TOWARDS A NEWER NOTION: NOTICING LANGUAGES FOR MATHEMATICS CONTENT TEACHING

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In this report, we revise and connect our approaches to mathematics teacher noticing and to the classroom language of the teacher for content teaching in the attempt: 1) to articulate mathematics teacher education knowledge from research on noticing and on language around a newer notion of noticing languages for content teaching; and 2) to apply the articulated knowledge to the design of content-specific materials oriented towards enhancing the development of noticing processes with primary school student teachers in mathematics teacher training programmes. We propose processes of identification, interpretation and decision on languages for content teaching aimed at reducing school learning challenges, and developmental work at the levels of specialised word names and content-related explanatory and exemplifying sentences.

PUTTING TWO APPROACHES TOGETHER

Professional teacher noticing has gained notable traction in research (see, e.g., the *ZDM* issue by Dindyal et al., 2021), and its development is considered important in teacher training programmes (Jacobs & Spangler, 2017). Mathematics teacher noticing refers to what mathematics teachers attend to in classroom situations and how they interpret their observations in order to make instructional decisions. While the focus on noticing mathematical thinking of students is distinctive, we strategically shift the focus onto noticing classroom languages for content teaching. We propose this shift in the context of progress and expansion of mathematics education research on language and teacher preparation (e.g., Shure et al., 2021). This said, relatively few studies have examined language responsiveness in mathematics teacher education (MTE) through networked collaboration of researchers who think about mathematics learning and teaching from different theoretical traditions.

In this report, we challenge and connect our respective approaches to mathematics teacher noticing and to the classroom language of the teacher for content teaching in the attempt: 1) to articulate MTE knowledge from research on noticing and on language around a newer notion of noticing languages for content teaching; and 2) to apply the articulated knowledge to the design of content-specific materials oriented towards enhancing the development of noticing processes with primary school student teachers in mathematics teacher training programmes. Following this introduction, in the first section we summarize some crucial parts of the sources of knowledge we build on. In the second section, we present a number of decisions adopted during our collaboration regarding the practical understanding of language-and-learner responsiveness for mathematics content teaching. We finish with the discussion of what comes next.

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NOTICING LANGUAGES FOR MATHEMATICS CONTENT TEACHING

The development of noticing in teacher training programmes

Sherin (2007) characterises teacher professional noticing as two groups of processes regarding: a) the selective attention to mathematics teaching and learning situations, and b) the knowledge-based reasoning allowing to make sense of what is attended to. In line with the decision processes introduced in Jacobs et al. (2010), we see teacher professional noticing as learning outcomes and related processes of: 1) identifying relevant aspects in mathematics teaching and learning situations; 2) interpreting these aspects according to knowledge of mathematics and mathematical pedagogies; and 3) taking teaching action decisions informed by the adopted interpretations.

The survey work in Fernández and Choy (2020) outlines research conducted on the development of noticing in mathematics teacher training programmes. This latter study also reviews the production of design strategies and materials that have been shown to support student teachers on what and how to notice. Different names, which reflect distinct theoretical lenses, are used to imply the material resources or documents aimed at facilitating noticing processes in the work with student teachers. A widespread strategy, however, is to provide tasks that consist of theoretical materials completed with illustrations of practice (Ivars et al., 2019). The theoretical materials provide linked knowledge of mathematics and mathematical pedagogies informed by mathematics education research. In this way, they offer theoretical lenses that are generally represented in the form of classroom situations (e.g., transcripts of teacher-students interaction) in which student teachers are asked to identify, interpret and decide on selected aspects of mathematics teaching and learning.

Language use for mathematics content teaching

Research on approaches to language as a resource in MTE has documented various relevant aspects to focus on in the classroom languages of teachers for content teaching (Planas, 2019). Within the sociocultural framing in Halliday (1985), Planas (2021) presents content-specific developmental work with secondary school teachers at the word and sentence levels of language. Drawing on Halliday (1978, p. 195), where a register is "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings", and strengthening the emphases on words and sentences, and on school learning, we consider three interconnected tools or resources in language for content teaching:

- *Naming*, or giving word names from content registers oriented towards reducing content learning challenges.
- *Lexicalisation*, or giving sentences with encoded explanations of content-related meanings oriented towards reducing content learning challenges.

- *Exemplification*, or giving sentences with encoded variations of content-related elements oriented towards reducing content learning challenges.

If we think of the common challenge of viewing fractions as numbers, at the level of words, the use in teacher talk of the name *terms* to refer to the numerator and the denominator, and of the name *number* to refer to fraction may be considered learner-responsive. These words can then be put into sentences with the potential function of explaining meanings in order to overcome the learning challenge, such as: *Fractions are numbers expressed in the form of a relationship between two terms.* From the perspective of variation theory (Marton et al., 2004), teacher talk can also produce sentences with the function of exemplifying variations, such as: *The size of one quarter is two if the whole size is eight, but it is three if the whole size is twelve*. This sentence would contribute to supporting the difficult understanding of the fraction size and the elements or facts that make it vary. Names, explanations and variations are, therefore, practical dimensions of naming, lexicalisation and exemplification. Their use in teaching can be approached as intersecting conditions of content languages oriented towards reducing specific learning challenges.

Particularising a language-informed notion of noticing

At the interplay of the two approaches presented above, we particularise a language-informed notion of mathematics teacher noticing: *noticing languages for content teaching*. Since the first author has researched the challenges faced by primary school students when learning fractions (e.g., González-Forte et al., 2020), we illustrate the theoretical and practical work around this newer notion specifically linked to this content. Considering the three processes in our approach to professional noticing, here illustrated for variations only, noticing languages for content teaching refers to:

Identifying mathematically relevant names, explanations and variations in languages for content teaching. Given, e.g., the situation of a teacher who is talking about the division of the unit into equal-size parts while drawing different rectangle models on the board, we want student teachers to develop the ability to *identify* the importance, in the language-responsive teaching of fractions, of using sentences to exemplify *variations of the shape of equal-size parts*, alongside other resources like drawings.

Interpreting names, explanations and variations in languages for content teaching with regard to their potential for reducing school learning challenges. Given the above-mentioned situation, we want student teachers to develop the ability to *interpret* the importance, in the language-and-learner responsive teaching of fractions, of *variations of the shape of equal-size parts* in order to help learners to challenge the frequent thinking of the equal-size parts of the continuous whole as always equal-shape.

Deciding language-and-learner responsive names, explanations and variations in languages for content teaching. Given the same learning challenge and a similar situation, now with a teacher who is talking about dividing the unit into equal-size parts and who draws one rectangle divided into equal-shape parts, we want student teachers to develop the ability to *decide* on the importance, in the language-and-learner responsive teaching of fractions, of alternative representations of rectangles based on *variations of the shape of equal-size parts* in order to challenge learners' thinking.

DESIGN OF MATERIALS TO ENHANCE PROCESSES OF NOTICING LANGUAGES FOR CONTENT TEACHING

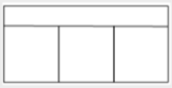
The ultimate objective in our collaboration is to enhance, in teacher training programmes, processes of noticing languages for content teaching aimed at resourcing school content learning. The consideration of appropriate materials and how to design them is thus key. On the one hand, we consider the design of preparatory theoretical documents that would guide student teachers in identifying relevant aspects of languages for content teaching, and in interpreting them in relation to knowledge of (school) mathematics and of learning challenges. On the other, we consider the design of representations of practice in the form of transcripts of either real or fictional languages of teachers in content teaching, together with prompting questions. The latter serve to identify, interpret, and take knowledge-based decisions as to which names, explanations and variations could improve the teaching languages in the transcripts.

A preparatory theoretical document

This document is designed to explain and illustrate the potential of language and some of their verbal tools for the teaching of fractions in the primary school. Operational definitions of naming (names or vocabulary within the school register of fractions), lexicalization (explanations of mathematical meanings regarding fractions) and exemplification (variations of elements related to fractions) are given with short instances of fictional languages for teaching fractions. Some of these instances intentionally miss opportunities of addressing learning challenges documented in the specialized literature. Table 1 reproduces two extracts that have been translated from the original document, one for naming (the names chosen are *equal-size parts*, *numbers* and *fractions*, and *nonequal equivalent fractions*), the other for exemplification (the chosen variations refer to *the size of the parts, and the pairs of fractions to be compared*). In each teaching situation, one instance is language-and-learner responsive (e.g., B2 or G2 are a model of more precise languages of fractions, which in turn respond to specific, well-documented and somehow predictable learning challenges), and the other instance (e.g., B1 or G1) is less responsive regarding the missed opportunities to situate the language within the content register more clearly and/or to address learning challenges. Moreover, each pair (e.g., B1 and B2) come with reflective questions on whether both instances would equally support school learners when facing the enunciated learning challenge.

Three teaching situations that illustrate the introduction of vocabulary		
A1. *If we eat one part of a pizza divided into four, we leave three parts uneaten.*	B1. *Which is larger, 2/3 or 4/5?*	C1. *One half is two quarters.*
A2. *If we eat one part of a pizza divided into four **equal-size parts**, we leave three parts uneaten.*	B2. *We have two **numbers**, 2/3 and 4/5. Which of the **fractions** is larger?*	C2. *One half and two quarters are **nonequal equivalent fractions**.*

LEARNING CHALLENGE
Understanding that a fraction is a number, and they are not two numbers separated by a slash.

Three teaching situations that illustrate the variation of examples		
G1. *This is a rectangle divided into four equal-size parts.* 	H1. *In 2/5 the parts into which the whole is divided are larger than in 5/7.*	I1. *Which is larger, 2/3 or 4/9?*
G2. *This is a rectangle divided into four-equal size parts.* 	H2. *In **1/5** the parts into which the whole is divided are larger than in **1/7**.*	I2. *Which is larger, **2/3** or **7/8**?*

LEARNING CHALLENGE
Understanding that the parts dividing a continuous whole are of equal area, that is, equal in size and variable in shape.

Table 1: Extracts of a translated version of the theoretical document

Short extracts of two fictional dialogues of one teacher with one primary school student each are included at the end of the preparatory document. These dialogues show the teaching of the equal-size condition of the parts in the part-whole relationship. The first teacher names the unit, the equal-size parts and the equitable sharing, amongst other specialised forms of vocabulary within the register, and gives explanations and mathematically relevant variations to help the student to overcome concrete learning challenges. The second dialogue serves to present a contrast. The teacher here misses several opportunities to use names, explanations and variations that support the learning of fractions. Although these dialogues are representations of practice, they are shown in the theoretical document to illustrate how names, explanations and variations represent intersecting tools in the classroom language of the teacher, rather than discrete elements working in isolation. Moreover, these dialogues situate words, words into sentences and sentences within the broader level of discourse.

A document for professional practice

This paired material contains three fraction comparison classroom situations which are represented through transcripts of interactions (dialogues) between one primary school teacher (Carlos, Patricia and Raquel) and one student each (David, Roberto and Lucía). The student teachers have the basic information at their disposal in the theoretical document and this allows them to engage in the three intended noticing processes. In that preparatory document, instances regarding fraction comparison are illustrated with respect to language-and-learner responsive names, explanations and variations, anticipating content knowledge and knowledge of school learning challenges.

The teachers' languages are designed to show different emphases on the use of names, explanations and variations. Carlos uses relevant names and explanations but does not offer variations that could challenge David's reasoning, which is biased by natural number thinking. Patricia uses relevant explanations and variations but does not offer names that could challenge Roberto's reasoning based on the difference between numerator and denominator. Raquel uses relevant names and variations but does not offer explanations that could challenge Lucía's reasoning based on choosing the fraction with the smaller denominator as the larger fraction. The student teachers have to read each dialogue and answer five questions focused on our noticing processes: Q1) *Identify*. What mathematically relevant names, explanations, and variations are used by the teacher? Q2. *Interpret.* What learning challenges may this talk help to reduce by means of these… names? (Q2.1) explanations? (Q2.2.) variations? (Q2.3.) *Decide*. Drawing on your answers, what other names, explanations or variations would support the learning of fractions? Choose a teacher intervention and propose a change.

Below we reproduce an English version of the dialogue between Carlos and David. Instead of two thirds, e.g., we write 2/3 because the teacher and the learner are supposed to say the names and to write them symbolically on the board. For clarity in this report, we mark the content-relevant names (except for names of fraction representatives such as two thirds) in bold and underline the explanations in teacher talk. In our design of this dialogue, the variations intentionally fail to support David when interrogating the validity of his reasoning. The natural number thinking bias here is the situated reference for the identification and interpretation of names, explanations and variations, whether explicit or absent, that would increase language-and-learner responsiveness in the teacher talk. Carlos misses the opportunity to introduce variations of the numerators and denominators of the fractions to be compared that would allow to question the understanding of these terms as natural numbers.

Carlos: I propose a challenge. I give you pairs of **fractions** and you compare the **fraction size**. Let's take 2/3 and 7/9. Which **fraction** is larger?

David: 7/9!

Carlos: All right, David. 7/9 is larger than 2/3. How did you come to it so quickly?

David: It's very clear. I didn't calculate anything.

Carlos: What did you know? Can you explain?

David: Yes. I saw the numbers. I mean, I know it because of the numbers.

Carlos: What **numbers** are you referring to?

David: Well, I am referring to numbers two and three, and numbers seven and nine. I always look at the two numbers... if they are bigger. Since seven is larger than two, and nine is larger than three, I know that 7/9 is larger.

Carlos: <u>But you have to remember that a **fraction** is a **number**, not two numbers separated by a slash. When you say numbers, you are actually referring to **terms**, the **numerator** and the **denominator**</u>. So, if we have **numbers** 1/4 and 5/9, which **fraction** is larger?

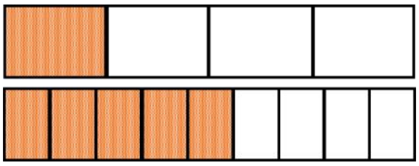
David: Now 5/9 is larger.

Carlos: How do you know?

David: Same reason. Numbers five and nine are larger, so that is the largest fraction.

Carlos: Could you make a graphical representation of both **fractions**? <u>Remember that we must represent **fractions** using the same **whole** for them to be **comparable**. Otherwise, they are **not comparable**</u>.

David: Yes. Here they are.



Carlos: So, was your comparison right? Is 5/9 larger than 1/4?

David: Yes, it is clear that 5/9 is larger. The larger the numbers, the larger the fraction.

Carlos: <u>Remember that a **fraction** is a **number** that expresses a relationship between two **terms**, the **numerator** and the **denominator**. These terms are **not comparable** like natural numbers. Which **fraction** is largest depends on the **quantity of equal size parts** dividing the **whole**, but also on the **quantity of parts taken**</u>.

LOOKING FORWARD THE NEXT COLLABORATIVE STEPS

We reported here the results of our collaborative study which seeks to theorise and prepare materials for developmental work on processes of noticing languages for content teaching in MTE settings. We anticipate that the introduction of the types of materials presented, covering a range of mathematical contents, will provide student teachers with basic professional knowledge and allow them to understand how the use of language can play an essential role in their teaching of mathematics. We expect mathematics teacher educators other than ourselves will use the material outputs of our study in their teaching. It is therefore important to initiate the implementation and evaluation of the materials in our contexts so as to explore the learning opportunities and challenges generated in MTE practice. Empirical insights stemming from the implementation of the materials in university classes will help to improve the materials, and to continue refining the theoretical tools and design processes.

We may have seemed to assume that mathematics teachers from other contexts will simply reuse the tasks we provided. On the one hand, the documents will need a careful meaning-responsive specialised translation if they are to be redesigned in a language other than the original. On the other, while student teachers may have developed basic knowledge of the mathematical contents, they may not be accustomed to producing explanations and reflecting on variations. While the preparatory document introduces explanations and variations, and hence involves some indirect teaching of them, it is not primarily designed to promote student teachers' learning or practising of the discursive practices embedded. Developing the ability to respond to critical uses and omissions of mathematically relevant explanations and variations in teachers' content languages may therefore become problematic. At some point in the collaboration, the studying and working with language tools at the levels of words and sentences will require specific attention and training in order to sequence explanations and variations at the level of classroom discourse and mathematical discourse practices. Further directions of work can still be planned within the granular levels of words and sentences. In the dialogue with Carlos and David, we foresee the potential of studying tools in language for giving sentences with encoded interrogations of content meaning.

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References

Dindyal, J., Schack, E. O., Choy, B. H., & Sherin, M. G. (2021). Exploring the terrains of mathematics teacher noticing. *ZDM-Mathematics Education, 53,* 1–16.

Fernández, C., & Choy, B. H. (2020). Theoretical lenses to develop mathematics teacher noticing. In S. Llinares & O. Chapman (Eds.), *International Handbook of Mathematics Teacher Education: Volume 2: Tools and Processes in Mathematics Teacher Education* (2nd ed.) (p. 3337–360). Brill.

González-Forte, J. M., Fernández, C., Van Hoof, J., & Van Dooren, W. (2020). Various ways to determine rational number size: An exploration across primary and secondary education. *European Journal of Psychology of Education, 35,* 549–565.

Halliday, M. A. K. (1978). *Language as social semiotic: The social interpretation of language and meaning*. Edward Arnold.

Halliday, M. A. K. (1985). *An introduction to functional grammar*. Edward Arnold.

Ivars, P., Fernández, C., & Llinares, S. (2019). Principles in the design of tasks to support pre-service teachers' noticing enhancement. In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.), *Proceedings of PME43* (Vol. 2, pp. 408–415). PME.

Jacobs, V. R., & Spangler, D. A. (2017). Research on core practices in K-12 mathematics teaching. In J. Cai (Ed.), *Compendium for Research in Mathematics Education* (pp.766–792). NCTM.

Jacobs, V., Lamb, L., & Philipp, R. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202.

Marton, F., Runesson, U., & Tsui, A. (2004). The space of learning. In F. Marton, & A. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 3–40). Lawrence Erlbaum.

Planas, N. (2019). Transition zones in mathematics education research for the development of language as resource. In M. Graven, H. Venkat, A. Essien, & P. Vale (Eds.), *Proceedings of PME43* (Vol. 1, pp. 17–31). PME.

Planas, N. (2021). How specific can language as resource become for the teaching of algebraic concepts. *ZDM-Mathematics Education, 53,* 277–288.

Sherin, M. G. (2007). The development of teachers' professional vision in video clubs. In R. Goldman, R. Pea, B. Barron, & S. J. Denny (Eds.), *Video research in the learning sciences* (pp. 397–410). Routledge.

Shure, V., Rösken-Winter, B., & Lehmann, M. (2021). How pre-service primary teachers support academic literacy in mathematics in a scriptwriting task encompassing fraction multiplication and division. *The Journal of Mathematical Behavior, 65,* https://doi.org/10.1016/j.jmathb.2021.100916.S