PRE-SERVICE TEACHER`S SPECIALISED KNOWLEDGE ON AREA OF FLAT FIGURES

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This study aims to characterise elements of specialised knowledge of a group of preservice teachers (PST) when solving area tasks. Emphasis is placed on the subdomain of Knowledge of Topics. The written justifications and procedures used in the resolution of one area task are analysed using mixed methods, including qualitative and quantitative analysis. The results indicate that PST who manage to respond to the demand of the task mobilise different registers of representation as well as procedures, justifications, properties, and geometric principles. Results suggest that the use of different representations in the resolution process has an instrumental value that allows other indicators of the subdomain of Knowledge of Topics to be mobilised.

INTRODUCTION

Teachers' knowledge of both content and its didactics has been studied from different approaches (e.g., Ball, Thames & Phelps, 2008; Carrillo et al., 2018). Particularly, we are interested in the content knowledge teachers possess, as it allows them to better understand and justify why they solve mathematical tasks in a certain way. Additionally, possessing content knowledge also allows teachers to know different ways of solving problems and teaching the content to their students (Shulman, 1986). We emphasize the importance of possessing knowledge of area measurement because this content can set the ground to understand other mathematical content in primary education, such as multiplication of natural numbers or fractions (Freudenthal, 1983). Despite the different applications that area measurement may have, numerous investigations conclude that PSTs do not have key content and pedagogical knowledge (Chamberlin and Candelaria, 2018; Simon and Blume, 1994), which has a negative impact on student learning. This study departs from the model of Mathematics Teacher Specialised Knowledge (MTSK) developed by Carrillo et al. (2018) and considers the relevance of the domain of content knowledge on area measurement with the objective to answer the following question: what is the specialised knowledge mobilised by PSTs when facing tasks involving the calculation of area? Thus, our study aims to characterise the Knowledge of Topics (KoT) mobilised by PSTs when solving tasks that require the use of diverse procedures.

THEORETICAL FRAMEWORK

Surfaces' measurement requires understanding and reorganizing the object that is going to be measured, as well as understanding different properties, concepts and procedures involved in measurement processes (Sarama & Clements, 2009).

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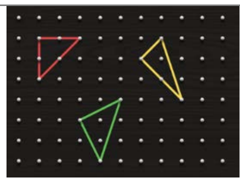
Therefore, it is not surprising that area measurement poses difficulties for PSTs. There are numerous studies that highlight such difficulties (Caviedes, de Gamboa & Badillo, 2021b; Chamberlin & Candelaria, 2018; Simon & Blume, 1994), which are mainly related to poor resolution strategies and limited acquisition of geometric properties. Such difficulties limit the ability of PSTs to propose examples and guide students' wrong answers (Runnalls and Hong, 2019). The tendency that PSTs have towards the use of formulas could be related to difficulties in using and coordinating the different registers of representation (e.g., geometric and symbolic) involved in the resolution of a given task, or else, to the lack of acquisition of geometric properties and principles involved in area measurement processes (Caviedes, de Gamboa, & Badillo,2021b; Hong & Runnalls, 2020; Runnalls & Hong, 2019). Knowledge of such conceptual elements could help PSTs to expand their range of resolution strategies while allowing them to justify what they do and why they do it (Caviedes, de Gamboa & Badillo, 2021b).

In order to understand and develop the different conceptual elements involved in solving area tasks it is necessary to consider the knowledge that PSTs have on such elements. In this sense, we adopt the analytical model of Mathematics Teacher Specialised Knowledge - MTSK (Carrillo et al., 2018), which determines the desirable components that PSTs should know for their future practice (Policastro, Ribeiro, & Fiorentini, 2019; Caviedes, de Gamboa, & Badillo, 2021b). Within the MTSK model, the KoT subdomain describes and makes it possible to distinguish the specific conceptual knowledge that is mobilised in the resolution of area tasks (see Table 3), and their relationships by means of interconceptual connections. Thus, KoT describes what and in what way mathematics teachers (or PSTs) know the content they teach.

METHOD

This study is situated in an interpretative paradigm and is part of a broader research that seeks to characterise the PSTs' specialised knowledge of area measurement. Content analysis (Krippendorff, 2004) is used to make a first interpretation of the PSTs' resolutions using the KoT indicators as analytical categories. In addition, a statistical implicative analysis (Gras & Kuntz, 2008) is conducted to explore relationships between different KoT indicators that PST mobilise in their resolutions. Data collection was carried out in the first term of the 2020-2021 school year. The participants were 147 PSTs enrolled in the third year of the Primary Education Degree at the Universitat Autònoma de Barcelona. The PSTs had had previous instruction on different procedures of area measurement as part of their study programme. A semi-structured open-ended questionnaire (Bailey, 2007) was designed to be completed individually. The PSTs were asked to justify each procedure in writing. To solve the tasks, PSTs could use manipulative materials (cut-outs as an annex to the questionnaire), as well as measuring instruments (except tasks 1, 2 and 3). The questionnaire was structured as follows: three tasks responding to contexts of equal partition, and comparison and reproduction of shapes (Tasks 1, 2 and 3); two measurement tasks (Tasks 4 and 5); one task of classification of statements and one task of the definition of the concept of area (Tasks 6 and 7); finally, one task of analysis of students' responses (Task 8). The PSTs had one week to answer the questionnaire and send it in pdf format. For sake of brevity, we present the analysis of two resolutions of Task 4 (Table 1).

Table 1. Task proposed to the PST group

Formulation	Graphic representation of the Task
Task 4: Look at the triangles constructed on the geoboard. **What is the area of each triangle? Which one has the largest area? Justify your answers using two or three different procedures.**	 (Compiled by authors)

Qualitative and quantitative analysis of PSTs` resolutions

Since we have not found any studies detailing the KoT indicators for area measurement processes, these have been constructed based on the results of a previous study postulating an epistemic configuration of the concept of area (Caviedes, de Gamboa & Badillo, 2021a). From this epistemic configuration, we define the KoT indicators to focus on the analysis of the PSTs responses to the task. Each indicator was adapted to the subcategories that the MTSK model proposes for KoT (phenomenology, representations, procedures, properties and principles, justifications, and intra-conceptual connections) and allowed a deductive coding of the PST responses, with the support of MAXQDA plus software. Table 2 shows the KoT indicators.

Table 2. Categories of specialised knowledge

KoT's categories	Indicators
Representations (R)	(R1) W*ritten:* use of adjectives such as "minor", "major", "double", "half", etc., related to surfaces. (R2) *Manipulative:* use of physical objects or dynamic geometry software. (R3) *Geometric:* use of convenient decompositions or partitions of known figures to calculate the area of unknown figures. (R4) *Symbolic:* use of the R^+ set to compare two or more surfaces, for counting units or adding up areas and-or for the indirect calculation of the area.
Procedures (P)	(P1) Compare two or more surfaces directly by total and-or partial overlapping. (P2) Compare two or more surfaces indirectly by cutting and pasting. (P3) Decompose in a convenient way, graphically or mentally, two

	or more surfaces. (P4) Carry out movements of rotation, translation, and superimposition of figures. (P5) Decompose surfaces into congruent units and/or sub-units to facilitate the process of measuring areas. (P6) Measure areas as an additive process by counting units or sub-units that cover the surface. (P7) Measure linear dimensions and use formulas.
Properties (Pp) and principles (Pr)	(Pp1) Use of conservation. (Pp2) Use of accumulation and additivity. (Pp3) Use of transitivity. (Pr1) Use of the fact that the unit of measurement can be divided into parts to facilitate the process of measuring. (Pr2) Use of the fact that every triangle is equidecomposable from a parallelogram. (Pr3) Use of the fact that the calculation of area is a matter of decomposing the figure into a finite number of parts so these parts can be put back together to form a simpler figure.
Justifications (J)	(J1) The overlapping method to compare two or more surfaces is useful for establishing equivalence or to include relationships. (J2) The mental act of cutting the two-dimensional space into parts of equal area serve as a basis to compare areas. (J3) The change in the shape of a surface does not change the area of the surface, as the figures can be decomposed and reorganised while keeping the same "parts". (J4) The area of the triangle is half of a square or a rectangle with the same base and height that contains it. Therefore, the formula of the triangle is base per height divided by two.

Figure 1 below shows examples of two PSTs (PST 7 and PST 133) that mobilise specialised knowledge. PST 7 uses written (R1), geometric (R3) and symbolic (R4) representations in the resolution process. As we can see, PST 7 mobilises (J2) and (J3) because she decomposes and reorganises triangles A and B into rectangles to later apply the area formula (P7). In addition, PS7 mobilises (J4) as she searches for the square containing triangle C to calculate its area by means of using formulas (P7). Geometric representations (R3) allow PS7 to decompose triangular surfaces by using auxiliary trace. Likewise, they allow PST 7 to use (P4) and (P5) in the case of triangle A, and (P3) and (P4) in the case of triangles B and C. The surface decomposition and reorganization procedures, allow PST 7 to implicitly mobilise (Pp1), (Pp2) and (Pp3) in addition to (Pr1), (Pr2) and (Pr3). This is because PS7 is able to accept that the area of a triangle does not change as its shape changes and that it is possible to simplify a resolution process by decomposing a figure and then rearranging its parts into a new figure. The resolution of PST 133 shows written (R1), geometric (R3) and symbolic (R4) representations. Figure 1 shows that (R3) allows PST 133 to use the auxiliary line tracing and to decompose the area around the triangles (P3) in order to find the legs corresponding to triangles B and C. This procedure allows PST 133 to obtain the length of the sides of triangles B and C by applying the Pythagorean theorem (P10) and (R4). Since the red triangle was located straight on the geoboard, PST 133 calculates its area by means of (P7). The comparison between triangles allows PST 133 to mobilise (Pp3).

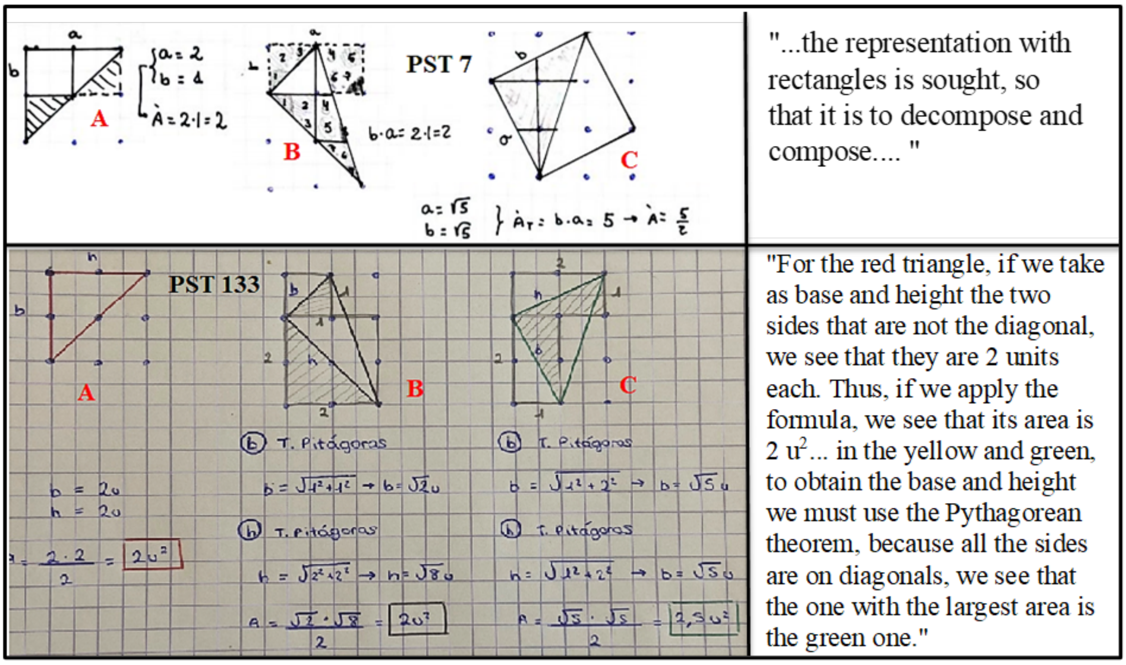


Figure 1. PSTs `resolution for Task 4

With the aim to identify relationships between KoT indicators, we performed a statistical implicative analysis. This analysis makes it possible to identify and organise quasi-implication relationships (implicative relationships between variables with a given probability) by means of a graph with arrows that relates the variables with the strongest implications at different levels and intensities. The quasi-implication between the variables A ➔ B indicates that, if PST respond affirmatively to A, they are likely to respond to B (although a relatively small number of responses may contradict it). That is, A ➔ B is equivalent to the set B not A being almost null (with the understanding that the set of observations A is almost contained in B). In this study, in the implicative graph, we use the arrow ➔ to indicate a quasi-implication according to the meaning described above. The variables considered for the implicative analysis are those arising from the qualitative analysis (presented in Table 3). In order to carry out the analysis, a value of 1 was assigned to each variable mobilised in the PSTs' responses and a value of 0 to each variable that was not mobilised in the PSTs' responses. The package C.H.I.C version 0.27 in the R console version 3.5.2 was used.

RESULTS AND DISCUSSION

Table 3 shows that PTS have a tendency to use symbolic representations (R4) and numerical procedures (P7). PST also struggle to solve Task 4 using different types of procedures. Geometric representations (R4) involving auxiliary line tracing, which allow the use of surface decomposition and reorganization procedures (P3), (P4), (P5), are used by a small number of PSTs. The same happens with the justifications, geometric properties and principles that support the above-mentioned procedures.

Table 3. Categories of specialised knowledge mobilised in Task 4

Code	Frecuency	Code	Frecuency	Code	Frecuency
R1	106	**P5**	38	**J2**	6
R4	141	**P3**	37	**P1**	5
P7	105	**NP**	34	**J1**	4
Pp2	121	**Pr1**	31	**Pp1**	6
R3	63	**J3**	11	**Pr3**	3
J4	44	**P4**	10	**Pp3**	6
P6	39	**Pr2**	7	**R2**	3

The implicative graphs in Figure 2 below (with 98% significance indicated by the red arrows and 95%, indicated by the green arrows) show different relationships between KoT subdomain indicators for those resolutions that make use of different procedures. Graph A (Figure 2) shows that PSTs using procedures related to isometric transformations (P4) make use of geometric representations (R3) by auxiliary line tracing and of procedures that require reorganizing and decomposing surfaces (P3). In turn, PSTs that make use of geometric decompositions (P3) use symbolic representations (R4), indicating a use of different procedures. The symbolic and written representations present a reciprocal relationship (R4➔R1/R1➔R4) due to the fact that a symbolic register is also a written register. Graph B (Figure 2) shows that PSTs simultaneously mobilise the properties of conservation (Pp1) and accumulation and additivity (Pp3). Both properties involve the use of geometric representations (R3), as PSTs decompose triangles by auxiliary line tracing, and subsequently rearrange them into a different figure (rectangle). We also observe that the use of (R3) also implies the use of (R1) and (Pp2), that is, the PSTs justify in written form the decompositions performed and the comparison of the triangles, in order to explain which has the largest area. The use of the transitivity property (Pp2) is also associated with the use of (R4), which indicates that PSTs make comparisons between triangles based on the numerical value of their areas. On the other hand (R1) implies the use of (R4) since both are written registers. Graph C (Figure 2) shows that the use of (J4) implies the use of (R4) and (R1), that is, PSTs justify by writing the relationship that exists between the area of triangles and squares or rectangles. The use of (J2) implies the use of (R3), as PSTs use decompositions to compare and relate the areas of triangles. Graph D (Figure 2) shows that the use of (Pr2) implies the use of (R3), i.e., PSTs use line tracing to decompose figures and identify that a triangle can be transformed into a rectangle. The use of (Pr1) implies the use of (R4) and (R1), which indicates that PSTs justify the decomposition of triangles into congruent units and subunits by means of written and symbolic registers. Again, we observe a reciprocal relationship between symbolic and written representations (R4➔R1/R1➔R4).

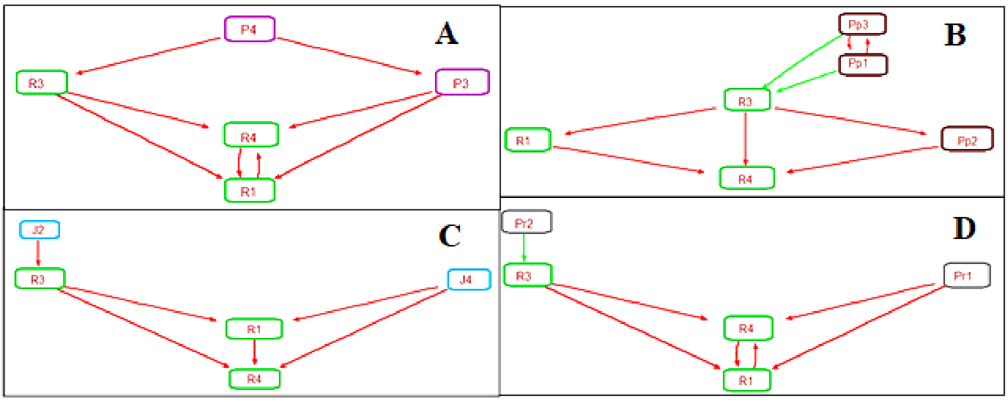


Figure 2. Implicative graph showing relationships between KoT indicators for Task 4.

The results of the qualitative analysis suggest that PSTs tend to associate area with the use of calculations and formulas, through the use of a symbolic register (Caviedes, de Gamboa & Badillo, 2021b; Chamberlin & Candelaria, 2018, Simon & Blume, 1994). Such a tendency explains why PSTs fail to mobilise conceptual elements linked to the measurement of areas, such as properties (Hong & Runnalls, 2020). Broadly speaking, the implicative graphs in Figure 2 show that representations are presented as a key conceptual element within the KoT indicators since they allow PSTs to use diverse resolution procedures. This suggests that representations have an instrumental and organizational value within the KoT subdomain indicators, that is, certain representations allow the use of certain procedures (or justifications, geometric properties, and principles) that would not be possible with the use of other representations. For example, the use of geometric representations allows PSTs to use surface decomposition and reorganization procedures, which would not be possible through the use of symbolic representations. The same geometric register allows the mobilization of properties of conservation and accumulation and additivity, which are not mobilised through the symbolic register. We consider that this instrumental value of representations could have implications for the didactic design of tasks that allow the development of specialised knowledge in PSTs.

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