STORIES OF OBSERVING, INTERVIEWING AND RESEARCHING IN COLLABORATIVE GROUPS TO DEVELOP MATHEMATICS TEACHING AND LEARNING

Laurinda Brown

University of Bristol, School of Education

Using the context of comparing and contrasting three papers from early in my career with three recent papers, I tell stories of my developing research practices, interviewing, observing and researching in collaborative groups focused on learning. Teachers supporting students to develop their stories of mathematics looks to the future with children working on solving real world problems, such as, climate change.

BEGINNINGS

I attended my first PME (Lisbon, 1994) without presenting a paper. I was a mature professional, having recently moved from being a mathematics teacher to being a mathematics teacher educator at the University of Bristol, School of Education (SoE). Since I still thought of myself as a teacher, it seemed natural for me to attend the *Teachers as researcher in mathematics education* working group led by Judy Mousley (RIP), Vicki Zack, and Chris Breen. They invited me to contribute in a small way to the following year's conference in Recife, Brazil, where I also presented my first research report. So began many fruitful academic collaborations and friendships with colleagues interested in the development of teachers of mathematics and, later, the development of mathematics teacher educators. In Recife, I found a theoretical position, enactivism, at a discussion group run by Rafael Núñez and Laurie Edwards, meeting another participant, David Reid (Goodchild, 2014), who became a close collaborator. There is not space to do justice to a development of the theory of enactivism in this plenary. From my present enactivist position, knowing is doing. What I do brings forth my world of knowing and my world of knowing is what I do. In enactivist terms, our history of structural coupling with our environment leads to patterned actions. Chairs may look quite different, but we recognise them as such. Basic-level categories are not actual behaviours, but they are how we label the same actions, such as a chair being for "sitting-on". The theme of this conference, mathematics education research, supporting practice, is central to my own research. In this plenary presentation, I look back over a life spent observing and interviewing mathematics teacher educators and teachers and students of mathematics, researching in collaborative groups and attending PME, asking what any implications there might be to take forward into the future of mathematics teaching and learning.

The title of this talk includes the word, "stories". What comes to mind when you think of this word? Many years ago, on reading a draft of a proposed PME paper, written by myself and Alf Coles, *The story of silence*: *Teacher as researcher – researcher as*

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teacher (Brown & Coles, 1996), a mentor of mine commented that we needed to change the title because readers would think the paper was fictional. For me the word "story" had a technical meaning. Bateson (1979) talks about the "pattern which connects" (p. 8), a story being "a little knot or complex of that species of connectedness which we call *relevance*" (p. 13). That paper, with story in its title, became the first in a series of PME papers documenting the research journey with Alf Coles. This presentation looks back over my research looking for distillations that are relevant, within this "*context*, of *pattern through time*" (p. 14). I do this by focusing on three aspects of my research practice that have been present across my work through time, interviewing, observing and researching in collaborative groups.

I will begin by focusing on three papers from early in my career to identify themes across all three aspects in each paper. I will then focus on three recent papers, each with insights mainly from one of the aspects to address where I am now. I will then look forward, briefly, to the future.

I am not seeking to share patterns from my own awarenesses, developed through my research journey, thinking that everyone needs to work in this way or do what I do. The intention is to interrogate how I see what I see and do what I do, perhaps expanding your space of the possible through what you might notice in the future. I map territory, asking questions about doing and being or how what I do informs what I think and believe, paraphrasing Bruner's (1990) "culturally sensitive psychology" (p. 16). I have been helped in the process of interrogating my early experiences through hour-long on-line meetings with Alf Coles, focused each time on a particular one of the first three papers. When we worked together on a one-year course with a group of prospective teachers at the University of Bristol, talking together after sessions became ritualised in what we came to call *reciprocal narrative interviewing* (Brown & Coles, 2019; 2020). We would take turns in questioning each other to uncover new awarenesses. It seemed natural to use this process to capture the origins of themes that have been with us since 1995 when we started to work together. Having both read the paper before meeting, we spent about 20 minutes talking through what we had been struck by. The rest of the hour was spent with Alf narratively interviewing me. Some extracts from the narrative interviews are included in the discussions of the papers that follow.

PAPER 1: "STOPPERS"

Firstly, I remembered a paper that was not written by me but by Joan Yates (1983), who worked at the University of Bristol, SoE as a mathematics teacher educator when I was a young teacher. In 1979, I was a teacher of mathematics in a school for students aged 11-18. After studying mathematics at university, in 1973-4 I completed a one-year post-graduate course to obtain a teaching qualification. I had been teaching for 5 years when I was invited to join a small group, three members, facilitated by Joan Yates. The work of this group was written up in a *For the Learning of Mathematics* (FLM) article (Yates, 1983) entitled *"Stoppers"*. "Stoppers", a label suggested by one member of the group, was an "aspect of our experience that had grown in importance,

in that it had engaged and captured our interest" (p. 35). We defined a stopper as "the moment when a pupil is no longer able to "cope" […] there is an observable breakdown" (p. 35). Although my focus in this presentation is on the processes of doing research, inevitably, given my interests, it is worth stressing that these processes are in the service of developing mathematics teaching and learning.

Joan talks about her work with a group of three teachers, who wished to remain anonymous. I was one of those teachers. Reading this paper again, strands of the researcher who developed out of this young teacher are apparent.

Joan had been to a conference run by Stenhouse and was now trying out the principles of teacher as researcher she had encountered there. Stenhouse (1996) wrote "curriculum research and development ought to belong to the teacher" (p. 142). He believed that "critical characteristics" (p. 144) of such a teacher could be:

> The commitment to systematic questioning of one's own teaching as a basis for development; The commitment and the skills to study one's own teaching; The concern to question and to test theory in practice by the use of those skills. (p. 144)

As Joan Yates writes in the paper, "our aim was to examine our own practice critically and systematically" (p. 35) echoing Stenhouse's words. In so doing we, the teachers, also acquired research skills. A particularly memorable, for me, article by Ginsburg (1981) looking at clinical interviewing included the sentence:

> Verbilisation can be misleading since the child may not have direct access to his [*sic*] cognitive processes or may have poor command over language. (p. 35)

In the paper, a substantial amount of the teachers' reflections on their children's mathematical thinking is included. My way of collecting data from 26 children, "To compare understanding of decimals without and with a calculator", is described. I ask the children to show me what they do in written form before any verbilisation. Here is the task for you try first, Order the following set of decimals from smallest to largest:

> 2.19, .888, 1.699, 2.2, 1.8989

Ask yourself whether you ask children to do this sort of task. In the UK at that time, most children would work at calculations, such as adding and subtracting, with decimals, but would not have been asked to put them in order of size. What did our children understand about the numbers they were calculating with?

> I asked [them] to space out and gave them a piece of rough paper each. I requested that they work on their own. On the blackboard I wrote: 2.19, .888, 1.699, 2.2, 1.8989. I gave the instruction: On one side of the paper I'd like you to put these numbers in order of size, from smallest to biggest and explain very clearly how you did it. They had ten minutes…most only needed five minutes. (p. 37)

Having collected the results together on the board (see Table 1, one child missed out 1.699) "to set up a visible reminder of different possibilities […] the ones where there were discrepancies were pointed out" (p. 37). What takes your attention in Table 1?

The students were asked to turn over the piece of paper and could "use a calculator to try to convince themselves, if necessary, and could talk" (p. 37).

		2.19	.888	1.699	2.2	1.8989
Smallest	1		26			
	2			21	1	3
	3			4		23
	4	17			9	
Biggest	5	9			16	

Table 1: Results of ordering the decimals.

The discussions that followed led to some students changing their minds about their previous answers. Is this process teaching or researching? Here, I recognise what I would now see as my need to base teaching and researching on something that the students or, later, prospective teachers do. I might interview teachers about their practices in the classroom, always, where possible, linked with observing lessons. Stenhouse (1975, p. 154) quotes Hamilton's eight propositions "which are of interest to all who are concerned to observe teaching", the second of which is "Students (or for that matter teachers) are never ignorant or know nothing". I believe passionately that children's and teachers' talk makes sense, from their perspectives. My task as teacher researcher is to learn their world. As a teacher, about the time of the "stoppers" research, I had some awareness of this when Dave Pratt and I, then both teachers, began a conversation at an Association of Teachers of Mathematics (ATM) weekend meeting at the start of a free evening. We were sharing our practices and began exploring differences heatedly. As we continued talking, we started sharing the details of our practices rather than labels, such as, structured, or open-ended. Observers, friends who kept returning to see if they could attract our attention, reported, later the same evening, being surprised that we each now seemed to be talking heatedly from the opposite position to earlier. Teachers in a particular context cannot be expected to see their classroom and report on it in the way that an experienced observer of many classrooms would. What is important is to hear what teachers say and see what they say through observing their classroom practice. I started to talk about the language a teacher uses about their practice being in the direction of their development or movement, what they were working on, talking in vectors rather than placing themselves on a continuum of practice that an observer could do.

In narratively interviewing me for this presentation, Alf drew attention to questions where my "thoughts were stimulated in the direction of" (Yates, 1983, p. 38):

> AC You offer two questions – When do I know something in mathematics? When do I know that another knows? Do those feel like questions that stayed with you?

LB How do I know that they know? We have talked about thata question in our writing. All it takes is a disturbance down the line and I need to question what I know. How do prospective teachers know what their children know? What do they do to find out? I think it's a motivating question for which you don't have to have an answer.

AC A shift was in recognising that question as one of a class of helpful questions that a prospective teacher can ask in the process of starting to accrue a range of behaviours in their teaching. Then the recognition, later on, was labelling that as a purpose, what we now see as a basic-level category. Here it's you using a purpose before you had that label.

LB When I first met you, I was inarticulate about my practice as a teacher educator. If I was going to be able to talk about what I did then I had to have words to talk with. In those early papers, whenever we gave an example of a purpose it was always "how do I know that they know?"

AC Here's you as teacher using that purpose – quite a precious example because quite quickly you go on to use it as teacher educator. Quite quickly you're supporting others to find their own purposes. Here's Joan having supported you to find that purpose for yourself.

Through Alf narratively interviewing me, we found an early example of a motivating question, "When do I know that another knows?", for my teaching that had emerged from the critical and systematic work in the group. At the time, it was a particular question but, over time, working with prospective mathematics teachers to identify their own motivating questions arising from the interrogation of their practice led to the labelling of such questions as purposes. These purposes became linked for me with basic-level categories in the enactivist literature in that they lead to patterned actions in the world of the classroom. I now recognise "stoppers" as a motivating label for the research group in the paper.

So, the three themes of the title are all represented in this first research paper: interviewing influenced by Ginsburg (1981), giving children something to do to support their verbalisation; close observation of children in classrooms as a teacher researcher and close observation of my own actions as a teacher in a critical and systematic way; and, researching in a collaborative research group with Stenhouse influences, research as exploration and personal transformation, using each other as critical friends to identify concepts with a direct link with practice to interrogate, in this case, "stoppers".

PAPER 2: THE INFLUENCE OF TEACHERS ON CHILDREN'S IMAGE OF MATHEMATICS

In 1988, I completed an MEd in Mathematics Education at the University of Bristol. I was also making the transition from being a teacher in a classroom to working in curriculum development at the Resources for Learning Development Unit (RLDU) (Llinares, Krainer & Brown, 2014, p. 602), supporting groups of teachers to develop

resources for mathematics classrooms. I was now a facilitator of collaborative teacher research groups, although the size of each group was at most ten teachers, in line with RLDU policy, rather than three. In 1995, I was full-time at the University of Bristol, SoE and wanted to submit a PME paper for the January deadline. I had written a paper for FLM (Brown, 1992), *The influence of teachers on children's image of mathematics*, which is the second paper that I have chosen, reporting the findings from my master's dissertation. Reporting on this research and extending it slightly provided the focus of my first PME paper (Brown, 1995).

The motivation for the dissertation research was "devising a set of instruments which might allow me to explore whether a particular teacher did, in fact, influence the image of mathematics of their pupils in the same way" (1992, p. 30). The individual's image of mathematics was the "personal theory (Kelly, 1955; Claxton, 1984) which an individual holds about mathematics at the present time" (p. 30), including "feelings, expectations, experiences and confidences" (p. 30). What I was looking for as an influence of the teacher on children's image of mathematics was a "common change or adaption of the pupils through working with the teacher" (p. 30). I worked with four teachers chosen as being effective by advisory teachers and heads of department, with contrasting structures in which they worked, for example, individualised learning or whole-class interactive. I did not believe that there would be a common image so, in limiting the number of children in the project to six for each teacher, I suggested they choose two who did respond to whatever they did with the class, two who did not respond and two to make up any imbalance in, say, gender.

Looking back, from a perspective of "knowing is doing", I find it hard to imagine why I so strongly thought that I would not see any common strand across the students and their teachers. Before giving my present position on the findings of this project, I want to say a little more about its design. I knew that I wanted to interview the students and the teachers, making the process as similar as possible. In trialling various protocols for the student interviews, it became apparent that simply asking what mathematics is to them did not work. We needed to do some mathematics together given potential issues with verbalisation. The Ginsburg (1981) article used in Paper 1 was important for interview strategies for doing mathematics. Each student chose one from 5 activities presented to them for us to work on together. In telling me about lessons they had experienced, the "work of Hoyles (1985) in *asking pupils to recall particular episodes"* (p. 31) was influential. In trialling the various protocols, I settled on two practices that seemed to support students in answering directly without the need to clarify. In early trials of questions, I had often been met by a blank face and "What do you mean?". The first practice was asking what appear to be long-winded questions, where the precise wording developed over time, such as:

> I am going to make some statements and, for each one, see what is brought to mind by what I say. Try to remember the event so clearly that you can tell me a story about what happened (a) Tell me about an activity you have done recently in a maths lesson, and, although you probably did not think so at the time, it is brought to mind now when I say,

> there you are, sitting in a maths lesson and what you are doing does feel like mathematics. (p. 31, italicised in orginal)

The second practice led to energised responses, what in the 1995 PME paper I called "provoked articulation" (Brown, 1995, p. 148) from the students:

> What I am interested in is your image of mathematics. So far you have indicated in your responses to the various statements and activities that maths is … Is there anything else you'd like to add that has not been covered so far to the question: What is mathematics to you? (p. 31, italicised in original)

As interviewer, I am attending carefully to what the interviewee is saying with the intention of, using as close to their words and phrasing as possible, feeding back to them my thoughts of, in this case, what they think mathematics is. The responses are what I, as researcher, see as robust evidence. Provoked articulation is energetic, "Yes, yes, yes, and …" or "No, no, no, what I think is …" and so on.

I was surprised at the time that whether the student responded to what their teacher did or not their image of mathematics was influenced by the teacher. I share statements from Teacher C and the six children (Figure 1) from the class as an example. There were other patterns for the other three teachers and their children. I was struck in all the interviews by the question, "Where are the children who don't like mathematics?" What did these teachers do that led to engagement? My answer now would be that the teachers had conviction in what they were doing.

I also wanted to observe lessons and planned one observation before the interviews and one after. In fact, I gained little from the post-interview observations since the children interviewed were intent on carrying on the interactions! In observing in Teacher C's classroom, I was struck by seeing a teaching strategy reportedly used previously in a primary school:

> A pupil offered an explanation of how they had begun to tackle a problem. The other pupils were invited to close their eyes and put up their hand if they had started in the same way. An alternative start was requested and the pupils again closed their eyes and put up their hands if this was their way of starting. The process continued with more information being collected and these different starts were then used for further exploration: *What was the aim of the people who drew the radius?* (Brown, 1992, p. 32)

These children experience mathematics through expressing their ideas and hearing other's ideas. The teacher is not the expert, "even Teacher C doesn't know all the answers" (Pupil C6), supporting students in "building their own frameworks, that are not necessarily your frameworks" (Teacher C). How could there not be a common strand of overlapping experience? The teaching strategy described above means that students hear multiple ways of starting to engage with a complex problem. They are not being presented with one method to learn. It is their task to build their own framework. Their experience is tackling complex problems but with the support of a range of solution strategies used by others. One student said, in the interview,

Pupil C2
I prefer having to solve it myself. It gives you that satisfaction of not having to take it from a book. I enjoy mathematics. I find it more of a challenge than a chore. The problem-solving exercises would help me because I could imagine how I felt and go logically through the steps.

Pupil C3
Mathematics is problem-solving. In Connect-4 I'd start by experimenting on a smaller grid to see if there's any pattern and be able to predict: maybe changing the number of counters which you have to make a row.

Pupil C1
You've got to actually solve things for yourself which aren't in a book. That's not really what I thought maths was going to be in the earlier years because that was just numeral sort of maths. You can relate it more to things outside, it's not just like a picture on the board, you can imagine it.

Teacher C: Influence through philosophy
I think the whole idea of a problem is that you model it and make it solvable. Mathematics is a framework and mathematics teaching is fun. Fun when you see the children building their own frameworks which are not necessarily your frameworks.

Pupil C4
I think maths is just applying stuff that you have learned in the lesson in reality.

Pupil C5
Maths is using what I already know like trigonometry and measurement.

Pupil C6
There was a real problem there. I understood what was happening and there were so many different types of maths used to find the final answer. Maths was numbers to me. I felt that in maths everyone knew the answer but as time's gone on I've discovered that even Teacher C doesn't know all the answers – so maths has changed – you can experiment.

Figure 1: Commonality of image *building frameworks* between Teacher C and pupils.

> There are several solutions to one problem. If you go round the class asking, you'll come up with six or seven. You can experiment […] like a real mathematician. (p. 32, italicised in original)

This strategy was one that I have used in my own teaching and told other people about. Thinking of questions for future research at the end of the paper, I thought I might look for what I called "transferable strategies" in future research. What has happened is that my focus has been on the meta-level to the actual strategies, motivating labels such as "how do I know that they know?" that lead to a range of teaching behaviours being employed flexibly, or "building frameworks" for Teacher C.

In our on-line discussion, Alf reported noticing, in the paper, me using a purpose, "Is this a classroom in which it's all right to be wrong?", as a "way of analysing teacher and pupil behaviours" in the early days of moving from being a teacher to visiting other people's classrooms, which was "a continual source of surprise, recognition, disturbance (resonance and dissonance) and consequently, through reflection, personal learning" (p. 29).

In the same way as Teacher C gets pleasure from his students building their own frameworks, as a mathematics teacher educator I watched my prospective teachers developing through identifying purposes, encouraging them to talk and write about stories from their classroom illustrating resonance or dissonance. Here is one such story as an example:

> I wrote the question on the board, asking for hands up for the answer. The first child gave me the answer to which I said "Correct". Belatedly, I asked if anyone else had anything different, but of course the children were then unwilling to offer an alternative answer that they now knew was definitely wrong. I realised immediately that I could not now see what the rest of the children had done. Since that occasion I have been attempting to gain answers from several members of the class […] An advantage in listing the variety of answers to a question is to show children that they are not alone in making a mistake and that others have had the same (or different) problems. Similarly, multiple equivalent answers can be highlighted whereas otherwise a child may feel that their answer is wrong just because it does not look identical or is in a different form. Hence the art, as a teacher, of "being expressionless" as a variety of answers are given to a problem appears to be a very useful one. (Brown, 2004, p. 6)

The identification of a purpose such as "being expressionless" is energising for the prospective teacher, supporting them in learning from their own experiences. I learnt that there is not one way to teach from this project, but that what is important is a conviction in an image of mathematics that provides a consistent classroom culture. In the narrative interview with Alf related to this paper, I was energised by other awarenesses about my research process provoked by the discussion. Here are three quotations, one on interviewing, one on observing and one on purposes:

> LB Them telling their stories and the talk is stories of [*pause*] Interviewing as people telling their stories.
>
> LB I think that's where my conviction came that you don't do interpretation in the classroom, you collect the data.
>
> LB When I was visiting prospective teachers to observe them teach, I would ask them what they wanted my focus of observation to be, what they were working on at the time, looking for a purpose, I suppose.

PAPER 3: THE STORY OF MATHEMATICS

When *The story of mathematics* (Brown, 2001) was published, I had been working at the University of Bristol, SoE for about ten years, taking part in several research projects. The project reported on in this paper arose from the conclusions of a research

report (Winter, Brown, & Sutherland, 1997) looking at *Curriculum materials to support courses bridging the gap between GCSE and A-level mathematics*:

> These comments encapsulate the importance of seeing the "story" of mathematics so that it has a coherence both in its teaching and in the experience of students. […] This finding is closely related to the mathematical competence and vision of individual, effective teachers. (p. 23)

The issue seemed to be that, due to teachers limiting their teaching to syllabus coverage for examinations at 16, students starting advanced level courses at 16+ had problems.

I am going to look at the research design of the story of mathematics project which led to a similar design on a successful major project in 1999, by which time my way of designing research feels stable. The findings of this project were first reported in a symposium at the American Educational Research Association (AERA) conference (Montreal, 1999), an invitation arriving after the conference for the set of papers to be published in a new educational policy, research and practice journal. The research questions for this project were:

- What are the stories of their mathematics for individual, effective teachers? How do their strategies and purposes in the teaching of mathematics support the doing of mathematics by their students?
- What stories are there within mathematics itself that can give an holistic sense of connections? What are the "big ideas" for mathematics?

Collaborative group

Three teachers were paired with three researchers forming a collaborative group. We met three times over about six months. At the first meeting we began the process of developing a common language through discussion of the research questions. My experience in the Stenhouse-influenced group (Paper 1) is apparent here. Between the first and second meetings of the group the researchers observed their teacher pair and each of the teachers was interviewed by me. The second meeting of the group focused on the interviews and observational data to consider sameness and differences in the teacher's practices. Common threads were identified through the mutual recognition amongst the teachers of similar practices and described through the developing common language. As researchers we were part of the conversation, feeding back anecdotes of related practice from our observations. In the last meeting, the teachers wrote lesson descriptions to illustrate the samenesses in practice which had been identified. The teachers' voices are given the space here, leading the conversations, with the labels, such as, same/different, being a "pedagogical tool through which mathematical ideas of order, inverse, pattern and structure can be explored" (p. 192) emerging from them, like "stoppers" in Paper 1.

Interviews

The interviews focused on the two research questions but for each "big idea" or teaching strategy mentioned there was an invitation to give an anecdote from a recent

lesson to illustrate what was meant by the language used. Interviewing as people telling stories again.

Observations

Each researcher then used the transcript of the interview of their teacher to inform their observations of the teacher's lessons. The researchers looked for evidence from practice to illustrate what the teacher had described during the interview. Here the practice I recognise is what I have come to call "staying with the detail". No interpretation in the observational record beyond the researcher identifying practice related to descriptions from the interview. At the point of recognition, the researcher aims to capture what is said and what is done, a process supported by video data in later projects.

Findings

In the design of the project, we had overlooked a central emergent theme from the data analysis: These teachers are able to make their teaching contingent upon the story of mathematics of their students. I return to Teacher C, his pleasure in his students building their own frameworks. Staying with the detail of these three teachers' practices led to a new awareness, showing a pattern rather than the isolated case of Teacher C. Transferable strategies are at the wrong level. What was central was the motivating labels, basic-level categories that could be linked to a complex set of developing mathematics and teaching strategies used by an individual teacher. These can look different whilst the same label is used. All three teachers work on complex connections within mathematics and mathematics teaching for themselves. Their stories of mathematics are not fragmentary. The gap between the students' experiences of mathematics between stages is bridged because the teachers' actions are the same and because the story of mathematics is the same for the teacher and the students. At a meta-level, it does not seem to matter what the story is, as long as there is one, and it exists in a process of learning contingent upon the voices of the students, who themselves know how to act in doing mathematics.

DISTILLATIONS FROM RECENT PAPERS: PATTERNS OVER TIME

A doctoral student, who was interested to read some of the writing of her supervisors, myself and Alf Coles, reported that she had found our recent work hard to access but had found the earlier papers inspiring, leading to me to discussing the first three papers at length. For the final three papers, I am going to focus on the distilled principles that inform our research practices, always focused on mathematics, as communicated in our latest writing. I have chosen three papers that each seem to focus centrally on one of the three aspects of this talk to share what they say about current practice. These distillations are patterns over time: learning through finding new basic-level categories from staying with the detail of experience; establishing a classroom or research group culture through metacommenting and attending to the voices of the teachers and students when researching in collaborative groups. I will then look to the future.

Interviewing

The writing, a book chapter, chosen for this section is entitled *Mapping the territory: Using second-person interviewing techniques to narratively explore the lived experience of becoming a mathematics teacher educator* (Bissell, Brown, Helliwell, & Rome, 2021). In writing this chapter with colleagues, I had been enactivist for 25 years. Its theoretical underpinnings are therefore enactivist. The focus of the paper is on the articulations of an expert teacher, moving from being in the classroom to supporting teachers in a national setting:

> In the moment there is no time for reflecting. In moving to a new job, therefore, we act using what we have done previously. […] Using what we have done previously in a new environment will be followed by adapting when what happens is not effective. (p. 207)

Three interviews seek to support the interviewee in identifying this process of adapting. I have a well-developed protocol for the interviewing process, including how the interviewer works contingently with the first-person accounts of the interviewee. The first three items in the protocol are based on Claire Petitmengin's (2006) work:

- Stabilising attention: A regular reformulation by the interviewer of what the interviewee has said, asking for a recheck of accuracy (often in response to a digression or judgement). Asking a question that brings the attention back to the experience.
- Turning the attention from "what" to "how" (never "why").
- Moving from a general representation to a singular experience. This is what we term "story", a re-enactment, reliving the past as if it were present. Talking out of experience, not from their beliefs or judgements of what happened, often involves the teachers in a move to the present tense. Staying with the detail is important, a maximal exhaustivity of description that allows access to the implicit. (p. 209)

These three items are extended into a fourth by Brown & Coles (2019). The fourth fundamental way of acting encapsulates our enactivist take on learning through adapting basic-level categories, what were originally described in the first three papers as "purposes", "motivating statements" or "questions" that accrue a range of behaviours:

- Getting to new basic-category labels: After dwelling in the detail, telling stories and exploring without judgement or digressions, the invitation is to elicit statements of what is being worked on. [...] In this way, new basic-level categories might be identified, such as the straplines (a word used in editing newspapers, memorable, usually less than five-word, phrases) from this research of "listening for" or "setting up the culture". These awarenesses, triggering and being triggered by the environment can allow adapted and new behaviours to emerge. (p. 209)

The density of this writing compared to the earlier papers is clear and gives some insight into our learning over more than 20 years. Working with prospective teachers, I had developed a story of how they learnt to teach and here I am applying those ideas to my learning in moving from being a teacher to being a teacher educator. For Alf and me, in our work together, learning is through changing or extending basic-level categories. The process goes back to those interviews in the MEd paper, of inviting the other to tell stories from experience, supporting them to stay with the detail of what happened, focusing on resonance and dissonance, without judgements, opening up the possibility of acting differently. New basic-level categories emerging seems to be energising for the learner, who can accrue a range of behaviours to use flexibly. Alf and I, in talking after sessions when we taught together on the course for prospective teachers, used this process, which we call reciprocal narrative interviewing, on each other, turn taking as interviewer and interviewee to develop insights and awarenesses, to ourselves learn. With teachers new to teaching, looking for patterns across a sequence of interviews staying with the detail of first lessons of a new year with groups new to them that year, I would be looking for when the basic-level categories had stabilised, when the teacher literally knew what they were doing in acquiring their story of mathematics (Brown & Coles, 2008).

Observing

Another book chapter, *Learning to teach mathematics: The lesson de-brief conversation* (Brown, Brown, Coles, & Helliwell, 2020) focuses on the observation of prospective teachers followed by discussions with them and their mentor after the lesson. It is important to stay with what the prospective teacher is able to notice, what we call staying with the detail, and this is linked to asking them what their focus of observation is before the lesson. A prospective teacher is not sure what they want the culture of their classroom to be but, in getting to a label such as "being expressionless" for your own behaviour, we have come to recognise what we call "metacommenting" by experienced teachers, especially in the early days of working with a new class. What is seen by the teacher as behaviours by students, such as, "getting organised" or "looking for a counterexample" are shared so that the students know what to do to do mathematics. Over time some of these behaviours are favoured by the students and the teacher does not then need to metacomment, since a culture of doing mathematics in that classroom has been established.

It is striking to me, in re-reading this chapter, how we, the mathematics teacher educators and authors of the writing, use fictionalised accounts of de-brief conversations. This seems a long way from being asked not to use "story" in the title of a paper because readers might think it was fiction. Markku Hannula (2003) had a paper published in FLM when I was editor, where he used fictionalised experiences and, in this chapter, we do the same, citing Hannula. As well as participants being anonymised, it is possible to bring together parts from several stories that focus on stressing important principles and practice.

Researching in collaborative groups

In 1995 I was funded to develop a Master's module based on the participants working in a collaborative group to develop their teaching. By 2011, the principles for running such a group were established. In the academic journal paper, *Differentiation from an advanced standpoint*: *Outcomes of mathematics teachers' action research studies aimed at raising attainment* (Coles & Brown, 2021, pp. 169-170), these were stated as:

- the group size should be less than or equal to 10,
- meetings should be spread out over an extended period of time,
- the teachers should come from a range of schools and be volunteers rather than conscripts,
- the leader of the group sets up a loose structure for meetings and time is given to each participant to discuss their emerging thoughts about their issue,
- the leader of the group gives individual readings in between meetings to support participants thinking about their issue, or there could be tutorials for participants between meetings linked to their Master's study,
- the leader(s) of the group will make one or more visits to each teacher's school to further support thinking about the issue and/or data collection, and, in some cases, the teachers visit each other's classrooms. (Brown & Coles, 2011, p. 865)

The paper focuses on a subset of teachers researching in a collaborative group with a focus on using higher-level content with groups of low attainers. The research was to be written up as part of a Master's module. One teacher gives the following reasons, grounded in her practice, for extending the syllabus for these children:

> [a student] was able to tell me that the square root of 49 is 7 two weeks after having studied Pythagoras' Theorem. […] Even though the student probably hasn't remembered how to use Pythagoras' Theorem, they have taken away how to square root. This was one of the factors that led me to think I should try teaching these students more of the difficult topics. Not just for them to try and grasp these harder topics, but because they might take something else away from it. (p. 175)

LOOKING TO THE FUTURE

In 2018 I travelled to Indonesia to give a keynote entitled, *Global needs: Rethinking teaching and learning mathematics for future changes*. I ended that talk saying:

> We need citizens with higher-order thinking skills to provide creative suggestions for whatever happens in an uncertain future. It just might be that cross-curricular teams of teachers working with students on real, complex problems will become the norm for schooling.

Teachers being led by their students who are building frameworks within real contexts that are not their frameworks makes me optimistic for the future. Here is one example of what is happening right now. Each individual student graduates from the Green School, Bali with a presentation. Watch Bronson Parish's (2018) presentation on ocean

flow, https://youtu.be/0ITm7v1yLGo. Many villages on his home island, Sumba, have no power or running water even though it is surrounded by water, continually moving. The presentation is his story of his practical ideas that he wants to be available to and affordable by anybody, harnessing the ocean's energy to desalinate water and support the growing of food in coastal communities affected by climate change, including building a working prototype with his father.

This is one student's journey solving a real problem in his community. Researching in collaborative groups with teachers gives me a vision of future schooling in which teachers are supporting children researching in collaborative groups to solve real problems in their communities.

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*Contact Laurinda.Brown@bristol.ac.uk for access to the papers marked with an asterisk.