TRANSFORMING CRITICAL EVENTS THROUGH SCRIPT WRITING IN MATHEMATICS TEACHER EDUCATION

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This paper proposes a teacher education strategy based on a combination of critical events and scripting dialogues. This strategy was used in two teacher education contexts in Israel and Greece with thirty-four prospective teachers (PTs). The PTs identified critical events in the context of their field experiences and transformed them into scripting dialogues with the aim to handle students' difficulties. The analysis focuses on the adopted pedagogical actions in the critical events and the scripting dialogues. The PTs used general and mathematics-specific actions to address students' difficulties promoting their conceptual understanding.

INTRODUCTION

In recent decades, there has been a growing interest in the use of critical events in teacher education (Stockero et al., 2020). In particular, critical events serve as a tool for teacher educators to develop PTs' *noticing* of students' mathematical thinking, considered as an important aspect of a teacher's expertise. Noticing entails attending to students' thinking, interpreting it and responding to promote further mathematical understanding (Jacobs et al., 2010). Research identifies a need for studies that focus on the *responding* dimension of noticing, that is, the potential teaching action (Santagata et al., 2021). Scripting dialogues is suggested as a new way of representing teaching practice in an imaginary way (Zazkis et al., 2009). In this paper, we propose a teacher education strategy that uses both critical events and scripting dialogues. PTs are engaged in transforming critical events from real classroom to scripting dialogues so that they become aware of different aspects of mathematics teaching and learning. Similar to other studies (Sun & van Es, 2015), in our study PTs selected critical events related to students' difficulties and attempted to address them in their scripting dialogues through specific teaching actions. We aim to explore the responding dimension of PTs' noticing of critical events related to students' difficulties while engaging in the transformation process.

THEORETICAL BACKGROUND

We take a socio-cultural perspective based on the role of tools and resources in mathematics teacher education facilitating teacher noticing, in particular, the nature of script writing and critical events as tools mediating PTs' noticing. Critical events are moments in which students' mathematical thinking becomes apparent and thus can provide teachers opportunities to delve more deeply into the mathematics discussed in the lesson (Stockero et al., 2020). Research indicates that PTs struggle to respond or

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provide teaching alternatives in ways that are built on students' thinking to promote learning (Sun & van Es, 2015) and that there is a need for studies getting a deeper insight on PTs' suggestions of alternative teaching actions and on the process of formulating them (Santagata et al., 2021). Research has suggested several structures for facilitating PTs' noticing. Often, PTs receive researcher-selected artefacts of practice, such as classroom video clips or narrative cases accompanied with prompts that focus attention on the details of student thinking and learning (Jacobs et al., 2010). In other cases, the PTs are asked to take part in producing the scenarios. For example, Zazkis et al. (2009) proposed the 'lesson play', where participants are presented with a beginning of a scripting dialogue between a teacher and students and are asked to continue the dialogue to resolve an issue they perceive as problematic. This provides insights into PTs' ways of understanding the mathematics in the situation and their images of the potential pedagogical considerations related with its learning. Still, other researchers invite PTs to analyze critical events selected by them in their field-work so as to promote reflection on their classroom experiences (Goodel, 2006).

Research indicates that in responding to students' difficulties, PTs tend to instruct the students what to do or correct occurring errors. These actions potentially remove the opportunity for students to make sense of errors and run counter to research recommendations for using errors as learning opportunities to foster understanding (Borasi, 1996). To classify the nature of PTs' responses to students' errors, Son (2013) suggested the term of Forms of address, distinguishing between two types: show-tell (teaching actions such as telling the definitions, explaining a procedure) and give-ask (e.g., pursuing mathematical explorations, inviting students to evaluate students' arguments). She further proposed three types of use of student error: (i) Active use, when using the student's error as a major tool for instruction, e.g., providing the students opportunities to explore why an error does not work. (ii) Intermediate use, when an error is addressed briefly as a stepping-stone to correct the student. (iii) Rare use, when not addressing the error at all or only remarking that a solution is wrong. In our study, we draw on Son's distinction among types of teachers' use of errors in their instruction and analyze the pedagogical actions in responding to students' errors identified in the critical events and the corresponding scripting dialogues written by the PTs. We use PTs' suggested rationales of developing the scripting dialogues as well reflections on the challenges they faced in the process and the learning outcomes, to better understand the PTs' engagement in the process. We aim to address the questions: (1) *What are the shifts in teaching actions that the PTs followed or suggested to handle students' difficulties from the initial critical event and the scripting one?* (2) *What is the PTs' rationale of developing the scripting dialogue? What are the challenges they faced in this process and what are the learning outcomes?* In this paper, our analysis addresses mainly RQ1. Through two case studies we provide some initial evidence in response to RQ2.

METHODOLOGY

The research took place in the context of two mathematics education undergraduate courses during the same semester in two universities: one in Israel and the other in Greece. The aim of the courses was to engage PTs in critical consideration of aspects of mathematics teaching as they emerge from the complexity of teaching practice in schools. Decisions concerning the design of the study and the data analysis were taken collaboratively through regular online meetings between the researchers. In the Greek context, 9 PTs participated in the study while in the Israeli context, 25 PTs. Enrolling the courses, the PTs had already attended a number of courses on mathematics education. In both contexts, the PTs experienced both University lessons and classroom observations. The structure of the intervention involved: identifying a critical event during classroom observations and illustrating it through a dialogue; providing the rationale of the selection (Why is it critical?); interpreting students' and teacher's actions and providing evidence; suggesting alternative teaching actions and developing a scripting dialogue between them (as teachers) and the students; providing rationale of the scripting dialogue; reflecting on the process (decisions, challenges, learning outcomes). PTs were asked to: address the above tasks in a written report; present briefly their work in the meetings; and discuss their scripting dialogues and the process of developing them in work-out sessions (3-4 PTs). The data consisted of: PTs' written reports; video recordings of the meetings including the work-out sessions; and designed resources (e.g., lesson plans, worksheets).

In this paper we analyze the data from the written reports in relation to one cycle of classroom observations. Initially, we used grounded-theory methods to analyze: the nature of critical events and their interpretation by PTs in relation to the teacher's actions and the students' thinking; the PTs' pedagogical actions in the scripting dialogues; the process of developing the scripting dialogue (rationale, decisions, challenges, learning outcomes). Then, we decided to focus on critical events related to students' difficulties as this was a dominant theme. Next, by combining grounded analysis and theoretical ideas related to teachers' response to students' difficulties we refined our coding of teacher pedagogical actions both in the critical event and the corresponding scripting dialogue to address the first research question. Finally, we analyzed specific cases of PTs to highlight the occurrence of the pedagogical actions as well as dimensions of the transformation process from the critical event to the scripting dialogue.

RESULTS

Most PTs (26 out of 34) selected critical events related to students' difficulties and attempted to address them in their scripting dialogues through specific teaching actions. Our analysis reveals a multiplicity of teaching actions indicating an active use of students' difficulties in the whole classroom as well as at the level of the individual students who faced the difficulty. In Table 1, we present the emerging categories of

pedagogical actions identified in the critical events and the corresponding scripting dialogues. We distinguished two main categories of actions: *general* and *mathematics-specific*. The total number of pedagogical actions in the scripting dialogues in both categories is increased in relation to the ones identified in the critical events. Part of the general actions concerned the teacher's interaction with individual students who had the difficulty (i.e. prompting students to explain and justify their solutions, and enabling the student to find her/his own error) while the rest of actions addressed the whole class. The latter indicates that the student's difficulty becomes a central point of discussion with the whole class (active use of students' difficulty) while the former shows a response to the individual student (rare or intermediate use of students' difficulty).

Prompting students either to evaluate other students' statements ("What do you think about Hala's idea?") or to explain and justify their solutions ("Justify your response") were dominant *general* actions both in the critical event (12 and 11 respectively) and the scripting dialogue (15 and 13 respectively). The general actions that appear to be more frequent in the scripting dialogues in relation to the critical events were: making a student's solution visible ("Come on the board to write your solution"); encouraging students to help their peers to overcome their errors ("try to explain Edi what his mistake is."); verifying understanding ("Is there something misunderstood before we continue?"); and enabling the student to find her/his own error (see case 2 below).

Pedagogical actions	**In critical events**	**In scripting dialogues**
General actions		
Prompting students to evaluate other students' statement	12	15
Making a student's solution visible	3	7
Prompting students to explain and justify their solutions	11	13
Prompting diverse solutions	3	5
Encouraging students to help their peers to overcome their errors	2	4
Verifying understanding	5	8
Providing time/homework to think about the error	3	4
Giving hints	6	6
Enabling the student to find her/his own error	6	10
Evaluating a student's answer	9	10
Total number of generic actions	70	82
Mathematics-specific actions		
Using representations and examples	4	8

Guided questioning to engage in a mathematical process	13	22
Modifying the task	1	5
Pointing to big mathematical ideas	5	6
Providing the correct solution	5	1
Providing explanation/exposition	7	9
Total number of math specific actions	35	41

Concerning the *mathematics-specific* actions, the dominant action was guided questioning to engage students in a specific mathematical process (see case 1 and case 2 below) both in the critical event and the scripting dialogue (13 and 22 respectively). The mathematics-specific actions that appear to be more frequent in the scripting dialogues in relation to the critical events were: using representations and examples (e.g., a drawing illustrating how many intersections can be between a circle and straight line) and task modification (see case 1 below). Also, in the scripting dialogues the PTs seem to avoid providing the correct solution. In most cases, transformation in the pedagogical actions is indicated through shifts from intermediate use of student's difficulty in the original event (i.e. student error is addressed briefly as a stepping-stone to correct it) to intermediate or active use of student's difficulty in the scripting dialogues (i.e. using student's difficulty as a major tool of their instruction, provide students' opportunities to discuss and test why a method doesn't work). PTs often brought to the scripting dialogues alternative teaching actions to those identified in the original events in most of the above categories.

THE PROCESS OF TRANSFORMATION – TWO CASES

We illustrate below how PTs modified the initial event to the scripting one through two cases of PTs (Elisavet and Nina) who made an active use of students' errors in their scripting dialogues introducing a large range of pedagogical actions as we have mentioned above. Elisavet and Nina offered 9 and 5 different actions respectively (Elisavet: 5 general, 4 mathematics-specific, Nina: 1 general, 4 mathematics-specific). Through the cases we also illustrate these pedagogical actions.

The case of Elisavet. Elisavet's initial event was taken from her observation of a lesson in Grade 11. The students had already worked on tasks to identify the center of circles of radius R situated either at O(0,0) or in another point of the plane K (x_0, y_0) using the circle equation in two forms. Elisavet considers this event as critical because of the students' difficulty to use the taught content in identifying the center of a circle in textbook tasks. In the initial dialogue, one student (S3) cannot provide a response. The teacher asks another student (S1) and through some guidance (i.e. he writes the formula as $x^2+y^2=5^2$) he recognizes the radius of the circle but not its center. S1 seems to be confused with the different forms of the circle equation. Then the teacher comments that "this is the simplest form of circle, we have referred to it many times"

and he asks another student (S2). S2 replies correctly. In the scripting dialogue, Elisavet adopts *general* pedagogical actions (e.g., *making a student's solution visible, enabling the student to find her/his own error*) and *mathematics-specific* ones such as reminding the relevant theory (*exposition*) and asking guiding questions (*guided questioning to engage in a specific mathematical process*). Below, we provide an extract from her dialogue with S2 and S3.

T: We know that the general form of the equation is $(x-x0)^2+(y-y0)^2 = r^2$. How can we find the center and the radius of the circle?
S3: The center, let K, have coordinates (x_0, y_0) and the radius will be r.
T: Fine. Now, in the special case where the origin of the axes is the center, what will be the point (x_0, y_0)?
S2: I will have $x_0 = 0$ and $y_0 = 0$.
T: Right. So, what is the equation of the circle in this case?
S2: $x^2 + y^2 = r^2$.
T: Fine. And what will be the center and the radius of the circle in this case?
S2: The center is the origin (0,0) and the radius is r.

The scripting dialogue differs substantially from the dialogue of the original event. Elisavet attributes to the teacher a more active role in using students' difficulties. The construction of the scripting dialogue was not an easy task for Elisavet. In her first attempt, she tended to talk more and then she made the dialogue more interactive. The crucial decisions she made concerned how the teacher could build on students' responses and direct them to develop their own way of thinking: "I think that the teacher should enable students to understand from the beginning the purpose of the task and from there to help them build step by step the final solution". Another difficulty that she experienced was to find appropriate explanations to students: "I had a hard time finding a way to explain to the student why it is wrong that the center passes through the origin". As regards her learning from this experience Elisavet seemed to have developed some awareness of the importance of exploring and developing students' understanding as well as the challenge that a teacher faces to take on-the-moment decisions.

The case of Nina. Nina's initial event was taken from her observation of a lesson in Grade 12 focusing on division of two complex numbers. The teacher solved a few examples on the board, and then asked a volunteer to solve the expression $\frac{1+i}{1-i}$. Amir approached and wrote $\frac{1+i}{1-i} = \frac{(1+i)(1-i)}{(1-i)(1-i)}$. The teacher asked him: "Did we learn to do it this way?" Another student responded: "We learned to multiply by the conjugate of the denominator". Amir responded: "It doesn't matter whether we multiply by the conjugate of the nominator or by the conjugate of the denominator". The teacher responded: "We multiply by the conjugate of the denominator". Nina considered this event as critical because of Amir's response, which reflects an incomplete mathematical thinking. She suggested that Amir cannot "see" his mistake because he does not understand the purpose of multiplying the fraction by the conjugant of the

denominator. In the scripting dialogue, Nina suggested alternative teacher actions (e.g., *guided questioning to engage in a specific mathematical process, making a student's solution visible, pointing to big mathematical ideas*).

> T: Come to the board. Try to multiply the fraction first by the conjugate of the nominator and then by the conjugate of the denominator…
> [Amir approaches the board and follows the teacher's instructions.]
> Amir: Wow, I was wrong. Multiplying by the conjugate of the nominator does not lead to anything meaningful… multiplying by the conjugate of the denominator resulted in a complex number in the standard form because the multiplication gave me a real number in the denominator.
> T: In conclusion, pay attention that we multiply by the conjugate of the denominator as an effective step for solving the task. It is not a random action. We choose this particular technique so as to receive a real number in the denominator and not to be left with a complex number in the denominator. Any questions before we continue?

The scripting dialogue written by Nina differs from the dialogue of the original event. Nina attributed to the teacher a more active role in using Amir's statement as a tool for her instruction. In contrast to the teacher in the original event, who addressed Amir's suggestion briefly as a stepping-stone to correct him, Nina provided a space for Amir to understand why his initial method is ineffective. Moreover, by inviting Amir to the board and by emphasizing the main ideas in her summary, Nina made the student's solution visible. Nina reported that it was crucial for her to enable Amir to understand the rationale for using a different method. She was uncertain about what would be a good way to do it and whether inviting Amir to the board would embarrass him. Eventually she decided that it would allow the other students to take part in the activity and support a norm that making mistakes is a constructive part of learning. Nina reported that the experience contributed to her developing some awareness of the multiple roles that a teacher fulfils simultaneously, in particular, building on student thinking and enabling students' participation in the learning process.

DISCUSSION

The pedagogical actions that PTs adopted to address students' difficulties in the scripting dialogues indicate an active use of these difficulties. Specifically, PTs attempted to engage all students in the class in discussing the difficulties through general and mathematics-specific actions. The general actions address mainly social norms and issues of participation while the mathematics-specific ones concern the development of students' conceptual understanding. The cyclic transformation process of critical events to scripting dialogues seems to have supported PTs in developing alternative teaching actions that effectively addressed students' difficulties. This finding is rather promising as the existing literature in the field speaks of the dominance of procedural ways by which PTs address students' difficulties tending to 'correct' the error (e.g., Son, 2013). Scrutinizing into the transformation process in the cases of Elisavet and Nina, we see that (a) student's difficulty becomes a central point of discussion with the whole class and (b) students' difficulty is handled in conceptual

and social terms. Both of them attempt to address social and mathematical aspects of classroom communication through common and different pedagogical actions, something that is challenging for the teachers (Skott, 2019). The analysis of the two cases suggests that the process of writing a scripting dialogue facilitated further PTs' sensitivity to students' difficulty: not only they focused on the difficulties, but also attempted to understand their sources and develop awareness of critical issues of mathematics teaching and learning. Further analysis of data including also the discussions among PTs in the two courses is expected to allow us getting a deeper understanding of the transformation process from the critical event to the scripting dialogue and its potential for the development of PTs' noticing of students' difficulties.

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