research has actually documented numerous *reasoning biases* or tendencies of school learners to confirm and retain meanings, experiences and beliefs that do not conform or that enter in negative conflict with mathematical content. We have already mentioned biases in the thinking of: the fraction parts of the continuous whole as equal-shape (Darrington, 2014), and the angle as static bounded region only (Mitchelmore & White, 2000).

We assume that reasoning biases remain behind important content learning difficulties, and accordingly propose work on noticing processes (e.g., ZDM issue edited by Dindyal, Schack, Choy, & Sherin, 2021) with mathematics teachers towards:

- Knowing common *reasoning biases* of school learners, and considering their importance in mathematics content learning.
- Identifying, interpreting, and deciding on *names into/and explanations* for mathematics content teaching aimed at reducing biased reasoning.

In the progressive thinking of how to make operative the theoretical framework and tools (see Figure 2), the issue of how to produce knowledge-based *names into/and explanations* is crucial. The amount of mathematical meanings associated to each curricular content is enormous, and hence in the work with teachers some criteria must be given for the effective selection of some names and explanations over others.

Otherwise, the framework tools may remain too open to be fully useful or manageable. Although conditions posed to the choice of words and sentences can generally be read as limitations to creative teaching, conditions regarding the content learning challenges to be addressed positively relate to teacher talk of higher language-and-learner responsiveness. The attention to particular reasoning biases can especially help teachers to gain knowledge-based autonomy and to produce content-related names into/and explanations aimed at reducing or preventing the biases in play.

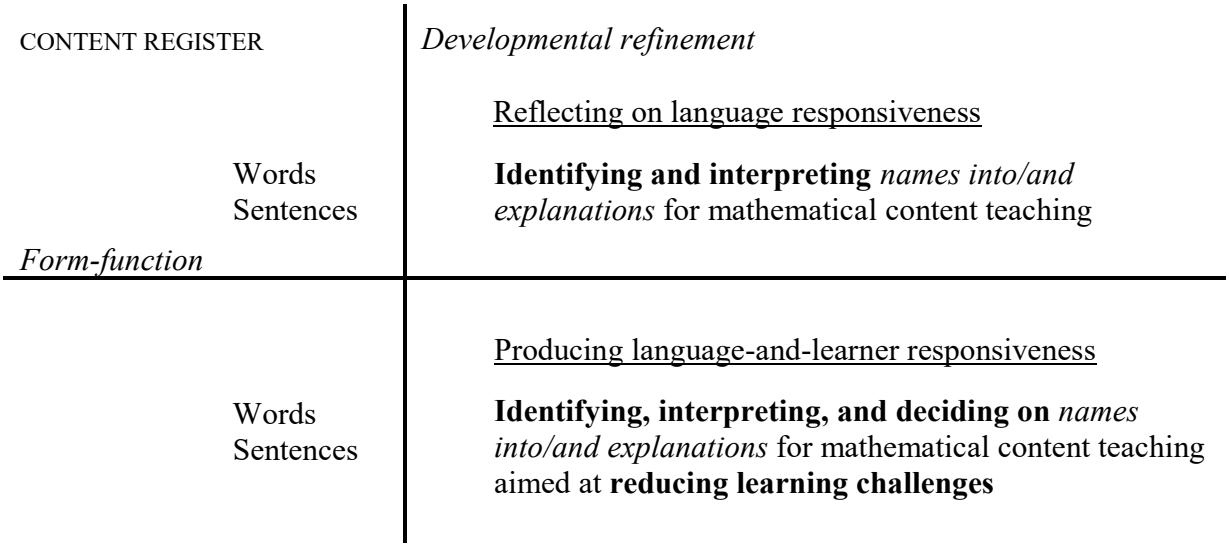

Figure 2: Successive versions of the developmental framework.

We cannot totally anticipate, accurately predict, or make a definite distinction of content teacher talk that will be learner-and-language responsive over the diverse interactional situations of a classroom lesson. Nonetheless, the curricular context and field-based knowledge can help to distinguish words and sentences which are expected to be responsive with respect to specific content learning demands and challenges.

In the upper secondary school classroom, for example, *the angle in between these lines measures one hundred and eighty degrees* is highly language-responsive, compared to *the angle in between these lines 'is' one hundred and eighty degrees*, or to *this is one hundred and eighty* (see Table 1). This explanatory sentence and the specialized names included, however, do not meet the particular challenge around the persistence of the static angle bias, compared to *the rotation from this line to this other line is half of a whole turn* —or to *the rotation about a point also makes an angle—.* Learner responsiveness makes these sentences qualitatively different (see Table 1). While all words and/into sentences in teacher content talk cannot be 'equally' responsive regarding particular registers and learning challenges, there must be some words and/into sentences offering opportunities for listening to specialized names and to explanations of mathematical meanings whose learning is possibly hindered by reasoning biases documented in field research as common and pervasive.

Teaching angles in the upper secondary school – **Static angle bias**			
Quality	Low	Medium	High
Language responsiveness	*This is one hundred and eighty*	*The angle in between these lines is one hundred and eighty degrees*	*The angle in between these lines measures one hundred and eighty degrees*
Learner responsiveness	*The angle in between these lines measures one hundred and eighty degrees*		*The rotation from this line to this other line is half of a whole turn*

Table 1: Examples of variability of responsiveness in teacher talk.

Although incorrect reasoning biases in a content domain are persistent in nature, and preventing, reducing or even eliminating them require the adoption of multiple directions, teachers need to develop the ability to identify, interpret, and decide on classroom talk that refers to, for example, the dynamic meaning for angle, or to the meaning of equal sizes of unequal shape for the fractional parts of a continuous whole.

One more example for developmental work would be the presentation to teachers of the equiprobability bias reasoning (Green, 1982), or the tendency of secondary school learners —but also younger and older learners— to believe that every process in which randomness is implied corresponds to a fair distribution, with equal probabilities for any possible outcome. Once the equiprobability bias was introduced and discussed, teachers would be able to notice that the probabilistic meaning of all the outcomes of an event being equally likely is not obvious or intuitive, or that semantic everyday associations operate in and interfere with the learners' thinking such as the physical meaning of equally likely or *physically equal*. The practice with them could then move towards identifying, interpreting, and deciding on talk for the communication of the probabilistic meanings encoded into names such as *equally likely* and into its distinction from *nonequally likely* in situations in which either A or B can occur, but one of them can be *most/more or least/less likely.* High responsive explanations to be considered would be: *They are all possible but five is the most likely outcome when you roll the die with the five painted twice*. In the project context of different intervention studies, we are engaged and making good progress in the production of materials (on fractions, angles, and probability teaching) for primary and secondary school teachers to gain knowledge on learners' specific reasoning biases, and professional noticing abilities at the levels of words and sentences within mathematical content registers.

MORE REFINEMENT, POSSIBLE EXTENSION

It is common to describe when the use of certain theoretical and practical constructs began in the literature, and then to draw on them, as if they were finished products, to conduct our investigations. In this report, we have addressed a framework in the middle of its conceptualization in research and developmental work with mathematics teachers

on language-and-learner responsive content talk in teaching. We have argued that to facilitate work with teachers it is necessary to clearly outline criteria for identifying, interpreting, and taking decisions on language-and-learner responsive languages of content teaching. When preparing and conducting developmental tasks around the teaching of a mathematical content and showing or asking for specialized word names and/into explanatory sentences, we thus need to provide criteria as to why these names and/into sentences can support the school learning of the content and meet learners' demands. We have proposed presenting to teachers well-documented content reasoning biases in order to guide their processes of noticing talk for content teaching.

Our theoretical and developmental project with mathematics teachers towards language-and-learner responsive content teaching remains unfinished in many respects. The realization of teacher talk, from the perspectives of explicit content teaching and reduction or prevention of learners' biased reasoning, requires further refinement and expansion work. The current framework integrates mathematics teaching that is responsive of mathematical content learning and mathematical language teaching with the sentence level linked to content-related explanations. Yet, this level can additionally be linked to examples or variations of content-related elements so that the following third tool in language is being examined:

- *Exemplification* or giving sentences with encoded variations of content-related elements oriented to reduce content learning challenges.

Furthermore, our project is grounded on the broader sociocultural interpretation of teacher talk as discourse, and hence on views that primarily focus on words and sentences once they are put to use or thought for use in situated communication. Rather than highlighted sporadically, the attention to word names and/into explanatory (and exemplifying) sentences should be blended and embedded in developmental work on mathematical discourse practices. While the tools of naming and lexicalization refer to discrete resources in the language system, our attention to these tools is shaped by social understandings of mathematical meaning making through participation in discourses that offer sustained opportunities of doing and talking mathematics. Regardless of strategic developmental orientations and analytical research decisions, there is not indeed a linear order in classroom practice from words to sentences, and from sentences to discourse, since mathematical meaning making is constructed and negotiated on a synergetic continuum across all levels of language.

By presenting the above-mentioned possibilities of extending the framework and of continuing the refinement of the theoretical and developmental tools, we hope to inspire other researchers to re-evaluate the importance of language-and-learner responsiveness in teacher talk, and perhaps to establish connections with their own frameworks for professional development on mathematical content teaching.

Acknowledgements

Research Project PID2019-104964GB-100 (Spanish Government), and Research Group GIPEAM 2017-SGR-101 (Catalan Government).

References

Adler, J. (2021). Content and context specificity matter in the 'how' of language-responsive mathematics teacher professional development. In N. Planas, C. Morgan, & M. Schütte (Eds.), *Classroom research on mathematics and language: Seeing learners and teachers differently* (pp. 77–100). Routledge.

Darrough, R. L. (2015). *Acquisition, utilization, and retention of foundational fraction concepts by middle grade students*. The University of Missouri at Columbia.

Dindyal, J., Schack, E. O., Choy, B. H., & Sherin, M. G. (2021). Exploring the terrains of mathematics teacher noticing. *ZDM-Mathematics Education, 53*, 1–16.

Green, D. R. (1982). *Probability concepts in 11-16 year old pupils*. Centre for Advancement of Mathematical Education in Technology, Loughborough University of Technology.

Halliday, M. A. K. (1978). *Language as social semiotic: The social interpretation of language and meaning*. Edward Arnold.

Halliday, M. A. K. (1985). *An introduction to functional grammar*. Edward Arnold.

Mitchelmore, M. C., & White, P. (2000). Development of angle concepts by progressive abstraction and generalisation. *Educational Studies in Mathematics, 41*, 209–238.

Morgan, C. (2021). Conceptualising and researching mathematics classrooms as sites of communication. In N. Planas, C. Morgan, & M. Schütte (Eds.), *Classroom research on mathematics and language: Seeing learners and teachers differently* (pp. 101–116). Routledge.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. Routledge & Kegan Paul.

Planas, N. (2019). Transition zones in mathematics education research for the development of language as resource. In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.), *Proceedings of PME43* (Vol. 1, pp. 17–31). PME.

Planas, N. (2021). How specific can language as resource become for the teaching of algebraic concepts. *ZDM-Mathematics Education, 53*, 277–288.

Planas, N., Morgan, C., & Schütte, M. (Eds.) (2021). *Classroom research on mathematics and language: Seeing learners and teachers differently.* Routledge.

Pournara, C., Adler, J., Pillay, V., & Hodgen, J. (2015). Can improving teachers' knowledge of mathematics lead to gains in learners' attainment in mathematics? *South African Journal of Education, 35*(3), 1–10.

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly, 23*(2), 139–159.