SPANISH RESEARCH ON MATHEMATICS EDUCATION

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INTRODUCTION

Didactics of mathematics as a research area

Mathematics education is a dynamic area of research in Spain. The Spanish Society of Research in Mathematics Education (SEIEM) now has around 240 members almost covering all Spanish universities. They work in different groups and use a variety of approaches to focus on the teaching and learning of mathematics in various contexts and at all educational levels, from pre-school to university. The development of this research community has taken place during the past 40 years, particularly during the last two decades.

An important date to trace this evolution is the Spanish law "for the university reform" of 1982, established during the first years of democracy after Franco's dictatorship (1939-1975). This law led to the recognition of Didáctica de las Matemáticas in 1984 as one of the "areas of knowledge" that structured university departments, which was a key factor in its consolidation as a scientific and academic discipline (Rico et al., 2002). Moreover, the law consolidated the integration of teacher education into universities, which provided institutional support to research in this area. The fact that teacher educators became members of university departments encouraged the production of doctoral theses and the constitution of research groups. Some educators already had a PhD in mathematics and reoriented their research in mathematics education. Others started their PhD abroad, mainly in France, but also in Italy, the UK or the US. In the late 1980s, PhD programmes in Didactics of Mathematics were initiated in the universities of Granada, Valencia and Autonomous of Barcelona, followed by other universities some years later. To run the programmes, universities established fruitful relationships with recognised international researchers and research institutions to offer doctoral courses and guidance with thesis supervision. The PhD candidates for these programmes were teacher educators, secondary school teachers, some recent graduate students in mathematics, and numerous students from Latin America who did not have the opportunity to carry out a PhD in mathematics education in their countries. The number of doctoral theses defended is a clear illustration of this short and intense evolution (Figure 1).

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 223-254). PME.

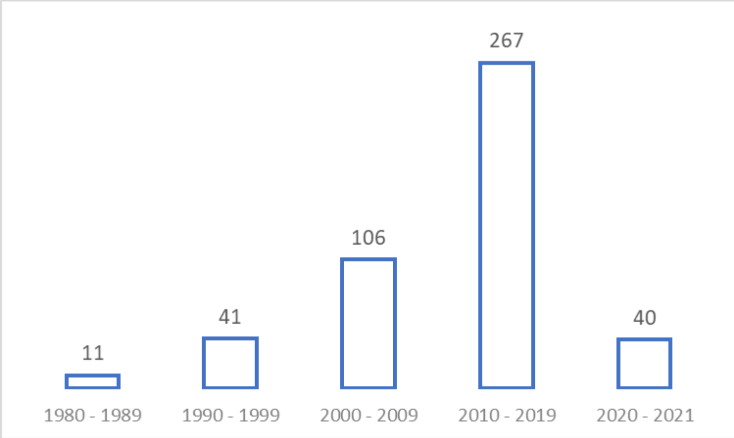

Figure 1: Number of doctoral theses in Didactics of Mathematics defended in Spain. (Source: our elaboration from https://dialnet.unirioja.es)

A terminological discussion about the denomination of the area also took place at that time: should it be Didáctica de las Matemáticas or Educación Matemática? The influence of the English-speaking communities enhanced the use of the latter, while the name traditionally used in teacher education and the continental European communities (France, Italy, Germany) pushed to maintain the former. Both expressions are used today in a quasi-interchangeable way, even though the Spanish society of researchers adopted "mathematics education" in contrast to the official denomination of the university knowledge area, which maintains the denomination of "didactics of mathematics".

The Spanish Society of Research in Mathematics Education

Born in 1996, the Spanish Society of Research in Mathematics Education (SEIEM, www.seiem.es) reached almost 100 members from 31 different universities only one year later, and, by 2021, it had more than doubled its membership. Among its objectives, it aims at maintaining a space for conceptual and methodological debate about research on mathematics education, encouraging the constitution of research groups and their collaboration, promoting mathematics education in research institutions and educational agencies, helping disseminate research outputs, and fostering the cooperation and exchange between research and practice throughout the educational system and other societal contexts. The SEIEM integrates ten thematic groups on "Learning geometry", "Teacher knowledge and professional development", "Didactic of statistics, probability and combinatorics", "Didactic of mathematics as a scientific discipline", "Digital environments", "Didactics of analysis", "History of mathematics education", "Early childhood education", and "Numerical and algebraic thinking".

During its 26 years of existence, SEIEM has been celebrating an annual symposium in different universities the only interruption being in 2020 due to the pandemic lockdown. The next one, the 25th symposium, will be held in Santiago de Compostela in September 2022. SEIEM symposia always include plenary sessions on specific research topics, the presentation of communications and posters, and time slots devoted to the thematic groups with more informal presentations and discussions. Together with the sister Portuguese Society, SPIEM, they organise a Summer School to contribute to young researchers' training, strengthen links between expert and junior researchers and foster cooperation between researchers from different universities in Spain and Portugal.

Finally, to "contribute to the advancement of knowledge of the processes involved in mathematics education and mathematics education research", the Society created a research journal in 2012, *Avances de Investigación en Educación Matemática* (AIEM, www.aiem.es) publishing two issues per year. It accepts papers in Spanish, English, Portuguese and French, and appears indexed in SJR and ESCI databases.

Since 2004, SEIEM is part of the Spanish Committee for Mathematics (CEMat), a re-structuration and extension of the Spanish IMU Committee, which integrates the different societies related to mathematics: the Royal Spanish Society for Mathematics, the Catalan Mathematical Society, the Spanish Society for Applied Mathematics, the Spanish Federation of Associations of Mathematics Teachers, the Spanish Society of History of Science and Technology, and the SEIEM. At the SEIEM's creation in 1996, the mathematician M. de Guzmán, who was the President of the International Commission on Mathematical instruction (ICMI) at that time, emphasised the need for the Society to channel its activity with a vision of integration. CEMat appears from this perspective as the common forum for all societies dealing with mathematics. It enables the collaboration between researchers in mathematics, researchers in mathematics education, teacher educators and mathematics teachers of all educational levels. The time has maybe come to open up to other Spanish educational societies and to establish links with other analogous associations in Europe and beyond.

International collaborations

As mentioned above, many of the research groups that started developing investigations in didactics of mathematics counted on the cooperation of international researchers who came to Spain to give research courses and seminars, or to host novice researchers in their universities. During this same period, many students from Latin America came to Spain for their doctoral studies in the newly created programmes. This situation helped maintain international collaborations and affiliations. Therefore, it is not unusual to find Spanish researchers in almost all international European and Latin American research organisations, and elsewhere in the world.

Spanish researchers have been involved in the European Society for Research in Mathematics Education (ERME) since its constitution in 1998, and regularly participate in its congresses (CERMEs). The Spanish contribution to CERMEs is notable, considering there has almost always been a Spanish researcher in the IPCs. Furthermore, the fourth congress was held in Girona, Spain, in 2005, chaired by M. Bosch, many Thematic Working Groups have had Spanish leaders and co-leaders

(Table 1), and there has been some participation in plenary activities, such as C. Batanero's plenary talk in CERME9.

Adult mathematics education
Affect and mathematical thinking
Algebraic thinking
Applications and modelling
Different theoretical perspectives in research in mathematics education
Geometrical thinking
History in mathematics education
Mathematical curriculum and practice
Mathematics and language
Mathematics education in multicultural settings
Teacher education
Stochastic thinking
University mathematics education

Table 1. CERME Thematic Working Groups with the participation of Spanish researchers as leaders or co-leaders

Since 2016, ERME has been promoting Topic Conferences (ETCs) organised on specific research themes by working groups associated with CERME conferences. Spanish contributions have been key in three ETCs. The first one is related to the International Conferences on the Anthropological Theory of the Didactic (CITAD), which has been held alternatively in Spain and France since 2005. The second ETC in which Spanish inputs are worthy of note is the International Network for Didactic Research in University Mathematics (INDRUM). Spanish researchers participated in its creation in 2015, collaborated in the organisation of the first INDRUM conference in 2016, co-chaired the third (2020) and fourth (2022) conferences, and possibly will organise the fifth one in 2024. The third ETC with Spanish researchers in its IPC corresponds to the ERME group about Language in the Mathematics Classroom, whose first conference was held in 2018.

Outside Europe, the most important Spanish collaborations are with Latin American research communities and organisations. Participation in the RELME conferences, CIAEM-IACME conferences (Inter-American Conference of Mathematics Education affiliated at ICMI) and engagement in the Latin American Committee of Educational Mathematics (CLAME) should be stressed.

Finally, broader conferences and organisations like PME, CIEAEM, or the ICMI itself, have always counted on Spanish participants assuming different responsibilities. Many are worth mentioning. First, the celebration of ICME8 (1996) in Seville and PME20 in Valencia the same year and the participation of M. de Guzmán (1991-2002), C. Batanero (2003-2006) and N. Planas (2021-2024) in the ICMI Executive Committee. Regarding ICMI thematic studies, C. Batanero, N. Planas and M. Bosch were IPC members of the 18th (2008), 21st (2014) and 24th (2018) ICMI Studies on Statistics, on Language Diversity, and on Curriculum Reform respectively. More recently, A. Gutiérrez has been designated as co-leader of ICMI Study 16 devoted to the learning and teaching of Geometry. Finally, T. Recio (2008-2016) was involved in The ICMI Klein Project and M. Bosch in the starting period of the ICMI AMOR project.

Determining the research areas

So far, we have discussed the emergence and development of research in didactics of mathematics in Spain from the perspective of researchers: their training, organisation and collaborations inside and outside the country. To present an overview of their productions, perspectives and work areas, we searched publications in some of the most relevant research journals released during the past 15 years, namely *Educational Studies in Mathematics*, *Enseñanza de las Ciencias*, *International Journal of Science and Mathematics Education*, and *Journal of Mathematics Teacher Education*. Contributions to past PME proceedings (2010-2021) were also included. This search led to identifying 13 work areas that were grouped into four sections, developed below.

The first and longest section presents research focusing on the learning of mathematical topics such as calculus, numerical and algebraic thinking, stochastic thinking, and geometry. The second section approaches language, discourse, and interaction, while the third one is related to teacher knowledge and professional development. The last and fourth area includes two theoretical approaches—the ATD and the OSA—that bring together researchers working on different problems from a common methodological perspective. Unfortunately, some topics remain outside the classification, like studies relating history to mathematics education, those approaching mathematics for students with special needs, the ones about attitudes and beliefs, or more global analyses about curricula and international comparisons. This a posteriori classification seeks to account for the diversity—and local coherence—of the investigations carried out by a community that integrates more than 200 researchers working from different theoretical perspectives and using different methodologies.

RESEARCH ON THE LEARNING OF MATHEMATICS

This section presents the Spanish research on teaching and learning mathematics that focuses on students' learning processes. It is divided into four parts, corresponding to the four mathematical areas whose learning processes have been most investigated in Spain over the past decades.

Research on learning calculus

Since 1997, when the research group for mathematical analysis education (GIDAM) of the SEIEM was created (www.seiem.es/grp/gidam.shtml), its members have been conducting research on the teaching and learning of calculus, in particular with ICT tools. Said research was carried out in secondary school and the first years of university education. Azcárate et al. (2015) present a review of the research produced by the various teams integrating the GIDAM. It involved different theoretical and conceptual frameworks, like Actions-Processes-Objects-Schemas (APOS), the onto-semiotic approach (OSA), and Advanced Mathematical Thinking (AMT), amongst others. The present summary provides a detailed review of the research performed by this research team. It is organised around the different topics approached: sequences and series, functions, limits, continuity, derivatives, integrals, ordinary differential equations (ODEs) and modelling.

Researchers from the University of Valladolid have conducted studies focusing on identifying and interpreting students' strategies to convert representations when working with real intervals (Pecharromán et al., 2019). Licera et al. (2019) used the framework of the Anthropological Theory of the Didactic (ATD) to study the problem of the uncertain status of real numbers in secondary education caused by the absence of an explicit approach for measuring magnitudes. Claros et al. (2016) used the phenomenology of Freudenthal, representation systems, and AMT to analyse the cognitive structure secondary school students develop when studying number sequences. Codes et al. (2013) studied university students' understanding of infinite series. As for functions, Ortega and Pecharromán (2010) used an instructional design to work on the properties of functions based on graphical representations, while Berciano et al. (2015) identified a significant learning improvement when working on the interpolation and extrapolation of functions graphically instead of algebraically.

The research group at the University of Granada has focused on aspects of the meaning of the concept of limit, considering its semantic and personal meaning, and as an element closely related to understanding. Fernández-Plaza et al. (2016), for instance, tried to characterise the meanings manifested at different levels of cognitive complexity, which allows integrating meanings expressed in definitions using the analysis of arguments.

Other researchers have stressed the importance of coordination in the processes in terms of the APOS theory, used to provide an understanding of limits (Valls et al. 2011), and have paid close attention to students' understanding of the derivative of a function (García et al., 2011; Orts et al., 2018; Fuentealba et al., 2019). The research groups at the universities of Alicante, Autonomous of Barcelona, and Seville analysed this topic in great depth, using the APOS theory. They tried to characterise different underlying structures of the derivative schema in terms of student ability to explicitly transfer the relationship between a function and its first derivative to the derivative function and the second derivative. This detailed analysis was performed both in secondary education and the first years of university education. Concerning differential calculus, Lucas et al. (2017) proposed a broader perspective to connect it with algebraic-functional modelling, using the ATD.

Ariza et al. (2015) characterised the development of engineering students' definition of the integral schema using fuzzy metrics to establish the level of development on intra-, inter- and trans- levels. Other topics that received attention involve the learning difficulties associated with integrals. Camacho et al. (2010) conducted a study to determine the difficulties students face in understanding the concept of the definite integral based on the study of areas of plane figures using symbolic calculus software. Boigues et al. (2010) presented an extensive study using fuzzy theory to analyse the validity of a scheme for the definite integral. González-Martín (2005) identified an epistemological obstacle combining two obstacles originated by the students' conviction that a finite (infinite) 3D figure has a finite (infinite) area or volume when learning the generalisation of the concept of integral (improper integral).

Researchers at the University of La Laguna have studied the derivative and its applications but in terms of the functions that are involved in solving ordinary differential equations and the modelling phenomena they organise. Camacho et al. (2012) studied the conceptual and cognitive processes that arise when introducing ODEs via problem-solving in a university freshman chemistry course, and Guerrero-Ortiz et al. (2016) analysed the difficulties students have interpreting ODE models that result when analysing growth phenomena using specific software designed for this purpose. From the ATD perspective, Barquero et al. (2019) focused on the modelling activity and the proposal of study and research paths considering an entire course of mathematics for engineers.

Research on numerical and algebraic thinking

In this summary, we address numerical thinking and algebraic thinking together, as they have elements and relationships in common. We highlight the main lines in which Spanish researchers have participated and which have given rise to relevant results and publications.

One of the research groups of the SEIEM is called Numerical and Algebraic Thinking. Its members' productions can be found in the proceedings of their annual meetings since 1997 (https://seiem.es/pub/actas).

A research group managed for decades by Luis Rico from the University of Granada has addressed topics in this research agenda (https://fqm193.ugr.es/). Different approaches (e.g. González-Marí et al., 2009) have been used in his publications. Enrique Castro led a research line on calculation and estimation (Segovia & Castro, 2009) and arithmetic problems (Castro & Frías, 2013). Other groups working on arithmetic problems are located at the universities of Extremadura (Gil et al., 2006) and Valencia (Gómez & Puig, 2014).

After Castro (1995), a research line emerged focusing on patterns and generalisation. It was developed through several research projects until 2013 (Cañadas et al., 2009; Molina et al., 2008). Since 2014, there have been three projects on algebraic thinking involving children aged 3-12 years (https://pensamientoalgebraico.es/en). These projects focus on different approaches to algebraic thinking (Ayala-Altamirano & Molina, 2020; Pinto et al., 2021; Ramírez et al., 2022; Torres et al., 2021), problem-posing in algebra (Cañadas et al., 2018; Fernández-Millán & Molina, 2016), and secondary school students' errors and difficulties with algebra (Castro et al., 2022; Molina et al., 2017). It is worth mentioning De Castro (2018) and Alsina (2016) with regard to patterns and algebraic aspects in early childhood education, involving children aged 3-6 years. Other results by these authors can be found on their ORCID pages (orcid.org/0000-0002-2246-5402; orcid.org/0000-0001-8506-1838). Moreover, at the University of La Laguna, there has been interesting research on students' errors and difficulties in algebra (Socas, 2007), and number sense (Almeida et al., 2014).

At the University of Valencia, there has been a tradition of research on historical studies, analysis of textbooks, and problem-solving, as, for example, Puig (2018) (orcid.org/0000-0002-7074-6110), or the books and book chapters by Gómez (e.g., Gómez, 2013) (www.uv.es/gomezb/Mispublicacionesdedominio publico.html).

As part of this Spanish agenda, some groups have focused on the learning processes of children with special needs, such as Down syndrome or autism, and the learning of arithmetic (Bruno & Noda, 2019; Polo-Blanco et al., 2021). They have also studied the learning processes of mathematically gifted students regarding pre-algebra (Gutiérrez et al., 2018). More publications about these topics can be found in: orcid.org/ 0000-0002-0154-8073, orcid.org/0000-0001-6425-6337, orcid.org/0000-0001-7187- 6788.

Research on students' stochastic thinking

Research on students' stochastic thinking was initiated in Spain in the 1980s at the University of Granada by a specific research group. It consists of seventeen researchers, and they have been involved in it all this time. They were pioneers in the field at a time when stochastics was not taught in primary school. Related research focused on the cognitive development of children and the reasoning biases in decision-making was carried out in the field of psychology. Other researchers became progressively involved in stochastic education at the universities of Cádiz, Jaén, La Laguna, Lleida, Valencia, and, more recently, Girona, Oviedo, the Basque Country, and Zaragoza. These teams have cooperated through the SEIEM and activities related to the International Association for Statistical Education, such as the IASE/ICMI Study (Batanero et al., 2011), in which an important group of Spanish participants took part. Their work on students' stochastic thinking is based on previous epistemological studies (e.g., Batanero, 2000) and textbook studies (e.g., Gea et al., 2015; Lonjedo et al., 2015; Serradó et al., 2005). Said studies identified new variables that had not been considered in psychological research and are relevant to the students' learning. For example, the results pointed to the multiple meanings of probability, which, until that time, had been limited to its traditional meaning in teaching and research, and opened up an unexplored research field related to the understanding of the frequentist and subjective meanings of probability. They also revealed the existence of semiotic conflicts in the interpretation of fundamental stochastic ideas. For instance, both textbooks and students mixed up the frequentist and Bayesian meanings of hypothesis testing, which is often interpreted deterministically (Batanero, 2000).

Part of the studies on reasoning and learning focus on university students, including several doctoral theses developed by statistics lecturers. This research helped to identify the main difficulties and errors related to topics such as association and correlation, hypothesis testing, normal distribution, confidence intervals, analysis of variance, random variables, and the central limit theorem. Teaching experiments based on the use of technology have also been designed and tested. For instance, Batanero et al. (1998) described the local, algebraic, unidirectional, and causal conceptions of statistical association, and showed the persistence of the latter after instruction. In another study, Batanero et al. (2004) reported the evolution of the personal meaning of normal distribution a group of students progressively acquired over several weeks of work using ICT tools.

Research on primary and secondary school students has considered the understanding of statistical graphs and tables, central position measures, probability, combinatorics, and sampling. Cañizares (1997) performed an extensive study with children aged 10-14 years, identifying the influence of age, combinatorial reasoning, and language comprehension in solving probability problems. Regarding combinatorics, the implicit combinatorial model was found to be a variable affecting strategies and errors in solving combinatorial problems (Batanero et al., 1997). Pallauta et al. (2021) pointed out the major difficulties of secondary school students in converting graphs into tables and interpreting tables. Batanero et al. (2020) demonstrated that, contrary to what was supposed in previous studies, students in compulsory secondary education understood the variability of small samples better than that of large samples. More recently, interest has grown in describing the emergence of intuitive stochastic ideas in early childhood and in offering activities to encourage children's statistical thinking (Rodríguez-Muñiz et al., 2021). One of the conclusions is that the first ideas about chance and probability occur much earlier than assumed in previous psychological research (Vásquez & Alsina, 2019).

Stochastics has also been reflected in teacher education research. For example, Alonso-Castaño et al. (2021) used problem-posing and problem-solving to analyse teachers' probabilistic knowledge; Berciano et al. (2021) connected Science, Technology, Engineering, Arts and Mathematics (STEAM) activities and early childhood prospective teachers' stochastic reasoning; González et al. (2011) summarised existing research on teachers' knowledge of statistical graphs, and Martins et al. (2012) and Serradó et al. (2006) analysed teachers' attitudes towards stochastics and its teaching.

In summary, Spanish research on stochastic thinking has been performed by an increasingly cohesive group of researchers for over 40 years across all educational levels and subjects.

Research on learning geometry

Research on teaching and learning geometry in Spain started with the study of the Van Hiele levels, characterisation of visualisation, and learning of proof. The Van Hiele model (Gutiérrez et al., 1991) has been the basis for several studies characterising students' reasoning in plane isometries, similarity, and organisation of 2D and 3D geometric objects. Guillén (2004) adapted the Van Hiele model to 3D geometry and presented learning activities that highlight the need to extend the characteristics of the Van Hiele levels to space geometry. More recent research extended the field of interest towards contexts of inclusion and mathematical talent (Aravena et al., 2016).

The processes students go through to articulate visualisation and geometric reasoning have been studied from primary school to university. Learning Trajectories for early childhood education on 2D and 3D geometry, patterns, and representations of itineraries have recently been studied. Researchers at the University of Alicante used configural reasoning to theoretically support that articulation (Clemente et al., 2015; Llinares et al., 2014, 2019) and Duval's theories to analyse grade 3 students' reasoning using polygons (Bernabeu et al., 2021). Both show the importance of visualisation for the development of students' classification and deduction processes in geometry. Results showed the influence of the figures provided in the problems and the subsequent modifications of those figures by the students for the development of their understanding of geometric concepts. This articulation has also been considered by the OSA (Godino et al., 2012) to advance in the establishment of skill levels in tasks requiring visual reasoning. It shows that there may be different cognitive configurations at each level, the levels depending on both conditions of the task and the visualisation skills required (Blanco et al., 2019). At the universities of Valencia and Granada, Gutiérrez et al. (2018) and Ramírez and Flores (2017) showed that visualisation is a cognitive process characterising mathematically gifted students. It is advanced for determining categories of the students' justification and proof abilities. Other publications about these topics can be found in: orcid.org/0000-0002-3292-6639, orcid.org/0000-0001-7187-6788, orcid.org/0000- 0002-8462-5897.

Technological environments and resources are facilitators for teaching and learning geometry. In the past decades, GeoGebra has been identified as a useful tool for proving propositions and exploring the understanding of geometric concepts and properties, such as properties of triangles, loci, and symmetry. In recent years, the development of students' argumentative skills in a GeoGebraTUTOR intelligent tutoring system environment has been explored. Paneque et al. (2017) showed that tutor-teacher-student interactions produced learning opportunities, inducing conjectures, and promoting the transition from empirical to deductive arguments. Automated reasoning tools of GeoGebra have also been explored from an educational perspective to contribute to increasing students' curiosity and critical spirit. Gómez-Chacón and Kuzniak (2015) studied how GeoGebra can influence students' geometric work. Their results showed a wide diversity of students' approaches because of variations in their interactions both with software and geometry. Other teaching experiments have been carried out with robots, 3D printers, and modelling, obtaining statistically significant improvements in computational thinking, as shown in Diago et al. (2021). The study of shapes in 3D geometry and their relationship with 2D geometry has been a major objective at the University of Almería, where the dynamic 3D geometry software NeoTrie VR of immersive virtual reality was developed (Rodríguez et al., 2021). Their results, together with those of researchers exploring augmented reality (Sua et al., 2021), seem to indicate substantial benefits compared to traditional methods.

Some researchers have focused on obstacles and difficulties encountered by students in learning geometry. González-Regaña et al. (2021) showed the conflicts pre-service teachers experienced in the transition from descriptions to formal definitions of polyhedra. Advanced aspects in vector geometry and its connection with algebra have been investigated (Borji et al., 2020), and teaching sequences have been proposed for the transition from the Euclidean to the Cartesian conceptions of a tangent. They aim to promote changes in the register of different coordinate systems and to encourage a dynamic and global conception of geometric loci (Gaita & Ortega, 2014). At lower educational levels, that connection has been investigated for the generalisation in geometric pattern sequences, including some results indicating that the difficulties in modifying the different interpretations and the lack of coordination between geometric and arithmetical structures could explain the difficulties students have with algebraic generalisation (Callejo et al., 2019).

Spanish researchers have also examined the importance of textbooks as a classroom resource, presenting a view of plane and space geometry, paying particular attention to compound proportion, lines and notable points of a triangle, and solids of revolution. Their results showed that there are few activities in textbooks aimed at explorations and formulation of conjectures and relationships. Recent research focuses on children's spatial orientation, measurement and visualisation in the study of areas of plane figures in primary school, translations at secondary school, and the transition from natural to axiomatic geometries at the university level. They also show that textbooks give preference to certain types of visualisations in area problems, thus preventing other ways of seeing (Marmolejo & González Astudillo, 2015).

RESEARCH ON LANGUAGE, DISCOURSE AND INTERACTION

In this section, we provide a brief overview of three lines of mathematics education research with emphasis on the following concepts: 1) language, 2) discourse, and 3) interaction. We argue that considered together, these three lines constitute a strong empirical context for the mathematics education research developed in Spain during the past two decades. This research views educational and professional practices as mediated by social aspects of teaching, learning, and thinking. However, they are differently rooted in sociolinguistics, cultural studies or psychology in conjunction with theories of mathematics education. A primary quality of mathematics education research and practice is thus *the social* (social mind/ practice/ context/ development/ work, in Planas & Valero, 2016), which cannot be dismissed without an important loss of meaning and coherence in knowledge construction. Another basic assumption is that language, discourse, and interaction are fluid, interrelated concepts. On the one hand, language cannot be studied in depth without looking into discourse, and, on the other, language itself is a process and product of social interaction. These three lines build up a common yet plural research agenda in which contexts of mathematics education are represented as linguistic, discursive and interactional social contexts.

Mathematics and language

Since the early work on classroom norms and language diversity in Gorgorió and Planas (2001), the line of classroom research on language and mathematics has documented challenges: for primary and secondary school learners of mathematics with migrant backgrounds and home languages other than the language of instruction; and for their teachers in trying to understand learners' thinking processes in the absence of full communication. Drawing on the integrated nature of the social, cultural, political and linguistic aspects of mathematics teaching and learning, language was initially characterised as a social tool in the processes of sharing meanings and values within the mathematics classroom. Since then, languages of doing mathematics within the classroom and deficit approaches in mathematics education research and practice have been questioned in ongoing projects.

Several empirical studies using bilingual and multilingual school lessons have reported how mathematical discourse practices of reasoning and explanation of content meaning are hindered by frequent shifts from one language to another to focus on linguistic accuracy in the language of instruction (Planas & Setati, 2009). These shifts in language are prompted by both teachers in whole-class discussions and peers in small group interactions. Findings have shown that access to and development of opportunities for mathematical learning in classrooms are mediated by distinct valorisations of the languages and mathematical meanings of some groups of students and the languages and mathematical meanings represented as appropriate in the school culture. Paying attention to lessons and their implications for pedagogies of mathematics instruction has motivated moves towards the study of mathematics teaching practices and the content of teacher talk in the classroom (Planas et al., 2018). A recent study identifies and interprets some of the language-based professional challenges of student teachers who are expected to support their pupils in processes of mathematics learning by exposing them to the teacher talk for understanding (Caro & Planas, 2021).

The development of this line of research has started to produce findings on teaching and learning useful mathematics in developmental work with secondary school teachers around mathematical-pedagogical aspects of the languages used when teaching content in the classroom. The study of the connection between content talk and mathematics teaching in the communication of mathematical meaning at the word and sentence levels, and in the enhancement of mathematical discourse practices have hence started to inform mathematics teacher education research and practice.

Mathematics and discourse

In this line of research, researchers have adopted the commognitive framework developed by Anna Sfard. It considers mathematics as a type of discourse including some special characteristics: word use, visual mediators, endorsed narratives, and routines. According to this framework, learning means changing one's discourse to become a participant in another discourse (for instance, the discourse of mathematicians, or the discourse of mathematics teachers).

Several studies have employed this framework to better understand how students perform mathematical tasks related to the mathematical process of defining. Gavilán Izquierdo et al. (2014) studied the changes in students' mathematical discourse when they described and defined mathematical objects (2D figures). They identified several types of situations depending on what changed (or did not change) in the students' discourse. Sánchez and García (2014) showed that two types of discourses coexist in the colloquial mathematical discourse of pre-service primary teachers when they describe and define 2D objects: socio-mathematical discourse and mathematical discourse. Those two types of discourses have their own features and norms. Their coexistence leads to the appearance of commognitive conflicts, which are encounters between participants in the discourse who use mathematical words in different ways. The resolution of these conflicts is one of the main sources of learning.

More recently, there have been studies on how pre-service primary teachers describe and define 3D geometric solids. Fernández-León et al. (2021) studied the routines the participants performed, which served to shed light on how undergraduate students define. They found that some students did not have a clear idea of what a definition is and that they sometimes mixed describing and defining. They also reported differences between the discourse of students when defining 2D figures described in Gavilán Izquierdo et al. (2014) and the discourse of students when defining 3D figures. The study of the routines of the students allowed the authors to infer the existence of commognitive conflicts between the discourse of pre-service primary teachers and the discourse of mathematicians. One of the conflicts was similar to a commognitive conflict identified in Sánchez and García (2014) that appeared due to a conflict between socio-mathematical and mathematical norms. Since the participants in the studies of Fernández-León et al. (2021) and Sánchez and García (2014) were pre-service primary teachers, their results may have implications for teaching and learning 3D geometry in primary schools.

Toscano et al. (2019) investigated the discourse of pre-service primary teachers when they solved didactic-mathematical tasks instead of mathematical tasks. By didactic-mathematical tasks, they meant real activities proposed by teacher educators with the aim of "bringing future teachers closer to the reality of the professional activities of a primary teacher" (Toscano et al., 2019, p. 3). The authors identified two different discourses: one in which pre-service teachers act as students solving a classroom task, and another one that resembles the primary teachers' discourse. This distinction is important because, for students to become teachers, they should become participants of the community of practice of primary teachers and adopt their distinctive discourse. However, Toscano et al. (2019) reported that few pre-service primary teachers showed signs of adopting a discourse resembling the discourse of teachers, which "could be due to the difficulty that the pre-service teachers have in assuming the role of a teacher, despite this being what the tasks demanded" (p. 12).

Alongside studies focused on pre-service teachers, there have been some analyses of the discursive activity of in-service mathematics teachers. Gavilán-Izquierdo and Gallego-Sánchez (2021) investigated the discursive activity of an upper secondary school teacher when she introduced the derivative. They identified several types of visual mediators and routines, which served them to deduce what resources secondary school students will have in their transition to university discourse.

The study of discourse conducted in local research has opened a door to a better understanding of what happens in classroom situations of teaching and learning mathematics and of mathematics education in university training programmes.

Mathematics and interaction

This third line of research recognises that students' mathematical learning is in part developed in the classroom interaction between students and teacher, and amongst students in communicative practice throughout involvement in mathematical tasks. The social quality of mathematics teaching and learning is thus basically examined from the perspective of the interactional processes in the classroom.

Two main research directions have been developed to study student-teacher and student-student interaction. In the case of the interaction between the teacher and the students, research has been focused on joint problem-solving in the classroom. An analysis system was designed, based on a detailed view of the interaction, which takes the cycle as a measure unit, each of which is categorised according to the processes promoted and the participation of the students and the teacher. Processes can be cognitive depending on the level of reasoning, or metacognitive depending on the reflective and regulatory capacity that creates awareness of one's own cognition. Participation of students and teachers is understood as the participation that each of them has in the construction of learning and knowledge (Sánchez-Barbero et al., 2019). This system of tools allows analysing if a teacher solving problems together with students in the mathematics classroom does so in a rather superficial way or in a genuine way. A superficial way emphasises the mathematical aspects of the problem to obtain the solution without forming a model or considering situational information. A genuine way implies a focus on the mathematical aspects of the problem and other aspects such as the intentional, temporal and causal structure of the situation, while also making sense of the students' mathematical reasoning (Rosales et al., 2012).

Moreover, the analytical system developed allows examining whether the type of problem influences the nature of the interaction between students and teachers for its resolution or not. Some studies showed there is little reasoning and low participation of students when solving routine problems (Rosales et al., 2012). The few studies including non-routine problems showed that student reasoning and participation, as well as metacognitive processes, increased (Sánchez-Barbero et al., 2019). Possible future research directions could aim at analysing further incidental specificities in the development of interaction, the nature of the student or teacher's questions, the forms of help, or the cycles started by students. Other research possibilities could be to compare the development of the interaction in relation with the knowledge of students, across different educational levels and subjects.

A study carried out using similar analytical methods and theoretical stances was also carried out at the university. Here, written productions of pre-service primary school teachers about their perceptions of their training process allowed interaction with the teacher educator about the development of the subject and the progress in pre-service teacher learning (Chamoso et al., 2012).

Concerning the interaction between students, Juárez Ramírez et al. (2020) analysed the influence of virtual forums on the modifications made by engineering students during a mathematical modelling project. The results showed that the level of interaction depended on how the experience was carried out, and the highest levels of interaction corresponded to the greatest improvements in the work of the students. In another study, the interaction between pre-service teachers enabled them to improve the mathematical tasks they had created (Cáceres et al., 2015).

All these different analyses and findings regarding interactional processes have contributed to a better understanding of how mathematics learning takes place. Proposals for learning improvement can thus be adopted through the planning, facilitation and enhancement of particular interactional processes in formal settings.

RESEARCH ON TEACHER KNOWLEDGE AND PROFESSIONAL DEVELOPMENT

In recent years, the interest in research into mathematics teachers' knowledge and professional development has increased in response to a perceived social need to improve the quality of teaching. A range of national and international research projects have brought researchers together in collaborative networks, such as RED 8 (Network 8: Mathematics Education and Teacher Training, (web.ua.es/es/red8educacion-matematica/red-8-educacion-matematica-y-formacion-del-profesorado.html), comprising researchers from eight different Spanish universities (Badillo et al., 2019), and RED MTSK (the Mathematics Teacher's Specialised Knowledge Network, https://redmtsk.net), which allows researchers from universities in Spain and Latin

America to cooperate. Broadly speaking, there are two major concerns framing the majority of studies in this area: (i) the identification, characterisation, conceptualisation and evaluation of the specific knowledge required for teaching, and (ii) the characterisation of teacher training programmes and the development of prospective teachers' competencies.

The range of different research aims requires the selection of specific conceptual frameworks within which to interpret results and make appropriate inferences from the conclusions, find answers to research questions, and propose new ones. Researchers more aligned with the concerns of (i) have tended to ground their work on models such as Didactic-Mathematical Knowledge and Competencies (DMKC), based on an onto-semiotic approach, or on models such as the Mathematics Teachers' Specialised Knowledge (MTSK) model. Researchers more in line with (ii) the activity of teaching itself and how appropriate competencies are developed have often based their work on the construct of professional noticing (Fernández et al., 2018). Framing one's work within theoretical models is not unique to mathematicians. Research studies are often situated within the scope of wider-reaching theories such as instrumental genesis, discourse analysis, or the ATD (Barquero et al., 2022). The section below describes various developments in research that are currently taking place in Spain and the contributions of theoretical frameworks to the advancement of research.

Research into the characterisation, conceptualisation and evaluation of teacher knowledge

The Mathematics Teaching Research Group (SIDM) at the University of Huelva has carried out research aimed at identifying the knowledge brought into play by teachers in the course of working through mathematical content in the classroom. The group's research involves the comprehensive analysis of classroom practice in collaborative research projects aimed at improving teacher education. The MTSK model (Carrillo et al., 2018) provides a holistic approach to the notion of specialised knowledge and comprises a system of categories which maximise its analytical potential.

The MTSK model enables the mathematics teacher's specialised knowledge to be interpreted at any educational level, or about any mathematical content. The group is currently carrying out research into the application of MTSK to teacher training (in terms of task design), and the work of mathematics teacher educators themselves, among other areas. The studies follow an interpretative paradigm and are based on case studies and teaching experimentation. Recent work has sought to connect studies and teacher knowledge from the perspective of MTSK with other theoretical frameworks for mathematics education, such as Mathematical Working Spaces (MWS), ethnomathematics, and didactic analysis.

The research group at the University of Barcelona place their work in the theoretical framework of the DMKC model for mathematics teachers. It is based on notions of the OSA to cognition and mathematical instruction (Godino et al., 2019). One key notion in this model is didactic suitability (DS), a checklist of six educational facets against which the suitability of a teaching procedure can be evaluated (by oneself or by someone else) to provide quality instruction and improve future implementations. The orientation of the group's research methodology is primarily qualitative. It is aimed at describing the practical argumentation and development of competencies of prospective and in-service primary and secondary school teachers, through the design and implementation of training cycles called experiments in the development of teachers' competencies and knowledge.

The GIPEAM research group has developed a line of research focused on characterising the mathematical and pedagogical knowledge shown by prospective primary teachers and in-service secondary teachers in training sessions by means of reflection in, for and about video excerpts of classroom practice. These qualitative studies are grounded in professional noticing and MTSK (de Gamboa et al., 2021).

Another line of research considers the role of knowledge in posing and solving problems in teacher education courses (Perdomo-Díaz et al., 2019; Piñeiro et al., 2021). This focus is currently being widened and applied to contexts of modelling and the integration of technology (Hernández et al., 2020).

Research has also been carried out on the affective dimension of prospective and in-service teachers concerning mathematics and its teaching and learning (Fernández-Cezar et al., 2020). Likewise, the relationship between mathematical and pedagogical knowledge and the affective domain has been studied to promote inclusive practices (Blanco et al., 2021).

Research linked to the analysis of classroom practice, the acquisition of teaching competency and professional development

Over the past few years, the GIDIMAT-UA group has developed training strategies for promoting professional noticing in mathematics teaching-learning contexts for primary and secondary teacher education programmes (Fernández et al., 2018; Fernández & Choy, 2020; Llinares, 2019). Some studies centre on defining learning trajectories as a means of developing prospective teachers' professional noticing of students' mathematical thinking (Ivars et al., 2020; Moreno et al., 2021; Sánchez-Matamoros et al., 2019). The methodological approach used in these studies is that of research cycles, in which learning environments are designed to integrate teaching records (diary-type notes, short video recordings, etc.), theoretical inputs taken from the research literature, selected to support the reasoning of the prospective teachers, for example, learning trajectories of specific concepts (Ivars et al., 2020), and guiding questions, to develop prospective teachers' professional noticing. The learning environments are considered as spaces for promoting teachers' reflection and broadening their understanding of their practice. The conceptual tools are theoretical knowledge. Some results have demonstrated that limited knowledge of mathematical content is likely to handicap the development of teaching competencies concerning organising mathematical content for teaching, and the interpretation of how students learn mathematics (Buforn et al., 2022). The use of teaching records allows prospective teachers to familiarise themselves with situations approximating classroom reality, and to develop the teaching competency of professional noticing, while also generating a professional discourse to describe classroom situations.

The theoretical models underlying the research into the affective domain in teacher education include both professional noticing and MTSK, models concerned with diversity training, and models seeking to develop functional thinking as a route into early algebraic reasoning (Oliveira et al., 2021). For those models based on MTSK, the studies aim to establish indicators for the knowledge implemented in teaching and learning mathematics with students that have special educational needs.

The use of technological tools (BlocksCAD) in teacher training has also been studied, and training strategies have been used following the Study and Research Path (SRP) model within the framework of the ATD (Florensa et al., 2021).

Another area which has recently made strides is research into teacher education through the development and implementation of STEAM activities. It seeks to analyse the practice of teachers using a methodology based on projects combining mathematics with other disciplines (Diego-Mantecón et al., 2021). The findings indicate a positive progression among teachers towards implementing integrated approaches.

End note

Collaboration through networks and research teams affects the results of the two main lines of research considered here. It combines the comprehensive analysis of the knowledge evidenced in in-service teachers' practice with the design of (pre-service or in-service) training tasks. In both cases—the analysis of classroom practice and task design—the researchers draw on theoretical models that interact with one another. Both domains follow a predominantly qualitative research paradigm in the form of case studies and teaching experiments. One implication of these studies is to transfer results to the design and evaluation of the effectiveness of teacher education programmes.

DEVELOPMENT AND ARTICULATION OF MATHEMATICS EDUCATION THEORIES IN SPAIN

In this paper, we synthesise the evolution of two research approaches in didactics of mathematics in Spain. They emerged intending to contribute to the development of didactics in the direction proposed by Guy Brousseau in the 1980s with the founding principles of the Theory of Didactic Situations in Mathematics (TDSM). They aspire to build a didactic science to deepen the study and understanding of didactic phenomena, i.e., those phenomena related to the production and dissemination of mathematical knowledge, and whose essential principle is the problematisation of mathematical knowledge. It abandons the assumption that didactics is exclusively concerned with the selection, sequence, and distribution over time of "given" mathematical content. It now postulates that the primary object of study is the mathematical activity itself. An epistemological approach is thus constituted, broadening the object of study of didactics, and hence extending the pedagogical-cognitive approach. The "Brousseaunian revolution" (Gascón, 2013) can be considered the *raison d'être* of the DMSC group (Didactics of Mathematics as a Scientific Discipline), set up in the SEIEM in 1998. It derives from the regular seminars organised since 1991 within the Inter-University Research Seminar.

Within the DMSC group, the Onto-Semiotic Approach to Mathematical Knowledge and Instruction subgroup emerged (Godino & Batanero, 1994; Godino et al., 2007, 2019), initially promoted by the FQM126 Group at the University of Granada. In parallel, the ATD (Chevallard, 1992, 2015) appeared, from the research group led by Josep Gascón at the Autonomous University of Barcelona, and Marianna Bosch at the University of Barcelona (BAHUJAMA Group).

This paper aims to present the most salient characteristics of the research developed in both theoretical approaches and their developments in recent decades. Their dialogue is still alive today, beyond their origins and the internal evolution in the DMDC group, as shown by the works published in *For the Learning of Mathematics* (Gascón & Nicolás, 2017; Godino et al., 2019) or in the course about dialogue between theories in the *Intensive Research Programme on the ATD* held at the Centre of Mathematical Research (Barcelona) in 2019.

Emergence and development of the OSA

Faced with the diverse theories in mathematics education, each addressing partial questions about teaching and learning, using different languages and theoretical tools, the Onto-semiotic Approach (OSA) aims to construct a theoretical system that allows addressing the epistemological, ontological, semiotic, cognitive and instructional dimensions involved in mathematics education in a unified manner.

Thus, in the OSA seminal article (Godino et al., 1994), an explanation of the meaning of mathematical objects and its relationship with other notions, such as concept, conception, representation, schema or understanding, is provided. An anthropological and pragmatist view of mathematics is assumed (Font et al., 2013). Therefore, the activity of people when solving problems is considered the central element in the generation of mathematical knowledge, as well as in its teaching and learning. It is further assumed that mathematics is not only a human activity but also an organised system of culturally shared objects. This is the reason why we need to analyse the various objects and processes emerging from the types of practices, as well as their structure. It is necessary to deepen our understanding of the problems of learning and teaching. Consequently, it was natural to develop an ontology and semiotics for the description of mathematical activity, as well as the processes of communication and production inherent to this activity. The pragmatist theory of the meanings of mathematical objects, as well as the typology of objects and processes developed, have served as the basis for the elaboration of a theory of mathematical instruction that considers the triple dialectic between content, teacher and students.

In addition, a theoretical DMKC model has been developed within the OSA (Breda et al., 2017). The global competency of analysis and didactic intervention of the mathematics teacher is believed to consist of five sub-competencies, which are associated with five conceptual and methodological OSA tools: *analysis of global meanings* (based on the identification of problem-situations and operative, discursive and normative practices involved in their resolution);, *onto-semiotic analysis of practices* (identification of the network of objects and processes involved in the practices);, *management of didactic configurations and trajectories* (identification of the sequence of interaction patterns between teacher, student, content, and resources);, *normative analysis* (recognition of the web of norms and meta-norms that condition and support the instructional process);, and *analysis of didactical suitability* (assessment of the instructional process and identification of potential improvements).

The problem of articulating theoretical frameworks is one of the central problems that gave rise to the OSA: the attempt to understand, compare, coordinate, and integrate theories used in mathematics didactics, such as the Theory of Didactic Situations (TDS) in mathematics (Brousseau), the Theory of Conceptual Fields (TCF) (Vergnaud), the Theory of Didactic Transposition (TDT) (Chevallard), the Theory of Semiotic Representation Registers (TRSR) (Duval), amongst others. Several articles in which these articulations are addressed (Godino et al., 2006), as well as a large number of articles with applications of OSA tools to various mathematical contents, have been published. These publications show the impact and dissemination of the OSA in different parts of the world. They are available at http://enfoqueontosemiotico.ugr.es.

Emergence and development of the ATD

The Anthropological Theory of the Didactic (ATD) originated as a research programme in the 1980s with the developments of the theory of didactic transposition and has evolved over the past 30 years (Bosch & Gascón, 2006). Today, approximately one hundred researchers collaborate in the development of the ATD, mostly in Europe, Latin America, Canada, and Japan. It is worth highlighting that the international congress of the ATD (CITAD), held since 2005, facilitates the discussion of the research advances. The dialogue between theories, curriculum issues and teacher education are recurrent discussion topics.

Since its beginnings, the ATD has adopted a broad institutional perspective. Creating, teaching, learning, and disseminating mathematics are human activities that take place in different institutional settings and through complex transposition processes. A general model of human activity is proposed using the key notion of *praxeology* (Chevallard, 1992). It constitutes the minimal unit of analysis of human activities and the knowledge used and generated. In the same direction as the TDS, when addressing a didactic problem related to certain knowledge (numeracy, algebra, negative numbers, calculus, among others), a fundamental step is to question the dominant epistemological models in the institutions involved, which provide a particular vision of the knowledge at stake. For this purpose, all the steps in the didactic transposition process are considered, and the so-called reference epistemological models (REMs) (Bosch & Gascón, 2006) are proposed as key methodological tools for providing didactics with an emancipatory point of view.

Closely linked to the previous developments, more recent work focuses on the transition between pedagogical paradigms. Chevallard (2015) characterises the current situation considering the dominant paradigm of “visiting works”, in contrast with a broader paradigm of “questioning the world” in which enquiring into problematic questions plays a crucial role. To investigate the conditions that allow the advancement from one paradigm to the other, some reference didactic models have been developed through the proposal of study and research paths (SRPs) (Bosch, 2018). Several SRPs have been designed and implemented at different educational levels, as well as in teacher education, identifying new needs in teaching devices and epistemological infrastructures (Barquero et al., 2021; Ruiz-Olarría et al., 2019; Sierra et al., 2012).

The ecological approach is a central research methodology in the ATD. It focuses on examining the conditions that encourage certain school activities to exist and grow, and the constraints that hinder them from existing and growing in certain institutional settings. In this ecological analysis, it is crucial to locate at which (mathematical, didactic, pedagogical, school, social) levels these conditions and constraints appear. The main aim is to study the range of possibilities offered by educational institutions and to anticipate difficulties when changes are introduced. Previous research (Barquero et al., 2019) has highlighted the usefulness of combining epistemological and ecological dimensions to compare different theoretical approaches, and to understand their impact on the formulation of research problems and the delimitation of the corresponding unit of analysis.

DISCUSSION

In 2008, S. Llinares, one of the former SEIEM presidents, explored what he called the “agendas of research in Mathematics Education in Spain” and found “a variety of perspectives and theoretical approaches used by researchers to try to answer the wide range of questions raised” (Llinares, 2008, p. 25). He also pointed out the need for strong theories from the point of view of their ability to explain the phenomena under study, putting the spotlight on the impact of research on the educational system and marking it as a task to be carried out.

Almost 15 years later, the variety of research questions and approaches remains, with the logical evolution that permeates any scientific field, partially as a consequence of the new social demands. Despite several local initiatives carried out by the SEIEM and the CEMat, the issue of the impact of research is still a pending task. Mathematics education in Spain has acquired a level of maturity, productivity and visibility that puts it on a par with neighbouring countries. However, the SEIEM status and the kind of influence it can provide to current teaching practice and institutions are sources of open questions that demand a lot of effort from the entire community, and certainly the development of new methods and theoretical perspectives.

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References

Almeida, R., Bruno, A., & Perdomo, J. (2014). Estrategias de sentido numérico en estudiantes del Grado en Matemáticas. *Enseñanza de las Ciencias*, *32*(2), 9-34.

Alonso-Castaño M., Alonso P., Mellone, M., & Rodríguez-Muñiz L. J. (2021). What mathematical knowledge do prospective teachers reveal when creating and solving a probability problem? *Mathematics*, *9*(24), 3300.

Alsina, Á. (2016). El currículo del número en educación infantil. Un análisis desde una perspectiva internacional. *PNA*, *10*(3), 135-160.

Aravena Díaz, M., Gutiérrez, A., & Jaime, A. (2016). Study of Van Hiele levels of reasoning in students from vulnerable secondary schools in Chile. *Enseñanza de las Ciencias*, *34*(1), 107-128.

Ariza, A., Llinares, S., & Valls, J. (2015). Students' understanding of the function-derivative relationship when learning economic concepts. *Mathematics Education Research Journal*, *27*, 615-635.

Ayala-Altamirano, C., & Molina, M. (2020). Meanings attributed to letters in functional contexts by primary school students. *International Journal of Science and Mathematics Education*, *18*(7), 1271-1291.

Azcárate, C., Camacho-Machín, M., González, M. T., & Moreno, M. (Eds.) (2015). *Didáctica del análisis matemático: Una revisión de las investigaciones sobre su enseñanza y aprendizaje en el contexto de la SEIEM*. Universidad de La Laguna.

Badillo, E., Climent, N., Fernández, C., & González, M.T. (Eds.) (2019). *Investigación sobre el profesor de Matemáticas: Prácticas de aula, conocimiento, competencia y desarrollo profesional*. Ediciones Universidad de Salamanca

Barquero, B., Bosch, M., Florensa, I., & Ruiz-Munzón, N. (2021). Study and research paths in the frontier between paradigms. *International Journal of Mathematical Education in Science and Technology*. doi.org/10.1080/0020739X.2021.1988166

Barquero, B., Bosch, M., & Gascón, J. (2019). The unit of analysis in the formulation of research problems: The case of mathematical modelling at university level. *Research in Mathematics Education, 21*(3), 314-330.

Barquero, B., Bosch, M., & Florensa, I. (2022). Contribuciones de los recorridos de estudio e investigación en la universidad. El caso de la formación del profesorado. *AIEM, nº 21*, 87-106.

Batanero, C. (2000). Controversies around the role of statistical tests in experimental research. *Mathematical Thinking and Learning, 2*(1-2), 75-97.

Batanero, C., Begué, N., Borovcnik, M., & Gea, M. M. (2020). Ways in which high school students understand the sampling distribution for proportions. *Statistics Education Research Journal, 19*(3), 32-52.

Batanero, C., Burrill, G., & Reading, C. (2011). *Teaching statistics in school mathematics—Challenges for teaching and teacher education: A joint ICMI/IASE study.* Springer.

Batanero, C., Godino, J. D., & Estepa, A. (1998). Building the meaning of statistical association through data analysis activities. In A. Olivier, & K. Newstead (Eds.), *Proceedings of PME22* (Vol. 1, pp. 221-242). PME.

Batanero, C., Navarro-Pelayo, V., & Godino, J. D. (1997). Effect of the implicit combinatorial model on combinatorial reasoning in secondary school pupils. *Educational Studies in Mathematics, 32*, 181-199.

Batanero, C., Tauber, L. M., & Sánchez, V. (2004). Students' reasoning about the normal distribution. In D. Ben-Zvi, & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 257-276). Springer.

Berciano, A., Anasagasti, J., & Zamalloa, T. (2021). Sentido estadístico en la formación de las y los estudiantes del Grado de Educación Infantil. Una aproximación desde un contexto de aprendizaje STEAM. *PNA, 15*(4), 289-309.

Berciano, A., Ortega, T., & Puerta, M. (2015). Aprendizajes de las interpolaciones gráficas y algebraicas. Análisis comparativo. *Enseñanza de las Ciencias, 33*(3), 43-58.

Bernabeu, M., Moreno, M., & Llinares, S. (2021). Primary school students' understanding of polygons and the relationships between polygons. *Educational Studies in Mathematics, 106*(2), 251-270.

Blanco, T. F., Godino, J. D., Sequeiros, P. G., & Diego-Mantecón, J. M. (2019). Skill levels on visualization and spatial reasoning in pre-service primary teachers. *Universal Journal of Educational Research, 7*(12), 2647-2661.

Blanco, M. T., Roel, V. G., Capone, R., Branchetti, L., & Gaio, A. (2021). Symmetry-art: a steam training workshop for primary school teachers. In M. Inprasitha, et al. (Eds.), *Proceedings of PME44,* (Vol. 2, pp. 102-110). PME.

Boigues, F. J., Llinares, S., & Estruch, D. (2010). Desarrollo de un esquema de la integral definida en estudiantes de ingeniería relacionadas con las ciencias de la naturaleza. *RELIME*, *13*(3), 255-282.

Borji, V., Erfani, H., & Font, V. (2020). A combined application of APOS and OSA to explore undergraduate students' understanding of polar coordinates. *International Journal of Mathematical Education in Science and Technology*, *51*(3), 405-423.

Bosch, M. (2018). Study and Research Paths: A model for inquiry. In B. Sirakov et al. (Eds.), *International Congress of Mathematicians* (Vol. 3, pp. 4001-4022). World Scientific.

Bosch, M., & Gascón, J. (2006). 25 years of the didactic transposition. *Bulletin of the International Commission on Mathematical Instruction, 58*, 51-65.

Breda, A., Pino-Fan, L., & Font, V. (2017). Meta didactic-mathematical knowledge of teachers: Criteria for the reflections and assessment on teaching practice. *Eurasia Journal of Mathematics Science and Technology Education, 13,* 1893-1918.

Bruno, A., & Noda, A. (2019) The concept of tens and hundreds in students with Down síndrome. *International Journal of Disability, Development and Education, 66*(2), 171-185.

Buforn, À., Llinares, S., Fernández, C., Coles, A., & Brown, L. (2022). Pre-service teachers' knowledge of the unitizing process in recognizing students' reasoning to propose teaching decisions. *International Journal of Mathematics Education in Science and Technology, 53*(2), 425-443.

Cáceres, M. J., Chamoso, J. M., & Cárdenas, J. A. (2015). Situaciones problemáticas auténticas propuestas por estudiantes para maestro. In C. Fernández et al. (Eds.), *Investigación en Educación Matemática XIX* (pp. 201-210). SEIEM.

Callejo, M. L., Fernández, C., & García-Reche, Á. (2019). Cognitive apprehension in visual pattern generalization problems. *Journal for the Study of Education and Development, 42*(4), 783-828.

Callejo, M.L., Pérez-Tyteca, P., Moreno, M., & Sánchez-Matamoros, G. (2021). The use of a length and measurement HLT by pre-service kindergarten teachers to notice children's mathematical thinking. *International Journal of Science and Mathematics Education, 20*, 597-617.

Camacho, M., Depool, R., & Santos-Trigo, L. M. (2010). Students' use of DERIVE software in comprehending and making sense of definite integral and area concepts. In F. Hitt et al. (Eds.), *Research in collegiate mathematics education VII (CBMS* 16) (pp. 29-61). AMS.

Camacho, M., Perdomo, J., & Santos, M. (2012). Procesos conceptuales y cognitivos en la introducción de las EDO vía la resolución de problemas. *Enseñanza de las Ciencias, 30*(2), 9-32.

Cañadas, M. C., Castro, Enc., & Castro, Enr. (2009). Using a model to describe students' inductive reasoning in problem-solving. *Electronic Journal of Research in Educational Psychology, 7*(1), 261-278.

Cañadas, M. C., Molina, M., & Del Río, A. (2018). Meanings given to algebraic symbolism in problem posing. *Educational Studies in Mathematics, 98*, 19-37.

Cañizares, M. J. (1997). *Influencia del razonamiento proporcional y combinatorio y de creencias subjetivas en las intuiciones probabilísticas primarias* [Unpublished PhD]. University of Granada.

Caro, A., & Planas, N. (2021). Estudio exploratorio con futuras maestras sobre lenguas matemáticas para enseñar la relación entre área y volumen. *AIEM, 19*, 117-131.

Carrillo, J., Climent, N., Contreras, L. C., & Montes, M. (2020). Using professional development contexts to structure prospective teacher education. In S. Llinares et al. (Eds.), *International handbook of mathematics teacher education* (Vol. 2, pp. 393-419). Brill.

Carrillo, J., Climent, N., Montes, M., Contreras, L., Flores-Medrano, E., Escudero-Ávila, D. ... Muñoz-Catalán, M. C. (2018). The mathematics teacher's specialised knowledge (MTSK) model. *Research in Mathematics Education, 20*(3), 236-253.

Castro, Enc. (1995). *Exploración de patrones numéricos mediante configuraciones puntuales* [Unpublished PhD]. University of Granada.

Castro, Enc., Cañadas, M. C., Molina, M., & Rodríguez-Domingo, S. (2022). Difficulties in semantically congruent translation of verbally and symbolically represented algebraic statements. *Educational Studies in Mathematics, 109*, 593-609.

Castro, Enr., & Frías, A. (2013). Two-step arithmetic word problems. *The Mathematics Enthusiast, 10*(1,2), 37-64.

Claros, F. J., Coriat, M., & Compaña, T. S. (2016). Estructura cognitiva y fenomenología: El caso de la sucesión convergente. *Enseñanza de las Ciencias, 34*(2), 87-105.

Clemente, F., y Llinares, S., (2015). Formas del discurso y razonamiento configural de estudiantes para maestros en la resolución de problemas de geometría. *Enseñanza de las Ciencias 33*(1), 9-27.

Codes, M., González-Astudillo, M. T., Delgado, M. L., & Monterrubio, M. C. (2013). Growth in the understanding of infinite numerical series: A glance through the Pirie and Kieren theory. *International Journal of Mathematical Education in Science and Technology, 44*(5), 652-662.

Chamoso, J. M., Cáceres, M. J., & Azcárate, P. (2012). Reflection on the teaching-learning process in the initial training of teachers. Characterization of the issues on which pre-service mathematics teachers reflect. *Teaching and Teacher Education, 28,* 154-164.

Chevallard, Y. (1992). Fundamental concepts in didactics: Perspectives provided by an anthropological approach. In R. Douady, & A. Mercier (Eds.), *Research in didactique of mathematics, selected papers* (pp. 131-167). La Pensée Sauvage.

Chevallard, Y. (2015). Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm. In S. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education* (pp. 173-187). Springer.

De Castro, C. (2018). El papel de las imágenes en el proyecto «¡A contar!» para el aprendizaje de las matemáticas en la educación infantil. *Unión*, *53*, 138-157.

De Gamboa, G., Badillo, E., Ribeiro, M., Montes, M., & Sánchez-Matamoros, G. (2020). The role of teachers' knowledge in the use of learning opportunities triggered by mathematical connections. In S. Zehetmeier, et al. (Eds.), *Professional development and knowledge of mathematics teachers* (pp. 24-43). Routledge.

Diago, P. D., González-Calero, J. A., & Yáñez, D. F. (2021). Exploring the development of mental rotation and computational skills in elementary students through educational robotics. *International Journal of Child-Computer Interaction, 100388.*

Diego-Mantecón, J. M., Prodromou, T., Lavicza, Z., Blanco, T. F., & Ortiz-Laso, Z. (2021). An attempt to evaluate STEAM project-based instruction from a school mathematics perspective. *ZDM - Mathematics Education, 53*, 1137-1148.

Fernández, C., & Choy, B.H. (2020). Theoretical lenses to develop mathematics teacher noticing. In S. Llinares & O. Chapman (Eds.), *International handbook of mathematics teacher education* (Vol. 2, pp. 337-360). Brill.

Fernández, C., Sánchez-Matamoros, G., Valls, J., & Callejo, M.L. (2018). Noticing students' mathematical thinking: Characterization, development and contexts. *AIEM, 13*, 39-61.

Fernández-Cézar, R., Hernández-Suárez, C.A., Prada-Núñez, R., & Ramírez-Leal, P. (2020). Beliefs and anxiety towards mathematics: A comparative study between teachers of Colombia and Spain. *Bolema, 64*(68), 1174-1205.

Fernández-León, A., Gavilán-Izquierdo, J. M., González-Regaña, A. J., Martín-Molina, V., & Toscano, R. (2021). Identifying routines in the discourse of undergraduate students when defining. *Mathematics Education Research Journal, 33*(2), 301-319.

Fernández-Millán E., & Molina, M. (2016). Indagación en el conocimiento conceptual del simbolismo algebraico de estudiantes de secundaria mediante la invención de problemas. *Enseñanza de las Ciencias, 34*(1), 53-71.

Fernández-Plaza, J. A., & Simpson, A. (2016). Three concepts or one? Students' understanding of basic limit concepts. *Educational Studies in Mathematics, 93*(3), 315-332.

Florensa, I., Bosch, M., & Gascón, J. (2021). Question-answer maps as an epistemological tool in teacher education. *Journal of Mathematics Teacher Education, 24,* 203-225.

Font, V., Godino, J. D., & Gallardo, J. (2013). The emergence of objects from mathematical practices. *Educational Studies in Mathematics, 82,* 97-124.

Fuentealba, C., Badillo, E., Sánchez-Matamoros, G., & Cárcamo, A. (2018). The understanding of the derivative concept in higher education. *EURASIA Journal of Mathematics, Science and Technology Education, 15*(2), em1662.

Gaita, C., & Ortega, T. (2014). Concepciones sobre lugar geométrico en estudiantes de arquitectura. *Educaçao Matemática Pesquisa, 16*(4), 1133-1145.

García, M., Llinares, S., & Sánchez-Matamoros, G. (2011). Characterizing thematized derivative schema by the underlying emergent structures. *International Journal of Science and Mathematics Education, 9,* 1023-1205.

Gascón, J. (2013). La revolución brousseauniana como razón de ser del grupo Didáctica de las Matemáticas como Disciplina Científica. *AIEM, 3,* 69-87.

Gascón, J., & Nicolás, P. (2017). Can didactics say how to teach? The beginning of a dialogue between the anthropological theory of the didactic and other approaches. *For the Learning of Mathematics, 37*(3), 9-13.

Gavilán-Izquierdo, J. M., & Gallego-Sánchez, I. (2021). How an upper secondary school teacher provides resources for the transition to university: A case study. *International Electronic Journal of Mathematics Education, 16*(2), em0634.

Gavilán Izquierdo, J. M., Sánchez-Matamoros, G., & Escudero, I. (2014). Aprender a definir en Matemáticas: Estudio desde una perspectiva sociocultural. *Enseñanza de las Ciencias, 32*(3), 529-550.

Gea, M. M., López-Martín, M. M., & Roa, R. (2015). Conflictos semióticos sobre la correlación y regresión en los libros de texto de Bachillerato. *AIEM, 8,* 29-49.

Gil, N., Blanco, L., & Guerrero, E. (2006). El papel de la afectividad en la resolución de problemas matemáticos. *Revista de Educación, 340,* 551-569.

Godino, J. D., & Batanero, C. (1994). Significado institucional y personal de los objetos matemáticos. *Recherches en Didactique des Mathématiques, 14*(3), 325-355.

Godino, J. D. Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM - Mathematics Education, 39* (1-2), 127-135.

Godino, J. D., Batanero, C., & Font, V. (2019). The onto-semiotic approach: Implications for the prescriptive character of didactics. *For the Learning of Mathematics, 39*(1), 38-43.

Godino, J. D., Cajaraville, J. A., Fernández, T., & Gonzato, M. (2012). Una aproximación ontosemiótica a la visualización en educación matemática. *Enseñanza de las Ciencias, 30* (2), 163-184.

Godino, J. D., Font, V., Contreras, A., & Wilhelmi, M. R. (2006). Una visión de la didáctica francesa desde el enfoque ontosemiótico de la cognición e instrucción matemática. *RELIME, 9* (1), 117-150.

Gómez, B., & Puig, L. (2014). *Resolver problemas. Estudios en memoria de Fernando Cerdán*. PUV.

Gómez-Chacón, I. M., & Kuzniak, A. (2015). Spaces for geometric work: Figural, instrumental and discursive geneses of reasoning in a technological environment. *International Journal of Science and Mathematics Education*, *13*(1), 201-226.

González, M. T., Espinel, M. C., & Ainley, J. (2011). Teachers' graphical competence. In C. Batanero, et al. (Eds.), *Teaching statistics in school mathematics—Challenges for teaching and teacher education* (pp. 187-197). Springer.

González-Marí, J. L., Rico, L., & Gallardo, J. (20009). Diversidad estructural y semiótica en el proceso didáctico de ampliación de los naturales a los enteros: Un estudio sobre comprensión en el campo de la relatividad aditiva. *Electronic Journal of Research in Educational Psychology, 7*(1), 309-340.

González-Martín, A. (2015). *La generalización de la integral definida desde las perspectivas gráfica, numérica y algebraica utilizando entornos informáticos. Problemas de enseñanza y aprendizaje* [Unpublished PhD]. University of La Laguna.

González-Regaña, A., Martín-Molina, V., Toscano, R., Fernández-León, A., & Gavilán-Izquierdo, J. M. (2021). El discurso de estudiantes para maestros cuando describen y definen cuerpos geométricos. *Enseñanza de las Ciencias, 39*(1), 81-97.

Gorgorió, N., & Planas, N. (2001). Teaching mathematics in multilingual classrooms. *Educational Studies in Mathematics, 47*, 7-33.

Guerrero-Ortiz, C., Mejía, H., & Camacho-Machín, M. (2016) Representations of a mathematical model as a means of analysing growth phenomena. *Journal of Mathematical Behavior, 42*, 109-126.

Guillén, G. (2004). El modelo de Van Hiele aplicado a la geometría de los sólidos: Describir, clasificar, definir y demostrar como componentes de la actividad matemática. *Educación Matemática*, *16*(3), 103-125.

Gutiérrez, A., Benedicto, C., Jaime, A., & Arbona, E. (2018). The cognitive demand of a gifted student's answers to geometric pattern problems. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (pp. 169-198). Springer.

Gutiérrez, A., Jaime, A., & Fortuny, J. M. (1991). An alternative paradigm to evaluate the acquisition of the Van Hiele levels, *Journal for Research in Mathematics Education, 22*(3), 237-251.

Gutiérrez, A., Ramírez, R., Benedicto, C., Beltrán-Meneu, M. J., & Jaime, A. (2018). Visualization abilities and complexity of reasoning in mathematically gifted students' collaborative solutions to a visualization task. A networked analysis. In K. S. Mix, & M. T. Battista (Eds.), *Visualizing mathematics* (pp. 309-337). Springer.

Hernández, A., Perdomo-Díaz, J., & Camacho-Machín, M. (2020). Mathematical understanding in problem-solving with GeoGebra: A case study in initial teacher education. *International Journal of Mathematics Education in Science and Technology, 51*(2), 208-223.

Ivars, P., Fernández, C., & Llinares, S. (2020). A learning trajectory as a scaffold for pre-service teachers' noticing of students' mathematical understanding. *International Journal of Mathematics Education in Science and Technology, 18*(3), 529-548.

Juárez, J. A., Chamoso, J. M., & González, M. T. (2020). Interacción en foros virtuales al integrar modelización matemática para formar ingenieros. *Enseñanza de las Ciencias, 38*(3), 161-178.

Licera, R. M., Gascón, J., & Bosch, M. (2019). Las tres dimensiones fundamentales del problema didáctico de los números reales. *Contextos de Educación, 26*(19), 13-26.

Llinares, S. (2008). Agendas de investigación en educación matemática en España: Una aproximación desde "ISI-web of knowledge" y ERIH. In R. Luengo et al. (Eds.), *Investigación en educación Matemática XII* (pp. 25-54). SEIEM.

Llinares, S., & Clemente, F. (2014). Characteristics of Pre-service Primary School Teachers' Configural Reasoning. *Mathematical Thinking and Learning, 16,* 234-250.

Llinares, S., & Clemente, F. (2019). Characteristics of the shifts from configural reasoning to deductive reasoning in geometry. *Mathematics Education Research Journal, 31,* 259-277.

Lonjedo, M., Huerta, M. P., & Carles, M. (2012). Conditional probability problems in textbooks: An example from Spain. *RELIME, 15*(3), 319-337.

Lucas, C. O., Gascón, J., & Fonseca, C. (2017). Raison d'être of the elementary differential calculus in the transition between secondary school and university. *Redimat, 6*(3), 283-306.

Marmolejo, G. A., & González Astudillo, M. T. (2015). Visual control in the construction of the area of two-dimensional plane figures, in textbooks. *Revista Latinoamericana de Investigación en Matemática Educativa, 18*(3), 301-329.

Martins, J. A., Nascimento, M. M., & Estrada, M. A. (2012). Looking back over their shoulders: A qualitative analysis of Portuguese teachers' attitudes towards statistics. *Statistics Education Research Journal, 11*(2), 26-44.

Molina, M., Castro, Enc., & Castro, Enr. (2008). Third graders' strategies and use of relational thinking when solving number sentences. In O. Figueras et al. (Eds.), *Proceedings PME32 & PME-NA30* (Vol. 3, pp. 399-406). Cinvestav.

Molina, M., Rodríguez-Domingo, S., Cañadas, M. C., & Castro, Enc. (2017). Secondary school students' errors in the translation of algebraic statements. *International Journal of Science and Mathematics Education, 15*, 1137-1156.

Moreno, M., Sánchez-Matamoros, G., Callejo, M. L., Pérez-Tyteca, P., & Llinares, S. (2021). How prospective kindergarten teachers develop their noticing skills: The instrumentation of a learning trajectory. *ZDM - Mathematics Education*, *53*(1), 57-72.

Oliveira, H., Polo-Blanco, I., & Henriques, A. (2021). Exploring prospective elementary mathematics teachers' knowledge: A focus on functional thinking. *Journal of Mathematics Teacher Education, 12*(2), 257-278.

Ortega, T., & Pecharromán, C. (2010). Diseño de enseñanza de las propiedades globales de las funciones a través de sus gráficas. *Enseñanza de las Ciencias*, *28*(2), 215-225.

Orts, A., Llinares, S., & Boigues, F. J. (2018). High School students' learning trajectories of the concept of tangent line. *Enseñanza de las Ciencias, 36*(3), 121-140.

Pallauta, J., Arteaga, P., & Garzón-Guerrero, J. A. (2021). Secondary school students' construction and interpretation of statistical tables. *Mathematics*, *9*(24), 3197.

Paneque, J. J., Cobo, P., & Fortuny, J. M. (2017). Intelligent tutoring and the development of argumentative competence. *Technology, Knowledge and Learning*, *22*(1), 83-104.

Pecharromán, C., Arce, M., & Conejo, L. (2019). Estrategias y errores de conversión entre representaciones de intervalos de la recta real. *Enseñanza de las Ciencias*, *37*(3), 169-187.

Perdomo-Díaz, J., Felmer, P., & Rojas, C. (2019). Teachers' mathematical tensions surfacing during the first session of a problem-solving professional development workshop. In P. Felmer et al. (Eds.), *Problem solving in mathematics instruction and teacher professional development* (pp. 359-378). Springer.

Pinto, E., Cañadas, M. C., & Moreno, A. (2021). Functional relationships evidenced and representations used by third graders within a functional approach to early algebra. *International Journal of Science and Mathematics Education*, doi.org/10.1007/s10763-021-10183-0

Piñeiro, J. L., Castro-Rodríguez, E., & Castro, E. (2021). Mathematical problem-solving in two teachers' knowledge models: A critical analysis. *Mathematics Teaching Research Journal*, *13*(1), 71-93.

Planas, N., Arnal-Bailera, A., & García-Honrado, I. (2018). El discurso matemático del profesor: ¿cómo se produce en clase y cómo se puede investigar? *Enseñanza de las Ciencias, 36*(1), 45-60.

Planas, N., & Setati, M. (2009). Bilingual students using their languages in the learning of mathematics. *Mathematics Education Research Journal, 21*, 36-59.

Planas, N., & Valero, P. (2016). Tracing the socio-cultural-political axis in understanding mathematics education. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The second handbook of research on the psychology of mathematics education* (pp. 447-479). Brill.

Polo-Blanco, I., González, M. J., Bruno, A., & González, J. (2021). Teaching students with mild intellectual disability to solve word problems using schema-based instruction. *Learning Disability Quarterly*. doi.org/10.1177/07319487211061421

Puig, L. (2018). Dos errores famosos en la Arithmetica Algebratica de Marco Aurel reconsiderados. En D. Ruiz-Berdún (Ed.), *Ciencia y técnica en la universidad: Trabajos de historia de las ciencias y de las técnicas* (Vol II, pp. 215-228). Universidad de Alcalá - Sociedad Española de Historia de las Ciencias y de las Técnicas.

Ramírez, R., Brizuela, B., & Blanton, M. (2022). Kindergarten and first-grade students´ understandings and representations of arithmetic properties. *Early Childhood Education Journal, 50, 357.*

Ramírez, R., & Flores, P. (2017). Habilidades de visualización de alumnos con talento matemático: Comparativa entre los test psicométricos y las habilidades de visualización manifestadas en tareas geométricas. *Enseñanza de las Ciencias*, *35*(2), 179-196.

Rico, L., Sierra, M., & Castro, E. (2002). El área de conocimiento de Didáctica de la Matemática. *Revista de Educación, 328,* 35-58.

Rodríguez, J. L., Romero, I., & Codina, A. (2021). The influence of NeoTrie VR's immersive virtual reality on the teaching and learning of geometry. *Mathematics*, *9*(19), 2411.

Rodríguez-Muñiz, L., Muñiz-Rodríguez, L., & Aguilar-González, Á. (2021). El recuento y las representaciones manipulativas: Los primeros pasos de la alfabetización estadística. *PNA*, *15*(4), 311-338.

Rosales, J., Vicente, S., Chamoso, J. M., Múñez, D., & Orrantia, J. (2012). Teacher-student interaction in joint word problem-solving. The role of situational and mathematical knowledge in mainstream classrooms. *Teaching and Teacher Education, 28,* 1185-1195.

Ruiz-Olarría, A., Bosch, M., & Gascón, J. (2019). Construcción de una praxeología para la enseñanza en la institución de formación del profesorado. *Educación Matemática, 31*(2), 132-160.

Sánchez, V., & García, M. (2014). Socio-mathematical and mathematical norms related to definition in pre-service primary teachers' discourse. *Educational Studies in Mathematics, 85*(2), 305-320.

Sánchez-Barbero, B., Calatayud, M., & Chamoso, J. M. (2019). Análisis de la interacción de maestros cuando resuelven problemas realistas conjuntamente con sus alumnos en aulas de primaria, teniendo en cuenta su experiencia docente. *Uni-pluriversidad, 19*(2), 40-59.

Sánchez-Matamoros, G., Fernández, C., & Llinares, S. (2019). Relationships among prospective secondary mathematics teachers' skills of attending, interpreting and responding to students' understanding. *Educational Studies in Mathematics, 100*(1), 83-99.

Segovia, I., & Castro, E. (2009). Computational and measurement estimation: Curriculum foundations and research carried out at the University of Granada, Mathematics Didactics Department. *Electronic Journal of Research in Educational Psychology, 17*(7), 499-536.

Serradó, A., Azcárate, P., & Cardeñoso, J. M. (2005). Randomness in textbooks: The influence of deterministic thinking. In M. Bosch (Ed.), *Proceedings of CERME4* (pp. 559-568). ERME.

Serradó, A., Azcárate, P., & Cardeñoso, J. M. (2006). Analyzing teacher resistance to teaching probability in compulsory education. In A. Rossman, & B. Chance (Eds.), *Proceedings of ICOTS-7.* IASE.

Sierra, T. A., Bosch, M., & Gascón, J. (2012). La formación matemático-didáctica del maestro de Educación Infantil: El caso de «cómo enseñar a contar». *Revista de Educación, 357,* 231-256.

Socas, M. (2007). Dificultades y errores en el aprendizaje de las matemáticas. Análisis desde el enfoque lógico semiótico. In M. Camacho et al. (Eds.), *Investigación en Educación Matemática XI* (pp. 19-52). SEIEM.

Sua, C., Gutiérrez, Á., & Jaime, A. (2021). Análisis de una actividad de visualización en un entorno de geometría dinámica 3d y realidad aumentada. In P. D. Diago et al. (Eds.), *Investigación en Educación Matemática XXIV* (pp. 579-586). SEIEM.

Torres, M. C., Moreno, A., & Cañadas, M. C. (2021). Generalization process by second-grade students. *Mathematics, 9,* 1109.

Toscano, R., Gavilán-Izquierdo, J. M., & Sánchez, V. (2019). A study of pre-service primary teachers' discourse when solving didactic-mathematical tasks. *Eurasia Journal of Mathematics, Science and Technology Education, 15*(11), em1762.

Valls, J., Pons, J., & Llinares, S. (2011). Coordinación de los procesos de aproximación en la comprensión del límite. *Enseñanza de las Ciencias, 29*(3), 325-338.

Vásquez, C., & Alsina, Á. (2019). Intuitive ideas about chance and probability in children from 4 to 6 years old. *Acta Scientiae*, *21*(3), 131-154.