IDEAS OF EARLY DIVISION PRIOR TO FORMAL INSTRUCTION

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Often young children develop ideas of mathematics before they formally meet them at school. Such is the case with early counting concepts. However, little is known about children's early ideas of division. The study reported here investigated the ideas of 114 children (5-6-years old) before they had received any formal instruction about division in their first year at school. A pencil and paper test comprising worded problems with diagrams was read aloud by the teacher. We analysed children's drawings on the diagrams. Results indicate that 74% of children could conceive of at least one division situation prior to any instruction. Some children (20%) could interpret quotitive and partitive division problems. Children drawing on diagrams can provide evidence of their conceptual interpretation of division problems.

INTRODUCTION

Our research interest in children's earliest multiplicative thinking (Cheeseman et al., 2020a), naturally led us to consider how aspects of division interconnect with early multiplication. In particular we were investigating the formation of equal groups from a collection of objects and the enumeration of the group structures. We played dancing games where children made groups of a specified number when the music stopped (Cheeseman et al., 2020b). Whether the thinking in the game was the basis of multiplication or early quotitive division – or both – intrigued us. It is often assumed that young children have no concepts of division before they are formally introduced to division at school. Our earlier research showed that many children achieve early multiplicative reasoning before it is formally taught (Cheeseman et al., 2020b). The research question we sought to answer here was: What concepts of division do young children develop prior to school instruction?

RESEARCH BACKGROUND

Early division

Previous studies have demonstrated that young children (4-5-year-olds) can model division problems using concrete materials before having any formal instruction (e.g., Carpenter et al., 1993), and that children's early understanding of division is underpinned by their experiences of sharing and allocating portions (Correa et al., 1998; Squire & Bryant, 2002). However, Correa et al. argue that an understanding of sharing is not the same as having an understanding of division, as division requires as understanding of the inverse relation between the divisor and the quotient. For example, if 12 sweets are shared between three friends they each receive four sweets, but if 12 sweets are shared between four friends they only receive three sweets each. In

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other words, the more people the fewer sweets each person would receive. This view was confirmed by Squire and Bryant (2002) and Ching and Wu (2021).

Squire and Bryant (2002) explored whether young children (5-8-year-olds) could distinguish between and recognise the role of the divisor and quotient in division problems and whether children found it easier to identify the quotient in a partitive rather than in a quotitive context. Each child was presented with a pictorial representation of the situation and required to provide a verbal response. The same problem (12÷ 4) was used for both experimental contexts. Whilst there was no difference mathematically between the two conditions, as the divisor and quotient were the same in both, the difference between the two conditions was the mental model of division children brought to the problem. The results suggested that children had two different schemas of action each of which was dependent on the nature of the problem context– sharing in partitive division, and forming quotas in quotitive division. The authors argued that providing children with different problem contexts and representations was important to help children to: recognise the dividend, divisor and quotient in a problem; think flexibly in given contexts; and develop a conceptual understanding of the multiplicative relationships. The work of these researchers relates to the present study where we examined children's understanding of different contexts using representations of division.

Recently published work of Ching and Wu (2021) reported that 5-6-year-olds could recognise and reason about multiplicative relationships in partitive and quotitive problems, and that explicit instruction is not a prerequisite for understanding division. Their results also showed that children performed better on partitive situations than quotitive, which resonates with earlier findings (e.g., Correa et al., 1998; Squire & Bryant, 2002). Ching and Wu's findings were particularly relevant to our study.

Matalliotaki (2012) used some similar methodological approaches to the research we report here. She examined 5-6-year-old kindergarten children's capacity to solve quotitive division problem prior to formal instruction. Each child was presented with six problems relating to the context of gloves, socks and football - three presented orally and the same problems were presented pictorially. Matalliotaki found that more children provided a correct response for the pictorial form than the oral (40% compared to 11%). The author found that the pictorial representation enabled the children to keep track of their thinking and collect pairs of objects by focusing on interpreting and coordinating the information in the problem.

Findings of these studies suggest that young children are capable of interpreting and solving partitive and quotitive division problems. Further, that creating mental models or schemas of action for these situations may assist children to recognise the relation between dividend, divisor and quotient – an important underpinning of division.

Assessing young children's mathematical thinking

Pencil-and-paper tests are not commonly used to evaluate young children's mathematical thinking. Such tests involve abstract ideas interpreted through words, diagrams and symbols. It is difficult for children to interpret the questions and to understand how they are required to respond. Written tests are considered inappropriate assessment tools for 5-6-year-old children due to the reading and writing difficulties they present. These difficulties are not confined to young children (White, 2005). For many years mathematics educators have been advocating for more authentic methods of assessing mathematical learning (Ball & Bass, 2000; Clarke & Clarke, 2004). We agree with the sentiments of these authors. However, based on earlier research (Cheeseman & McDonough, 2013) we know that carefully constructed and meaningful pencil-and-paper tests can be used successfully to elicit young children's thinking.

Of course, as an assessment of knowledge and skills the pencil-and-paper test reported here is limited in its scope as a tool to reveal division concepts. Nevertheless, children's responses as shown by their drawings on the test, have provided some interesting data which give insights into children's thinking.

METHOD

The drawn responses to a pencil-and-paper test protocol (Streit-Lehmann, 2019) were analysed to produce the results reported here. Detailed descriptions of the instrument, the sample, and the data analysis follow.

The assessment instrument consisted of two separate forms, each was a page three worded problems. One form had three quotitive division situations where children had to portion objects into equal groups; the other page had three partitive division situations where children had to share all objects equally. Each worded problem on the quotitive division test used matched numbers on the partitive division test (12 ÷3, 7 ÷2, 22 ÷4) but the problem contexts were different to elicit different thinking. Inclusion of remainders and numbers beyond the children's usual curriculum range (e.g., 22) indicated that the problems became progressively more difficult, yet the language of the test was kept as simple as possible and constructed in short sentences. The protocol required the teacher to read the problems aloud to the children. Diagrams are an element of pencil-and-paper tests known to be difficult for students to interpret (Smith et al., 2011; van den Akker et.al., 2006). As a result, every attempt was made to use simple diagrams recognizable to the children.

An opportunistic sample of students, who were in their first year of school in Australia and had not formally been introduced to division, was obtained by personal links to the second author. From the larger sample of student responses, a randomized sample of 114 students was selected based on: two completed tests for each child, and representation from Government, Catholic and Independent sectors, and multiple classes, representing data across 20 classes from 10 schools.

Categories of responses were proposed for coding and each researcher independently coded a small sample of students' responses. Through discussion, agreement was reached about the interpretation of the children's drawings and the thinking that each type of response indicated. Where there were several acceptable solutions we defined what we considered as "correct". A revised coding system was then applied to the data and results were entered into a spreadsheet. Findings reported here focus on the facility children exhibited with the problems on the test.

FINDINGS AND DISCUSSION

The major finding of the study concerned young children's ability to interpret division contexts. Only about one quarter (26%) of children were unable to provide a correct response to any of the six division worded problems. The corollary is that, 74% (almost three quarters) of children in our sample could provide a correct response and draw their thinking to at least one division worded problem. These children showed some awareness of division, prior to instruction.

Next, we present the percentages of correct solutions for each problem.

		Problem context (calculation)	Correct solution only	Correct solution and representation
Quotitive	Q1	Apples in bags of three (12 ÷3)	67%	43%
	Q2	Socks in pairs (7 ÷2)	51%	25%
	Q3	22 children with 4 at a table (22 ÷4)	41%	32%
Partitive	P1	Candies share between 3 jars (12 ÷3)	64%	41%
	P2	Donuts share between two (7 ÷2)	53%	38%
	P3	4 children sharing 22 cards (22 ÷4)	33%	9%

Table 1: Percentages of correct solutions for each worded problem

Table 1 displays the correct written solutions only and the correct solutions matched to a correct drawn representation of the solution for each diagram. Writing a correct numerical solution only, either as a numeral or text, was easier for the children than drawing their thinking to match their solution. For each question roughly one third of children who could answer numerically could not draw their correct solution (see Table 1). The exceptions to this finding were Q2 *the socks in pairs* (7 ÷2) where half of the children were unable to draw pairs of socks, and P3 *where four children shared 22 cards* (22 ÷4) for which two thirds of children could not share the cards equally. These questions (Q2 and P3) required careful examination to find possible explanations for these results. We noted that drawing the socks in pairs (7 ÷2) was surprisingly difficult for 68% of children.

We hypothesise that the diagram given in the worded problem (Fig. 1) unintentionally made the drawing of solutions more difficult. Perhaps children were trying to match a "left sock" with a "right sock", or perhaps the illustration of an additional pair of socks was misleading. All we know is that drawing three pairs of socks with one left over was difficult.

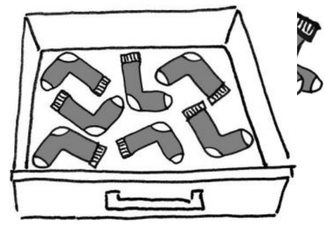

Fig. 1 Socks in a drawer

In Table 2 the numbers and percentages of children who recorded correct solutions matched to correct illustrations to both the quotitive and partitive versions of each division calculation are shown.

Calculation	Quotitive division	Partitive division	n=students (%)
12 ÷3	Q1 Apples in bags of three	P1 Candies shared into 3 jars	23 (20%)
7 ÷2	Q2 Socks in pairs	P2 Donuts share between two	17 (15%)
22 ÷4	Q3 22 children, 4 at a table	P3 4 children share 22 cards	2 (2%)

Table 2: Students correct in both quotitive and partitive worded problem

Twenty-three of 114 children (20%) were able to demonstrate their correct solution to Q1 and P1 (see Fig. 2 for an example).

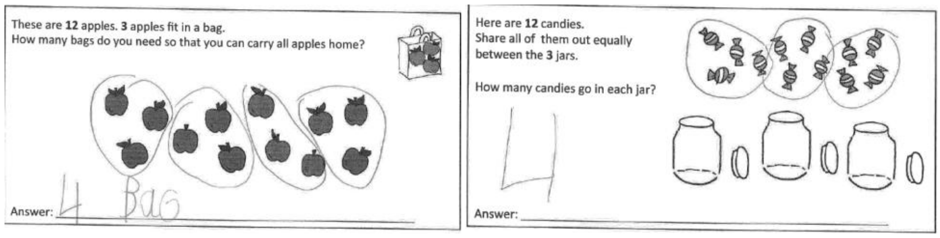

Fig. 2 Child 17's response to quotitive and partitive problems for 12 ÷3

This child used a grouping strategy for each problem and recorded a correct response. Of note is that she recorded 'bag', which shows a connection to the problem. For P1 other children drew lines from each candy to a jar, reflecting one-to-one sharing.

Seventeen children (15%) were able to correctly solve and draw a solution to Q2 and P2 (see Fig. 3 for an example). Child 33 used a grouping strategy for the partitive problem rather than an action of sharing or drawing a line from each donut as might be expected, and also recorded 3 on each plate. Of the 23 children who responded correctly for P2, 13 acknowledged the half, by dividing the remaining donut in two with a drawn line.

Fig. 3 Child 33's response to quotitive and partitive problems for 7 ÷2

Two children (2%) successfully solved and drew a solution to Q3 and P3 (see Fig 4 for an example).

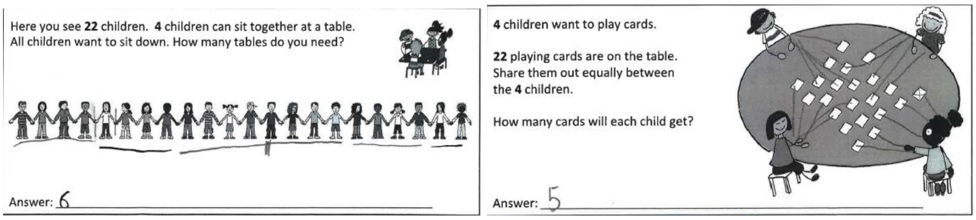

Fig. 4 Child 87's response to quotitive and partitive problems for 22 ÷4

Unlike the previous examples in which the children used grouping, child 87 used lines to distinguish the groups in Q3 and lines to show the distribution of the cards to each child in P3. The other child who successfully solved both problems used circles to show grouping in each instance. The fact that only two children successfully solved both problems highlights the complexity of these contexts.

Three major findings result from this study.

- Almost three quarters (74%) of children showed some awareness of division prior to instruction. They could provide a correct response and draw their thinking to a least one worded division problem.
- Twenty percent of young children could interpret both quotitive and partitive division problems. These results extend Matalliotaki's (2012) findings by focusing on both conceptual forms of division.
- Children's drawing on diagrams can provide evidence of their conceptual interpretations of division problems.

CONCLUSION

The limitations of the reported empirical study include: its assessment method – i.e., the use of a pencil and paper test to investigate the thinking of young children, the randomised sample of responses was taken from data gathered in a relatively small geographical region, and the results are indicative rather than broadly generalisable.

We posed the research question: What concepts of division do young children develop prior to school instruction? Our findings indicate that some young children (26% of those we analysed) displayed no knowledge of division via a pencil-and- paper test.

This is unsurprising as the children had not formally met division and the test method was unfamiliar to many of them. Of interest, was the fact that 74% of children in this study were able to conceive of either a quotitive or partitive division context for at least one worded problem. We have found no other research evidence that has reported similar findings with 5-6-year-old children. Our results indicated that the context determined the way children solved each worded problem. This finding echoes the work of Squire and Bryant (2002) and Ching and Wu (2021). Our study raises questions about the suitability of particular worded problems researchers select and the diagrams we use with children. For example, the static objects depicted on paper are difficult for many children to use to represent their dynamic thinking about partitive division. We contend that it may be simpler to draw a loop around a quota of objects to divide them into a new unit, than to organise lines to represent sharing objects one by one to a person or target. Therefore, the action of drawing could impact on the results of a study and this finding requires further study.

The results presented here make us keen to continue to investigate young children's conceptual development of division. The main implication from our findings is to question the assumption that young children have no early multiplicative concepts - including rudimentary ideas of division. We are conscious that there is a lot that we are yet to discover about early mathematics concept development in young children. We encourage fellow researcher to investigate such thinking. Our argument is not about arithmetical calculations of division problems but the interpretation of contexts that require either partitive or quotitive thinking, as almost three-quarters of the children whose tests we examined could display some conceptual awareness of division, prior to any formal instruction.

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