TEACHER LEARNING ABOUT EXEMPLIFICATION IN GEOMETRY THROUGH LESSON STUDY

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We explore aspects of Malawian teachers' learning in their first encounter with Lesson Study (LS) professional development. Experienced secondary mathematics teachers from two schools participated in a theory-guided LS focused on geometry. Using data collected during the first LS cycle, we examined dimensions of variation of geometry examples made available, and changes in example sets. Findings show teachers' take-up of two dimensions of variation in the initial lesson plan, with the third dimension coming into focus during lesson reflection. We argue that theory-guided LS can support teachers to strengthen their knowledge and use of example sets in geometry.

INTRODUCTION

In Malawi, geometry learning is considered as very challenging (Ministry of Education Science and Technology [MoEST], 2020). One of the aspects of teaching geometry is using diagrams that exemplify geometric objects and properties. Exemplification has been advocated as an important teaching practice (e.g. Adler & Alshwaikh, 2019; Watson & Mason, 2006), suggesting possibilities for supporting the teaching of geometry. The broad question we pursue in the LS project is: how can we organise professional development (PD) to support teachers' learning of exemplification in geometry? PD using Lesson Study (LS) is relatively new in Malawi and to date mainly conducted with primary mathematics teachers and teacher educators (Fauskanger, Jakobsen & Kazima, 2019). In LS, teachers undertake collaborative research to reflect on and improve their teaching (Lewis et al., 2006). As a PD practice, LS has been adapted and implemented in many countries and in geometry (Fujii, 2014; Huang & Leung, 2017). We build on these studies and respond to the call for theory-guided LS by adapting and using a Mathematics Teaching Framework (MTF) that structured LS in algebra in low-income South African secondary schools (Adler & Alshwaikh, 2019) to introduce LS in secondary level geometry in Malawi. MTF includes a focus on exemplification and draws directly from variation theory (e.g. Marton & Tsui, 2004) to enhance generalising about an object of learning through focusing on what changes (variance) amidst what remains the same (invariance) across an example set (Watson & Mason, 2006). Building on Adler & Alshwaikh (op cit), we will argue that LS is a productive context for learning exemplification as a mathematics teaching practice, here in the context of geometry. We focus on variation in diagrams, geometric examples in our terms, as they are vital for enhancing learners' geometric reasoning (Al-Murani, Kilhamn, Morgan & Watson, 2019; Huang & Leung, 2017). The specific questions addressed in this paper are: 1) what dimensions of variation in examples do

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the teachers make available, and 2) how do these dimensions unfold in successive lesson plans over a LS cycle? We begin by describing the MTF framework and the LS model used.

MATHEMATICS TEACHING FRAMEWORK (MTF)

MTF is a lesson planning, observation and reflection tool developed from the Mathematical Discourse in Instruction (MDI), an analytical framework for describing and evaluating the quality of mathematics made available in teaching (Adler & Alshwaikh, 2019; Adler & Ronda, 2015). MDI draws from key tenets of socio-cultural theory, and thus a view of mathematics as an interconnected and hierarchical network of scientific concepts, and teaching/learning as goal directed and mediated (Vygotsky, 1987). The starting point of MDI/MTF is that teaching/learning, is always about 'something' which in Marton and Tsui's (2004) language is called the 'object of learning', and the work of the teacher entails bringing that 'something' into focus – its mediation (Adler & Ronda, 2015). The object of learning is what learners are expected to be able to know and do at the end of the lesson, which in our case is establishing and applying the exterior angle of a triangle theorem. In MTF, the object of learning is mediated by the teaching practices of exemplification, explanatory communication, and learner participation. Exemplification includes and distinguishes examples, tasks and representations as semiotic mediational means. In geometry, however, a diagram can be viewed as both an example and a representation. Thus, MTF required adaptation to clarify how patterns of variation in example sets can be described in geometry. We draw on the constructs of dimensions of variation and the range of change (Watson & Mason, 2006). The features of diagrams that vary constitute dimensions of variation, and the extent to which they are varied is the range (Al Murani et al., 2019; Watson & Mason, 2006).

In this paper, we describe three possible dimensions of variation in geometric diagrams each of which was made available in the lesson plans we analysed: angle measures, complexity, and orientation of diagrams. We describe the range of change in orientation as between standard or non-standard. For example, standard orientation means that a triangle is drawn in its prototype position, i.e., a triangle ABC drawn with vertex A on top and vertices B and C on the bottom and horizontal side BC extended to E to form exterior angle ACE. Non-standard orientated triangles are those drawn in atypical orientations e.g. with two vertices on top and one on the bottom with one of the top sides extended to form an exterior angle. The range of change for diagram complexity is between basic (if it does not require decomposition to do calculations or proof) e.g. one triangle with one or more exterior angles. A diagram is complex if it requires decomposition, for example a diagram comprising overlapping triangles with an exterior angle of one triangle being an interior angle of another triangle.

METHODOLOGY

Malawian secondary mathematics teachers' challenges in teaching geometry are both in content and pedagogy (MoEST, 2020). Teachers were thus introduced to both geometry content using MTF and to LS mode of PD in a two-day PD with secondary mathematics teachers from two schools. After the workshop, teachers from each school met to decide on problems that they each wanted to focus on in their LS. The Malawian LS proceeded as illustrated in Figure 1 below. There were two initial planning sessions (LP1A and LP1B), followed by teaching 1, reflection 1 and lesson planning 2, then teaching 2, followed by reflection 2 and lesson planning 3. One knowledgeable other (first author here – KO1) participated in all stages of the LS cycle and video recorded the sessions. During LP1A session, there was minimal input from KO1 as teachers discussed their choices of examples. We aimed to identify the aspects of MTF from the PD session that teachers had initially taken up and included in their plans on their own. All authors (as knowledgeable others KO1, 2 and 3) commented on the plans, and KO1 discussed the comments with the teachers LP1B planning session.

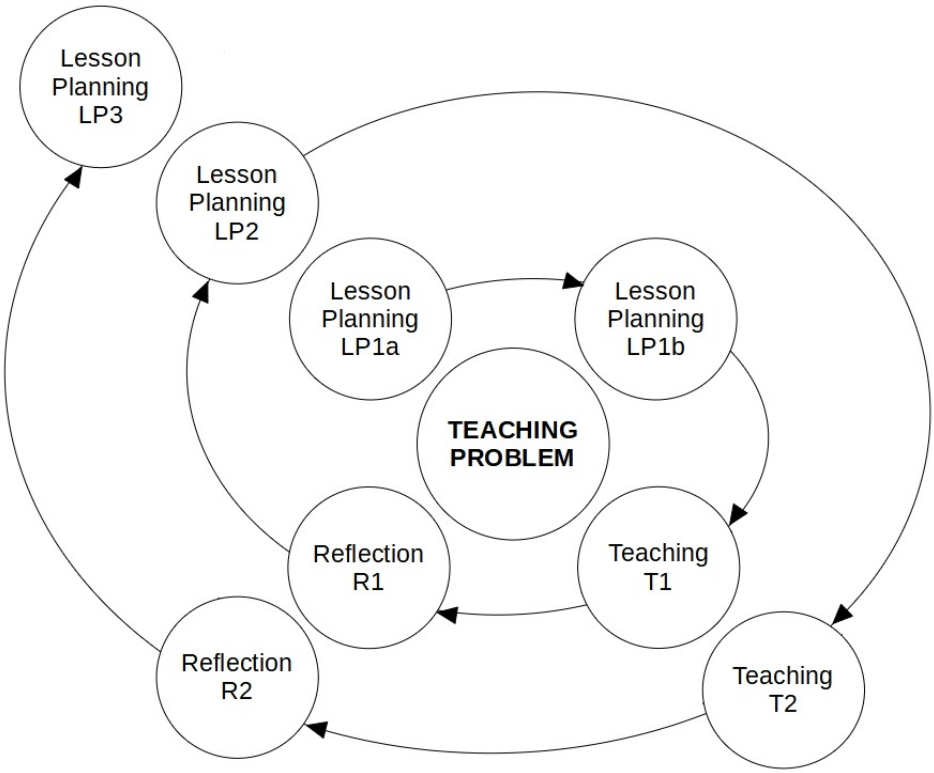


Figure 1: Lesson Study Cycle.

In this paper we focus on one school and conduct content analysis on a) transcripts of lesson planning sessions 1A, 1B and 2; b) written lesson plans; and c) transcripts of reflection 1 session. We began by analysing the whole set of examples across each lesson plan to examine the dimensions of variation made available and the range of change in each dimension. We then compared examples in all plans to identify what

the teachers maintained, added or removed. We simultaneously examined the transcripts for the planning sessions and lesson reflection sessions for an in-depth understanding of teachers' rationales for changes made in the example sets.

RESULTS

The teachers decided that the problem they wanted to take for LS is the challenges that learners face in understanding the relationship between the exterior angle and the two opposite interior angles of a triangle. They specifically mentioned that students were not able "to form equations with interior and exterior angles of a triangle". After the PD workshop, they were encouraged to plan a lesson, using the MTF as a guide, that could bring this relationship into focus with learners. We describe the examples planned by teachers in LP1A, LP1B and LP2 sessions using Table 1. We have not shown example space for LP3 because it is like that of LP2.

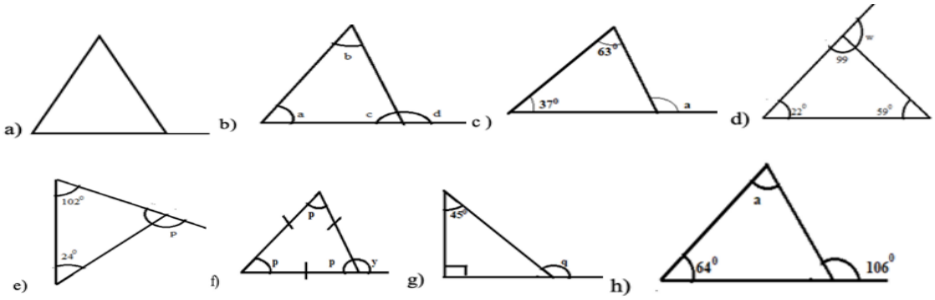
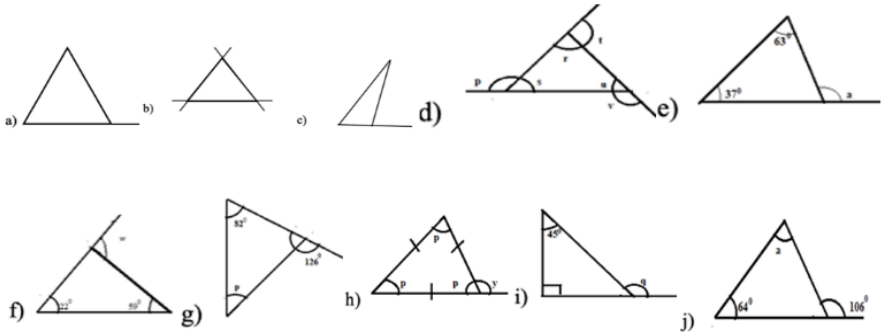
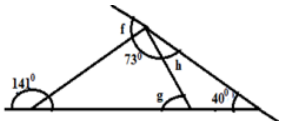
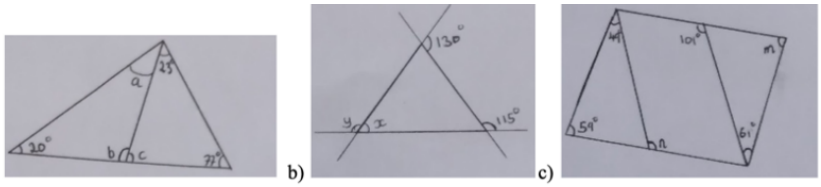
LP1A	
LP1B	
LP2	Examples a, b, c, d, e, g, h and i are the same as in LP1B but another example was added as shown 
Assessment examples for LP1B and LP2	

Table 1: Examples for LP1A, LP1B and LP2.

In each lesson plan, examples were to be used for different tasks. For example, in LP1A examples a and b were to be used for empirical activity of measuring angles to derive the theorem, and all the other examples were used for applying the theorem to calculate

measures of angles. While more can be discussed about the tasks, in this paper, we have backgrounded them to focus on the examples. Analysis of the example space in LP1A (shown in Table 1) show variation in two dimensions: the orientation of the diagrams in terms of the position of the exterior angle, and variation of the angle measure to be calculated. We view this as learning initiated by the PD workshop as there was no contribution from the KO in relation to examples at this stage. As Table 1 shows, the third dimension of varying the complexity of the diagram was not in focus in LP1A as the example space contained only simple diagrams of a triangle with one exterior angle with varied orientation and angle measure to be calculated. In LP1B, KO1 suggested that the teachers consider using several examples with varied orientation and measures of interior and exterior angles to derive the theorem, and to consider including complex examples in the example space to enable building to generality through different dimensions of variation. As shown in Table 1, the teachers took up these suggestions, adding examples b and c for doing empirical activity, and examples a, b and c in the assessment section in LP1B to LP2. So, from LP1A to LP1B, the learning in terms of variation in examples was initiated by KO1 through the suggestions on varying the diagram orientation and complexity.

In contrast, from LP1B to LP2, changes made were initiated by teachers' observations from their teaching of lesson 1 and assessing the learners at the end of the lesson. In LP2, teachers removed some examples from LP1B. In the discussion during LP2, teachers paid attention to rushing of the teaching of the application examples. They agreed to drop examples f and j from LP1B because they were like examples e and g respectively, and added a new complex example shown in Table 1. The similarity was in terms of angle measures to be calculated and the difference was the orientation of the diagrams. The inclusion of complex example in LP1B resulted from their discussion during reflection 1 (see transcript below) on student responses to a complex diagram in the assessment task given at the end of the lesson.

T represent teacher, and KO represent knowledgeable other.

87. T1: The triangles that we gave them (in the assessment tasks) are different in complexity. Different from the examples we did during the lesson.

88. T4: Mmm, exactly.

89. KO: How different were they from the examples you did in the lesson?

90. T1: Like triangle number one, in the example (a), we didn't have that kind of diagram, there are three triangles in fact in the first diagram if we were to count them. There was no triangle that had a line inside (in the lesson examples).

91… KO: Within another triangle?

94… T1: So that was somehow a challenge because during the lesson, the exterior angle wasn't inside the triangle. But here we see angle b is interior to one triangle and is also exterior on the other triangle.

603. T4: So, in terms of what we need to do differently, we discussed that on the examples, there's need for an example where the exterior angle is also inside the … bigger triangle.

From line number 87 to 94, T1 refers to examples a and c under the assessment section in Table 1 to explain the gap between examples used for teaching and those used for assessing learners. He explained that while the examples used during the lesson were simple, they contained only one triangle and one exterior angle, the examples used for assessment were complex because there was an overlap of at least two triangles, making some of the exterior angles to be embedded in a bigger triangle. In line 603, T4 explains that they would address this gap by including a complex example where an exterior angle of a triangle is embedded in another triangle. They agreed to include complex example in LP2 to ensure that the learners engage with complex examples during the lesson, thus reducing the gap between the lesson examples and assessment examples. Thus, through the teaching of their lesson, the teachers learned what worked well through the example space and what did not and made changes to improve on what did not work well. We infer from their changing choices on example spaces that those dimensions of variation appeared to make sense to the teachers, and they used these to reflect on their own example spaces.

Of further interest to us in terms of the teachers' use of the MTF for working on their teaching, in the focus group interviews (data also not presented here) at the end of the cycle, the teachers talked about using the MTF in their planning to compare the examples from the five different prescribed textbooks to identify the textbook that contained varied examples, and constructed some of their own to produce an example set that contained all the variations that they were looking for.

DISCUSSION AND CONCLUSION

Our intention for introducing the teachers to MTF was that their discussions during lesson study, that is, lesson planning, teaching, and lesson reflection would be informed by MTF, resulting in enhancing their learning to improve the quality of teaching geometry. As the findings have shown, the teachers were able to take up the exemplification aspect of MTF in a substantial way, mirroring findings in Adler and Alshwaikh (2019). From the first lesson planning session, the teachers showed that they were developing a deeper understanding of how dimensions of variation in the selected examples could be infused into their own classroom practices to benefit student learning. As the teachers worked with the examples, they also gained new insights into the mathematical content. For example, teachers realised that a triangle could have up to six exterior angles and not only three exterior angles as indicated in some of the textbooks that they use. While developing their own examples to enhance different patterns of variations, the teachers noticed that exterior angles that are formed from a common vertex of a triangle are vertically opposite and so equal. We therefore add that the moments of constructing, critiquing and revising example sets using variation provide teachers with mathematical content learning opportunities as well.

In the lesson reflection transcripts, we also noticed that the teachers used variation of examples as an analytical tool for reflecting on the quality of their teaching. One clear indicator of this was their noticing and then addressing the gaps in the examples space spread across teaching and assessment. Therefore, the findings show that MTF was used to structure discussions during planning and reflection in terms of choice and use of examples (Adler & Alshwaikh, 2019). We regarded the teachers' attention to analysing and discussing what varied and what remained the same in the example spaces as knowledgeable choice of examples by the teacher.

In conclusion, in this study, we explored aspects of what teachers learned in their first encounter with LS type of PD by examining the possible dimensions of variations of geometry examples made available, changes in the example sets, and how these changes come into focus. The findings reveal that the teachers quickly picked up two dimensions of variation during the PD workshop and implemented them in their initial plan. The third dimension of variation came into focus through the knowledgeable other and through lesson reflection on learners' assessment examples. In conclusion, the paper contributes to the confirmation of prior work on exemplification in algebra and builds on it by locating it in geometry through introduction of LS form of PD in Malawi.

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