LAYERING METHODOLOGICAL TOOLS TO REPRESENT CLASSROOM COLLECTIVITY

Lynn McGarvey[1], Florence Glanfield[1], Joyce Mgombelo[2], Jennifer Thom[3], Jo Towers[4], Elaine Simmt[1], Josh Markle[2], Brent Davis[4], Lyndon Martin[5], Jérôme Proulx[6]

[1]University of Alberta; [2]Brock University, [3]University of Victoria, [4]University of Calgary, [5]York University, [6]Université du Québec à Montréal

Our research is guided by the question: "How might we observe, document, display, and analyze data from a collective systems perspective?" In this research forum, we share new research tools for studying mathematics classrooms, highlight opportunities for observation and analysis by layering these tools, and then illustrate how the layering of tools allows for visual distinctions across lessons and classrooms.

INTRODUCTION

For nearly 30 years, the researchers in this forum have worked individually, in subgroups, and as a collective to explore, analyze, and report on data related to collective action in mathematics classrooms (e.g., Davis & Simmt, 2016; Martin & Towers, 2015; Thom & Glanfield, 2018). For the past eight years, we have engaged in a methodological research project with the goal of exploring how we might observe, document, display, and analyze classroom data from the perspective of collective systems. That is, we intentionally shift the unit of analysis from individual students to the classroom as one 'body.'

This research forum builds on our previous PMENA working group (McGarvey, et al., 2015), NCTM research symposium (McGarvey et al., 2017), PME-42 research forum (McGarvey, et al., 2018), and PMENA colloquium (Thom, et al., 2021). These forums have been essential for engaging with the research community. Through the comments, criticisms, and suggestions received, we have taken up, expanded, extended, and revised our work. Here, we review our work to date, share new methodological tools, and examine, discuss and debate these tools as well as the insights gained about mathematics classrooms and lessons when these tools are layered.

BACKGROUND AND THEORETICAL FRAMING

This project is rooted in complex systems research—an approach to inquiry that investigates how relationships between parts of a system can give rise to collective behaviours. Examples of complex systems include birds flocking, ants foraging for food, weather systems, the Internet, and many others. Each complex system arises from the layering of biological, social, societal and environmental subsystems (Davis & Simmt, 2016). Complex systems are challenging to model and difficult to predict, but

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 177-201). PME.

are often understandable in retrospect. A key aspect of complex systems is the dialectical entanglement of the system and its environment (Varela et al., 2017).

Our overriding project goal is to develop methodological tools to better understand the dynamics of the classroom as a collective whole, rather than continuing to treat classroom interactions as a series of distinct individual contributions. While we do not discount the value of research that explores individual understanding, teacher actions and decision making within classroom contexts are often based on the teacher's sense of the class as a whole. We choose to understand the whole by developing tools where the unit of analysis is the whole class, rather than individuals within it. Because of this, we need different analytical tools.

Grounded in diverse yet complementary frameworks that include complexity science, network theory, embodied cognition, and enactivism, our work attempts to conceptualize the entire classroom as one collective agent. In our work, we explore and generate new techniques for representing, analyzing, and interpreting group activity by making use of modelling techniques to represent classroom lessons as a visual whole. In doing so, we highlight one or more features of classroom collective action all-at-once without attributing actions or utterances to specific individuals. This approach is useful for observing particular aspects of a system that may contribute to its global traits.

At PME-42 in Sweden, we presented four methodological tools under the metaphor of "vital signs" including utterance distribution, actions on the board, a mathematical ideas network, and the dynamics of ideas based on the Pirie-Kieren (P-K) model (Pirie & Kieren, 1994). We found the use of "vital signs" to be useful way to foreground a particular feature of classroom activity, while recognizing that such tools must be layered in order to provide a more robust indicator of the health of the body. Utilizing feedback, suggestions, and criticism received at PME-42 and at other forums, we developed two additional tools including Lesson Activity Mapping and Bodymarking, and revised the Dynamics of Ideas into two related tools: Persistence and Movement of Ideas. We focus on these tools in this forum.

Lesson Activity Mapping visually represent the collective actions and interactions in the classroom, such as whole class lecture, small group discussion, individual seatwork, along with the focus of interaction, such as problem solving, sharing solutions, providing explanations, and so on. The second tool, Bodymarking, emphasizes the collective gestures and gaze orientation of the class. Third, we have substantially revised the dynamics of ideas, so that it can be visually interpreted in the same way as the other tools. The other advancement of our work has been to establish a visual standard for all of the tools so that they can be layered, so that we may, for example, examine (Non)Actions on the Board, Bodymarking, and Lesson Activity Mapping for one lesson simultaneously. We can also compare this set of tools across different lessons to illustrate holistic differences in patterns, and point to key moments in a lesson.

Our project is intended to be exploratory as we continue to examine the potential for conceiving of classroom collectives as adaptive and self-maintaining complex systems. As such, we use complex systems as an interpretive frame for observing, analyzing, and comparing mathematics lessons and classrooms.

COMMON DATA

As in other presentations and publications, we use the TIMSS videos (timssvideo.com) as a common source of data in which to explore, develop, and illustrate the methodological tools (see McGarvey et al., 2018). The advantage of using the TIMSS videos is that they are publicly available and capture classroom activity in a way that is common in mathematics education research. That is, there is a video of the full lesson based on a single camera following the teacher and a set of transcripts. In addition, the TIMSS resources provide "lesson graphs" that outline the lesson activities and timing (see Figure 3).

For this forum, the tool developers were asked to analyze several TIMSS videos. In this paper researchers describe their methodological took using one or both of the following two lessons: (1) Solving Inequalities (JP4) in Japan, and (2) Exponents (US3) in the United States. The two lessons are approximately 50 minutes in length and there is a similar number of students in each class (i.e., 35 and 36 respectively). The lessons offer contrasting features including the physical arrangement of desks (i.e., rows and grouped desks), and style of teaching (i.e., whole class and small group). Figures 1 and 2 offer a storyboard for each of the two lessons.

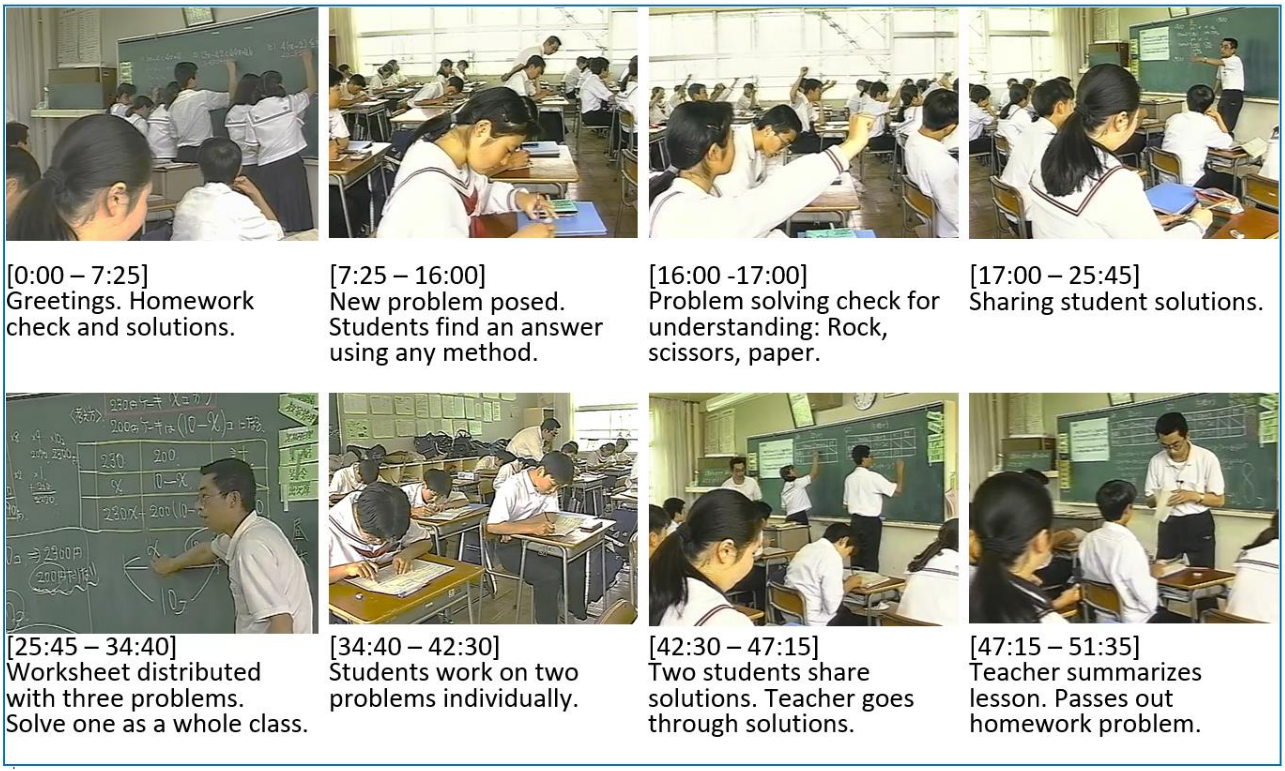


Figure 1: Japan lesson storyboard

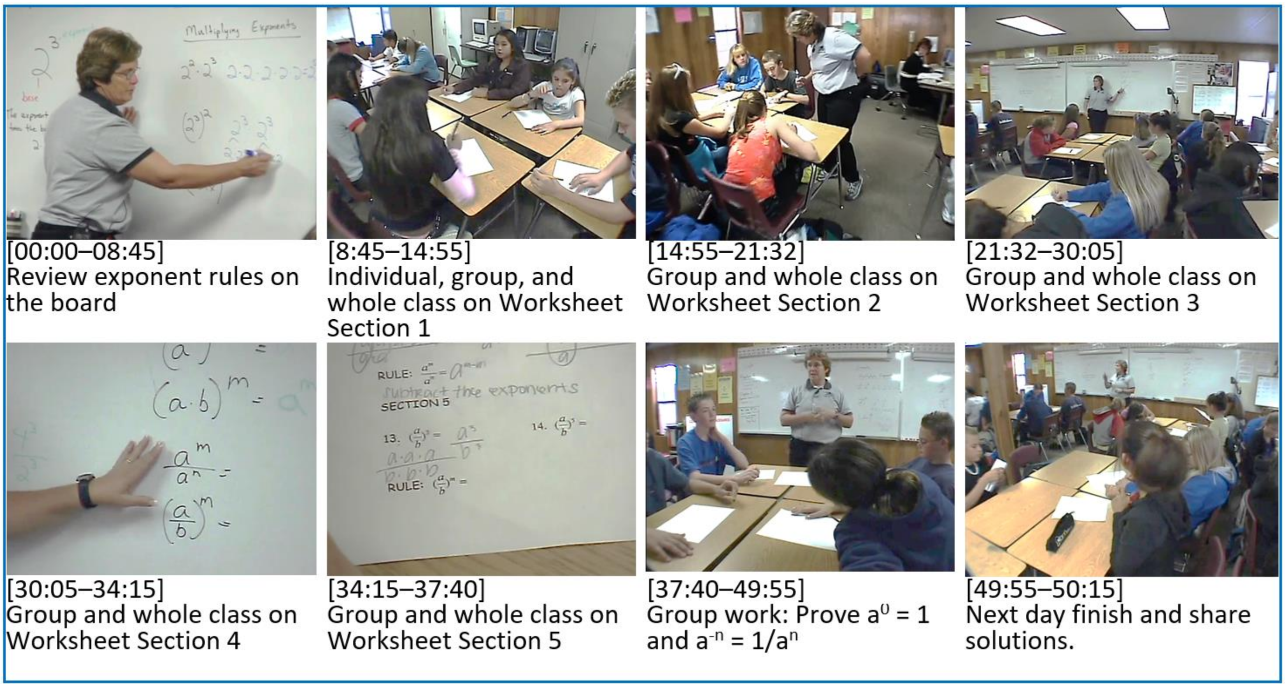


Figure 2: US lesson storyboard

Analysis of the two lessons for each of the three tools presented in this forum are illustrated below. They include (1) Lesson Activity Mapping; (2) Bodymarking; and (3) Persistence and Movement of Ideas.

LESSON ACTIVITY MAPPING

Elaine Simmt and Lynn McGarvey

University of Alberta

INTRODUCTION

The general impetus for our research team is to explore and develop methodological tools that model some aspect of classroom collectivity as a visual whole. The tools developed to date range from modelling relatively simple and specific aspects of the classroom to more complex features. When contemplating what aspect might have value when making comparisons across lessons with different content, tasks, and discourse styles, we landed on some common lesson structure questions: Are students engaged in whole class, small group, or individual activity? Where is the locus of control for engaging in the task? That is, are students generating solutions or applying learned processes? And what is the source or central focus of the activity? Are they preparing to engage in a task, engaged in a task, sharing solutions, and so on? By layering these forms of engagement, we could model general lesson structures.

The structure of mathematics lessons is an ongoing area of interest in mathematics education. In fact, the 1995 and 1999 TIMSS Video Studies brought considerable attention to the variability in lesson structures worldwide (e.g., Hiebert et al., 2003). Attention to a number of features, such as public and private work, mathematical and non-mathematical engagement, the types of mathematical activities, and so on. For the most part, an aggregate of lesson features for each country were described using descriptive statistics, and resulted in lesson patterns or "scripts" for each country. For example, the script ascribed to Japanese lessons included: presenting a problem, having students working individually or in groups, discussing various methods for solution, and summarizing key conclusions. In comparison, U.S. lessons were describe by an acquisition/application script based on the pattern of reviewing material, teacher demonstration, practice, and seatwork (Hiebert et al., 1996; Stigler & Hiebert, 1999).

The contrast in scripts spawned new reform-based lesson structures, such as Launch-Explore-Discuss (Stein et al., 2008) that emphasized "Task-First" rather than "Teach-First" approaches (Russo & Hopkins, 2017). However, as we might expect, the lesson patterns ascribed to entire countries are much more varied when not reduced to a general form, and that it may be more useful to make comparisons at the level of the "lesson event" (Clarke, et al., 2007). We considered several aspects of the lesson structures valuable to our work in modelling classrooms as collectives, and explored how to visualize these features under the vital signs metaphor.

BACKGROUND

We developed the Lesson Activity Mapping tool using the video, transcripts, and "lesson graphs" provided as resources for the TIMSS videos (see Figure 3 for a portion of the US3 lesson graph). Lesson graphs are one-page summaries of the activities in each lesson. As seen in Figure 3, the lesson graph chunks the class period into timed segments (left column) making distinctions between public and private class work (right column). Information about the mathematics content, and teacher and student actions are also provided. In most instances, the segments are described as either "Public Class Work" or as "Private Class Work." Public class work includes such activities as reviewing homework, sharing solutions, posing problems, class discussions, teacher demonstrations, and so on. Private class work typically referred to individual seatwork where students were working independently on problems from the textbook or worksheets provided.

Under each public or private block, the lesson graph provides a brief description of the activities or tasks in that timeframe, and general descriptions of the teacher and student actions. As we can see in the US3 lesson graph (Figure 3), in the first 9.5 minutes of the lesson the teacher announces the topic of the day, comments on a teaching aid she is using to represent exponents, and then demonstrates how to multiply exponents using three examples.

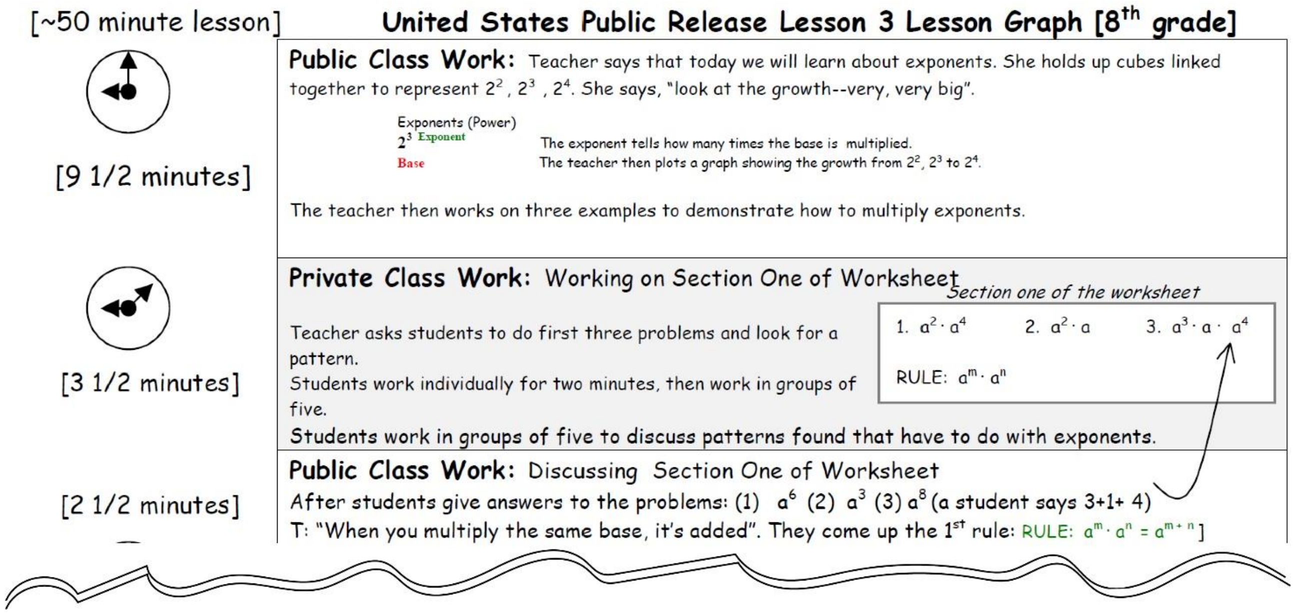


Figure 3: US3 Exponents lesson graph

We found the lesson graphs to be valuable summaries of the lesson activities. In particular, the distinction between public and private work contributed to our view of collective activity in which two or more actors offered actions and utterances to others, in contrast to when students acted (primarily) on their own.

The other component we believed would contribute well to collective modelling of lessons are broad-based activity segments. Activity segments, described by Stodolsky (1988) are the "instructional or managerial" aspects of a lesson that have an intended purpose and with relatively clear starts and stops (p. 11). Different activity types or variations of activity segments have been explored including "setting up," "working on," "sharing," and "demonstration (Stigler et al., 1999); reviewing, demonstrating, practicing, correcting/assigning (Clarke et al., 2007); and "development,", "student work,", and "review of student work" (Kaur, 2021). The lesson graphs also showed the general activity segments for each lesson.

METHOD AND CODING

For the purposes of generating a new tool or vital sign, we chose to visually represent lesson structures based on public/private indicators, as well as demarcated activity segments. Because we are specifically interested in collective activity, we coded for two categories of public class work, whole class and small group engagement, as well as private work according to the following descriptors:

- *Public-Whole* (dark green) involves periods of time where information and ideas were at the level of the whole class.
- *Public-Group* (light green) includes periods of time where information and ideas were discussed at the level of a sub-group of the class in which two or more students were involved. The public-group periods were coded when students were directed to work with a group. That is, the grouping was an intentional

aspect of the lesson. It did not include periods in which students turned to one another for brief moments.

- *Private* (yellow) refers to time periods where individuals worked predominantly independently. Again, this was an intentional aspect of the lesson where the teacher instructed students to work on their own

For activity segments we selected four categories of actions: non-mathematical, presenting, engaging, and sharing.

- *Non-mathematical* (grey) refers to segments of the lesson that involve activities such as greetings, announcements, moving into groups, and other forms of housekeeping.
- *Presenting* (pink) involves segments where information is offered in preparation to engage in work or reviewing and summarizing completed work. These segments included the presentation of tasks, procedures, instructions, worked examples, explanations, and question-answer interchanges.
- *Engaging* (blue) includes periods where most students in the collective were actively involved in a task.
- *Sharing* (chartreuse) segments are predominantly a reflection on completed tasks by providing solutions.

Figure 4 provides the colour coding used for the two sets of codes. We placed the activity segments at the top and the public-private segments at the bottom.

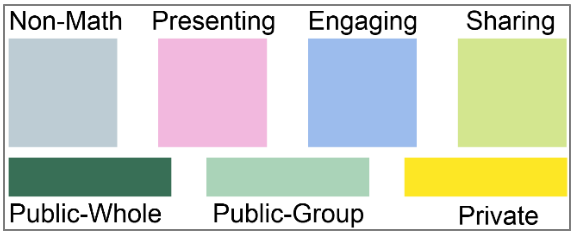


Figure 4: Colour codes used for activity segments (top) and public-private (bottom)

Although other researchers have examined activity segments using additional codes we felt that visually it was important to limit the number of segments. Our goal is to seek out lesson patterns broadly, rather than detailed descriptions of the lessons.

As with the other tools, we examined the lesson in 15-second segments and coded for public-private and for the activity segment. Rather than overlap codes within a 15-segment time period we chose to code to the nearest 15-second mark. This was intended to provide a clearer picture of the overall lesson pattern. Creating a standardized visualization allows us to layer the Lesson Activity Mapping tool with other vital sign visualization tools from the same lesson, enabling the researcher to look for critical points in the lesson.

RESULTS

Using the public-private and activity segment codes above we coded the United States (US3) and Japan (JP4) lessons (see Figure 5).

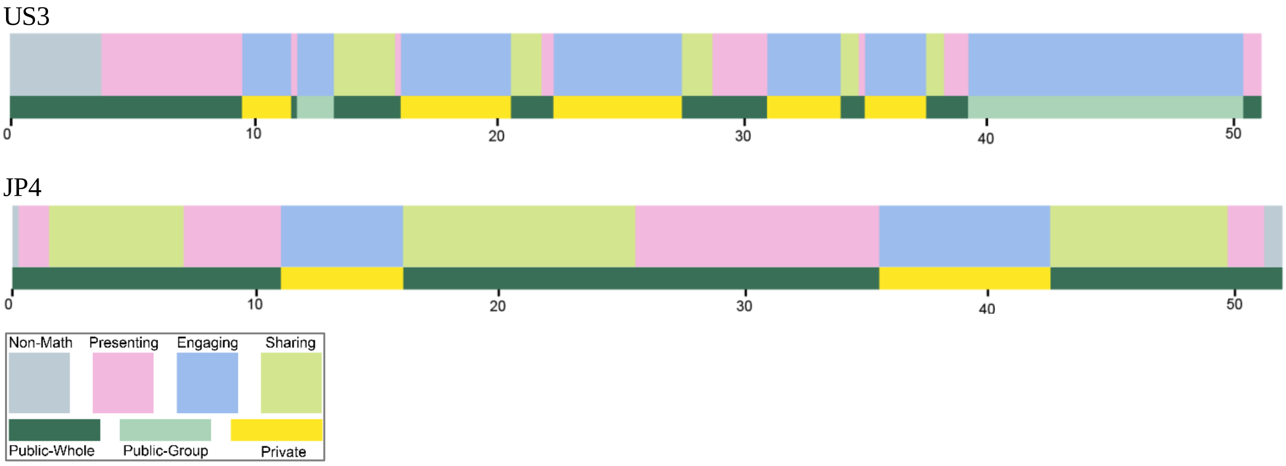


Figure 5: US3 (top) and JP4 (bottom) Lesson Activity Mapping based on public-private and activity segments

The visualization tool shows a number of common features. For these two lessons, presenting and sharing activity segments always occur with public-whole class engagement. Engaging is most often paired with private work; however, there are two instances in the US lesson in which the teacher explicitly tells the class to work with their group. In these instances, we see engaging with public-group work.

When the two lesson structures are viewed as a whole, we see a number of visual differences. In the lesson graph provided for the US3 lesson, it shows the lesson in twelve segments. However, when coding at 15-second intervals we see twenty-one different segments. In this particular lesson the majority of the lesson was based on students completing sections of a worksheet, so we see the teacher present information to prepare students to answer the question in a section; the students complete the section individually or in small groups, and the solutions to those sections are shared. This results in a frequent cycling through of presenting-engaging-sharing.

The JP4 lesson shows eleven segments. The number of mathematical segments (nine) matches with the number of segments shown on the lesson graph provided. (The additional two segments are non-mathematical occurring at the beginning and end of the lesson.) We see much longer segments, for all three types of lesson activities.

Another contrast between the two lessons are how the lessons begin and end. While the US lesson begins with presenting information to prepare students to complete the worksheet, the Japan lesson begins with a sharing of solutions to the homework from the previous night. The US lesson ends with a lengthy period of engagement while the Japan lesson ends with sharing solutions and a brief period of presenting in which the teacher summarizes the key insights of the day.

We also notice the difference in the pattern for engaging in public and private work. The US lesson shows frequent cycling between public and private while the Japan lesson is predominantly public, except for two periods where students work privately and independently on a problem.

CONCLUDING REMARKS

The visualization offered through the Lesson Activity Mapping tool provokes questions for us: What is happening when the activity moves from public to private? How do the patterns of shifting back and forth impact the development of mathematics in the lesson? What are the patterns that exist for other lessons? Is the pattern an artifact of the content of the lesson or pedagogical distinctions of the teacher or culture? Do different patterns of activity segments and public-private actions point to different lesson structures already known in the mathematics education community?

We acknowledge that examining only two lessons does not provide warrants for generalization. Rather, presenting these two lessons illustrates that the visualization of a lesson can stimulate questions for the mathematics education researcher.

We believe that the visualization offered by the Lesson Activity Mapping tool offers possibilities for analysis of multiple lessons by the by the same or different teachers, topics, grade levels, and cultures. By making such comparisons, the visualizations can help us identify overarching patterns representing the dynamics of the system. There is a trivialization of the situation with this tool; however, using it across multiple examples as a way to make visual comparisons offers an opportunity to identify new insights and questions.

To conclude we would like to reiterate that the purpose of the Lesson Activity Mapping tool is to observe lessons in ways we have not before and to see things that may have gone unnoticed. Finally, when using this tool in conjunction with others, we can identify what might be interesting moments in the lesson, and as we look across content and contexts we may be able to identify dynamics of lessons that help us better understand learning systems.

BODYMARKING

Jo Towers and Josh Markle

University of Calgary Brock University

THEORETICAL CONSIDERATIONS

Bodymarking is a methodology and tool for understanding collective action through the ways bodies gesture in the classroom. Observing, tracking, and analyzing this kind of movement is prominently featured in the theories and approaches collected under the rubric of body pedagogics (Shilling, 2017), an embodied approach to the study of various cultural practices, including in formal teaching/learning contexts. In such educational contexts, researchers have tracked specific teacher movements and the paths teachers have taken as they move around classrooms (Andersson & Risborg, 2018; 2019), as well as other embodied phenomena, including gaze and gesture (Kaanta, 2012). Spurred by the availability and affordability of eye-tracking technology, gaze, in particular, has become a well-studied phenomenon, especially in the mathematics classroom, and is frequently used as a means of measuring student engagement and the pedagogical priorities of teachers (e.g., Abrahamson et al., 2015; McIntyre et al., 2019; Seidel et al., 2021). Building on this work we have chosen to focus on the kinds of actions and gazes that we believe might best illuminate collectivity in mathematics classrooms and to attempt to capture, with a digital tool, these everyday aspects of classroom life.

BODYMARKING TOOL DESCRIPTION

The Bodymarking tool focuses on six everyday classroom actions, which we denote as strands, including intentional movements of the hand or body, gaze, writing, and other kinds of tool use. We argue that each of the strands, which we discuss in detail below, captures a distinct gestural expression in the classroom.

Following a taxonomy used in other fields, such as neuroscience and neuropsychology, gestures can be characterized as either transitive (i.e., involving tool use) or intransitive (i.e., not requiring tool use). We have chosen to observe and record both types of gesture through Bodymarking. Strands associated with transitive gestures involve tool-oriented actions in the classroom. These include writing in public spaces (Boardwork), writing in private spaces, (Writing), and the use of other tools, such as mathematics manipulatives (Manipulating Tools). We argue these are three of the most prominent means of interacting with the material world in the mathematics classroom.

As described in Mgombelo et al.'s (2018) "Vital sign 2: (Non)actions on the board"—whiteboard, chalkboard, computer screen, etc.—often orients classroom action. In the Japanese lesson, for example, we see the board used by the teacher to convey information and by students to engage in problem solving. In contrast to Mgombelo et al.'s vital sign, we only remark on engagement with the board through the addition or subtraction of material, whether it be by a student or teacher. Our interest lies more in the distinct cadences of public work and the complex ways it couples with other classroom phenomena, not the nature of any one particular engagement. We are similarly interested in the ways the classroom works privately, which we capture through the Writing strand, and how it engages other materials in the environment, either publicly or privately.

We have also chosen to observe three distinct gestures that do not require the use of tools—Pointing, Hand and Body Movement, and Shared Gaze—to focus on as aspects of collective action in the classroom. These kinds of gestures have been frequent objects of study in mathematics education (e.g., Alibali et al., 2014), and more generally, have been shown to play a fundamental role in learning (Novack & Goldin-

Meadow, 2015). We focus on intentional movements of the hand or body, such as when a student raises a hand to ask a question, counts out a sum on their fingers, or measures a length with outstretched arms. By intentional, we mean gestures that we interpret as emerging out of the interactional domain of the classroom. This includes gestures that intimate actions, describe abstract ideas, and orient the gesturer or others; they can be deliberate, communicative gestures, or the kind of unconscious gesturing that often accompanies speech in conversation. Though these gestural movements may be spontaneous, they are not random. In the Bodymarking process, we do not record movements we interpret to be random or reflexive, such as when a student taps their foot.

Though the Hand and Body Movement strand could be considered inclusive of actions such as pointing and gazing, we conceive of pointing and gazing as specific kinds of gesture worthy of closer scrutiny and have therefore separated these from the other hand and body movements we track. Though pointing is clearly a particular kind of hand gesture, we believe it often functions in ways other hand and body movements do not. Our emphasis on pointing speaks to our interest in understanding how actors in the classroom are oriented by and toward each other and their environment at the collective level. In studying interaction in the context of virtual spaces for collaboration, Luff et al. (2013) noted that if "there is one collaborative activity that exemplifies the embedded character of practical action then it is reference, and in particular, pointing" (p. 2). Moreover, as Cooperrider (2021) noted, pointing often goes beyond the directing of attention to include a host of iconic and communicative phenomena. Finally, pointing is unique with respect to the other two strands denoted as intransitive in that it can be incorporated with tool use. By focusing on the phenomenon of pointing, not just its physical instantiation, Bodymarking can capture the complex ways we use the material environment to orient ourselves.

How we conceive of gaze in Bodymarking is similarly nuanced. Gaze is an increasingly studied phenomenon in the context of mathematics education (see Strohmaier et al., 2020) and is frequently associated with quantifying measures of visual attention. In work more closely aligned to our use of gaze, Abrahamson et al. (2015) sought to identify emergent patterns of sensorimotor activity, including gaze, and mathematical discourse. We are particularly interested in studying collective action and so in the Bodymarking tool we capture instances of shared gazing, those that involve all or most of the class and those that may only involve small groups.

THE CODING PROCESS

Using TIMSS video as source material, we recorded observations for each of the six strands at 15-second intervals for the duration of each lesson. We coded entirely without sound or subtitles, an approach also adopted by Wilmes and Siry (2021) in their study of multimodal interaction in the science classroom. Although the object of their study is a better understanding of how students enact science, and so is explicitly focused on a lesson's content, they note that viewing video with no sound allows the researcher to "draw analytical focus…to the embodied aspects of interaction" (Wilmes & Siry, 2021, p. 79). In this sense, their approach is consistent with ours: we are not so much interested in the mathematical content intimated by an individual's iconic gesture, for example, but rather what the cadences of actions at the classroom scale can tell us about the states and dynamics of collective knowing.

For each of the strands except Shared Gaze, we recorded only the occurrence of a gesture, not its frequency. For example, there is no distinction made between a 15-second interval in which only one instance of pointing occurs and a 15-second interval in which there are many. If a gesture is observed, we assign a colour-code to the relevant strand for that 15-second interval (Figure 6). For the Shared Gaze strand, we code a 15-second interval as one of two colours (see Figure 6) if we determine that the shared gaze occupies at least half of the interval.

Bodymarking Strands	15 Sec. Intervals	Description of Strands
Pointing		Using fingers or objects, such as a pencil, to focus attention on aspects of written work, identify key ideas or missing steps, etc.
Hands/Body		Gestures involving the hand while not engaged in pointing; bodily movement, such as modeling distance with outstretched arms.
Shared Gaze (Focused)		Majority of individuals in frame share a common object of gaze
Shared Gaze (Diffused)		Common objects of shared gaze are multiple or only orient a minority of individuals in frame
Boardwork		Involves addition and/or removal of work by an individual in public view, such as on a chalkboard, whiteboard, or overhead projector
Writing		Addition and/or removal of work in private view, such as on a student's worksheet or notebook
Manipulating Tools		Using any sort of tool, including a calculator, working with manipulatives, etc.

Figure 6: Bodymarking Strand Colour-Coding and Descriptions

As described above, only one strand, Shared Gaze, requires further distinction. For that strand, we distinguish between two types of gazes, those shared by most or all of the individuals during an interval (dark brown) and those shared by only some individuals during an interval (light brown). It is worth noting an important limitation of the TIMSS video data: we are constrained by the view of the camera. To address this limitation when coding for this strand, we only consider individuals shown in the camera's view in discerning gazes shared by most and some. If an interval depicted four individuals all gazing at a single object, it would be coded as Shared Gaze (Focused); if only two of the four individuals were gazing at the object, but the other two were each looking at something else, it would be coded as Shared Gaze (Diffused). The purpose of the Shared Gaze strand is to tell us something about how the collective gaze of the classroom is oriented: is the whole classroom oriented by a common project? Are small groups of individuals focussed on a multiplicity of objects? To best capture this ebb and flow, we colour code the distinct gazes along a single strand, Shared Gaze.

Applying the Bodymarking tool to both of the Japanese and US lessons yielded 332 and 357 unique observations, respectively, across the six strands (Figure 7).

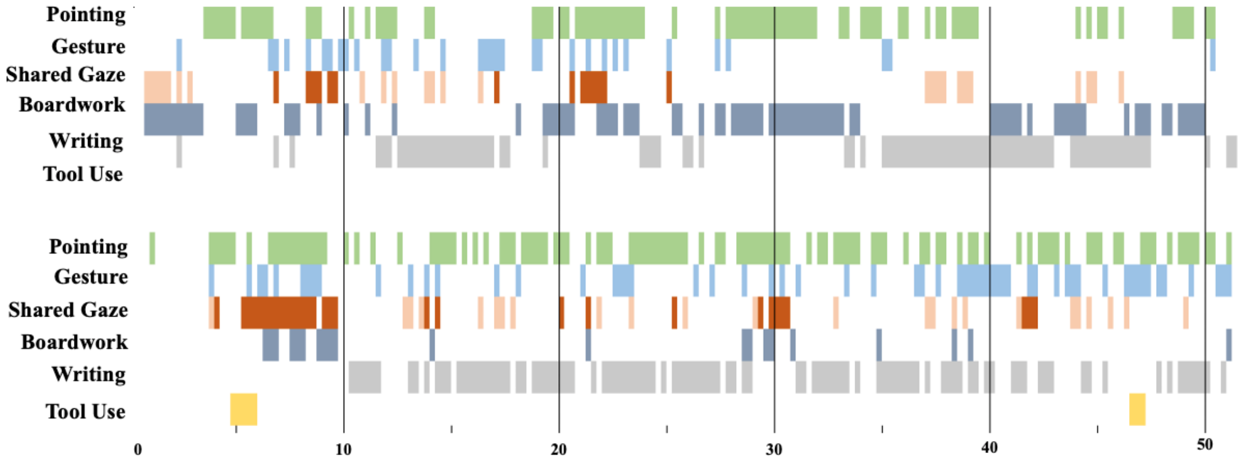


Figure 7: Bodymarking Visualizations for Japanese (Top) and US (Bottom) Lessons

DISCUSSION

The lesson storyboards shown in Figures 1 and 2 reveal the ubiquity of everyday actions, such as pointing and gazing in the classroom. Although we observe and record these individual gestures through the Bodymarking process, the name we have chosen also points to our interest in marking out the ephemeral body of the collective as it emerges through classroom action. For the two lessons in this analysis, we found our attention drawn to two phenomena. The first concerns the ways in which Shared Gaze couples with other actions in the classroom. We would expect Shared Gaze (Focused), shown in dark brown, to occur naturally alongside other strands, such as Boardwork, and this is evident in Figure 7. What stands out to us is the observation that Shared Gaze (Focused) is the only strand that did not frequently occur in the absence of the others. That is, Shared Gaze (Focused) is almost always coupled with at least one other strand. In fact, there is only one 15-second interval, early in the Japanese lesson, in which Shared Gaze (Focused) occurs in the absence of other strands. This leads us to question how moments in which there is a focused gaze shared by the classroom differ from those in which there are multiple objects of shared gaze or none at all. Moreover, we are interested in what those moments might tell us about how the actions captured by the other strands couple with each other and with gaze.

A second phenomenon of interest involves the potential for observing cadences of classroom action over the course of a lesson. Figure 8 highlights three intervals in the Japanese lesson in which Boardwork is prominently featured. Intervals A and B show Boardwork coupling with intransitive gestures, such as Gaze and Pointing, while interval C shows it coupling with a transitive gesture, Writing. The intervening periods in which Boardwork is absent depict unbroken blocks of writing.

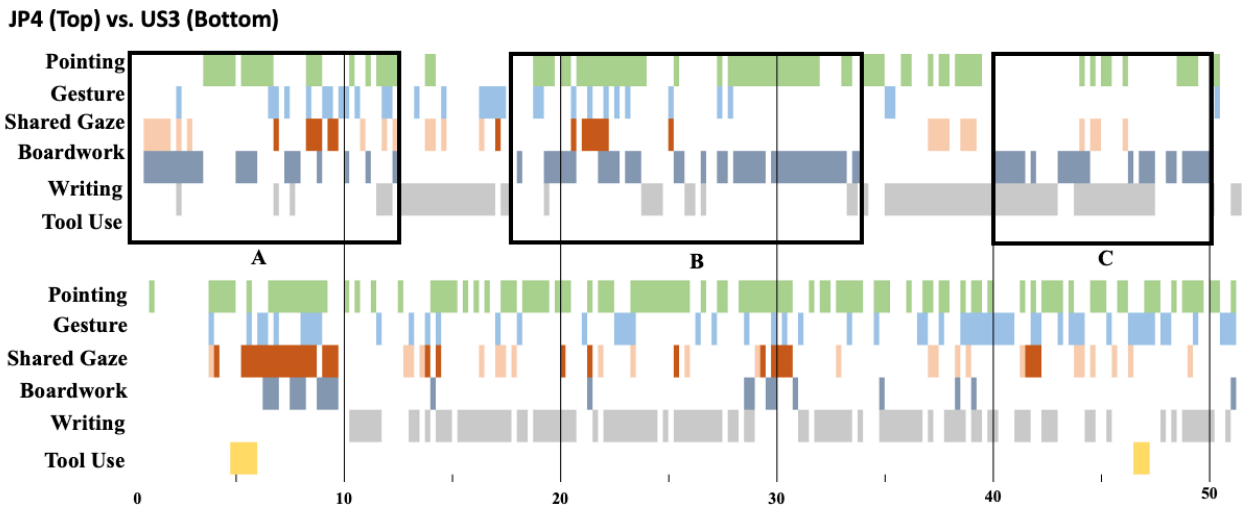


Figure 8: Intervals of Boardwork and Other Gestures

The cadence of the Japanese lesson contrasts with the US lesson, which depicts no discernable rhythm. To be sure, this is a function of classroom pedagogy—the Japanese lesson, for example, alternates introducing new content with practice, while the US lesson involves small group work on a problem set for most of the period—but we argue the Bodymarking visualizations may yield additional insight when applied to a larger set of classroom data. How might similar pedagogies manifest in different settings? What could variations within a lesson, as depicted in intervals A, B, and C, tell us about the way collective action emerges in the classroom? And how are they reflected in other vital signs?

CONCLUDING REMARKS

Bodymarking offers a means of visualizing everyday classroom action. By focusing on both transitive and intransitive gestures, such as writing and pointing, respectively, we argue it has the potential to provide insight into the collective engagement of the material world. Moreover, in attending to how these gestures couple with one another, and how those couplings are reflected in other vital signs, Bodymarking may provide insight into how collective knowing and doing emerges in the mathematics classroom.

PERSISTENCE AND MOVEMENT OF IDEAS

Jennifer S. Thom and Florence Glanfield

University of Victoria and University of Alberta

CONTEXT

In 1989 Pirie and Kieren introduced a model (Figure 9) of a dynamical theory for the growth of mathematical understanding. The authors characterized mathematical understanding as an embodied process that was inherently dynamic, levelled but non-linear and recursive (Pirie & Kieren, 1994). The model featured eight nested yet unbounded levels: Primitive Knowing as "the starting place for the growth of any particular mathematical understanding" (p. 170); Image Making as the activity by which to "make distinctions in previous knowing and use it in new ways" (p. 170); Image Having as the "use [of] a mental construct about a topic without having to do the particular activities which brought it about" (p. 170); Property Noticing as the action by which to "manipulate or combine aspects of images to construct context specific relevant properties" (p. 170); Formalising as activity which "abstracts a method or common quality from the previous image dependent know how which characterised noticed properties" (p. 170); Observing as "reflect[ing] on and coordinat[ing] formal activity and express[ing] coordinations as theorems" (p. 171); Structuring as involving "formal observations as a theory" (p. 171); and Inventising which entails the "break[ing] away from preconceptions ... and creat[ing] new questions [that] might grow into a totally new concept" (p. 171).

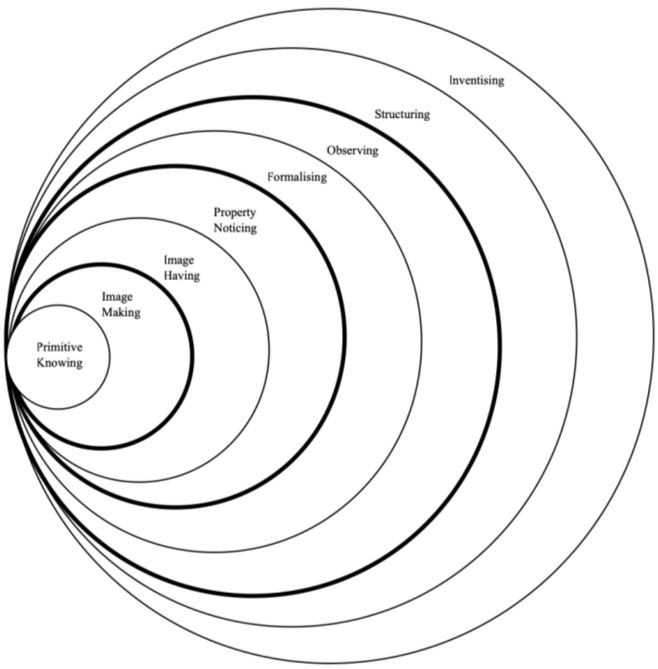


Figure 9: Pirie-Kieren Model

The nested structure of the model shown in Figure 9 reflects each level as including all inner levels as well as being integral to all outer levels.

To date, the Pirie-Kieren model/theory has predominantly been used to illuminate the understanding of individual students. In contrast, we use Pirie and Kieren's model/theory to attend to the emergence and dynamics of ideas at the collective level, in mathematics classes, as suggested by Thom and Glanfield (2018); Kieren and Simmt (2002); Martin and Towers (2003; 2015); Davis and Simmt (2003); and Pirie and Kieren (1994).

Using the JP 4 TIMMS lesson we first identified concepts and ideas within the lessons. We identified the level at which the ideas emerged, monitored the ideas as they were (re)iterated or elaborated upon, and tracked during each lesson as they moved back and forth across the different levels of the model. There were two ideas in the Japan lesson around the concept of inequality. Idea 1 (I1[JP]), the first idea to emerge, involved the procedure(s) used to solve an inequality. Idea 2 (I2[JP]), the second idea to emerge, related to how an inequality expression could be used to model a specific context.

DYNAMICS OF IDEAS TOOL

In the first iteration of the Dynamics of Ideas Tool we used the Pirie-Kieren theory to code the mathematical ideas within the lessons and map the emergence, (re)iteration(s), elaboration(s), and the dynamics, or movement back and forth, of those ideas onto the Pirie-Kieren model, according to the eight levels (as seen in Figure 10 which shows the mapping of the first 17 minutes of the Japan lesson). Five minutes 33 seconds of this period were not coded. Two minutes 57 seconds consisted of going over homework related to I1[JP]. I1[JP] emerged, and for the most part, stayed at the *Formalising* level. The balance of time, 8 minutes 30 seconds, was spent on I2[JP]. Interestingly, I2[JP] arose in manners that were not specific to any one level in the model but indeed, clearly beyond *Image Having*. To distinguish these events, we mapped the moments in which I2[JP] occurred *Beyond Image Having* as dotted spheres on the boundaries between levels. In addition to this, and unlike I1[JP], I2[JP] moved across levels, back and forth, from *Primitive Knowing* through to *Formalising*, and *Beyond Image Having*.

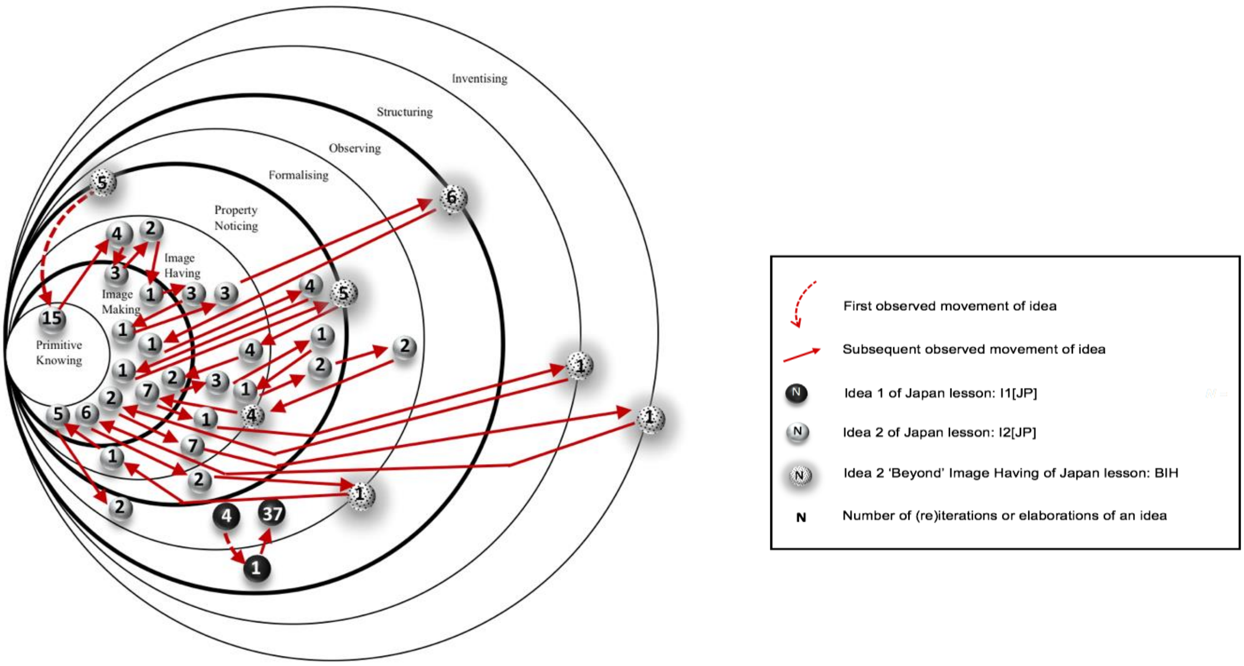


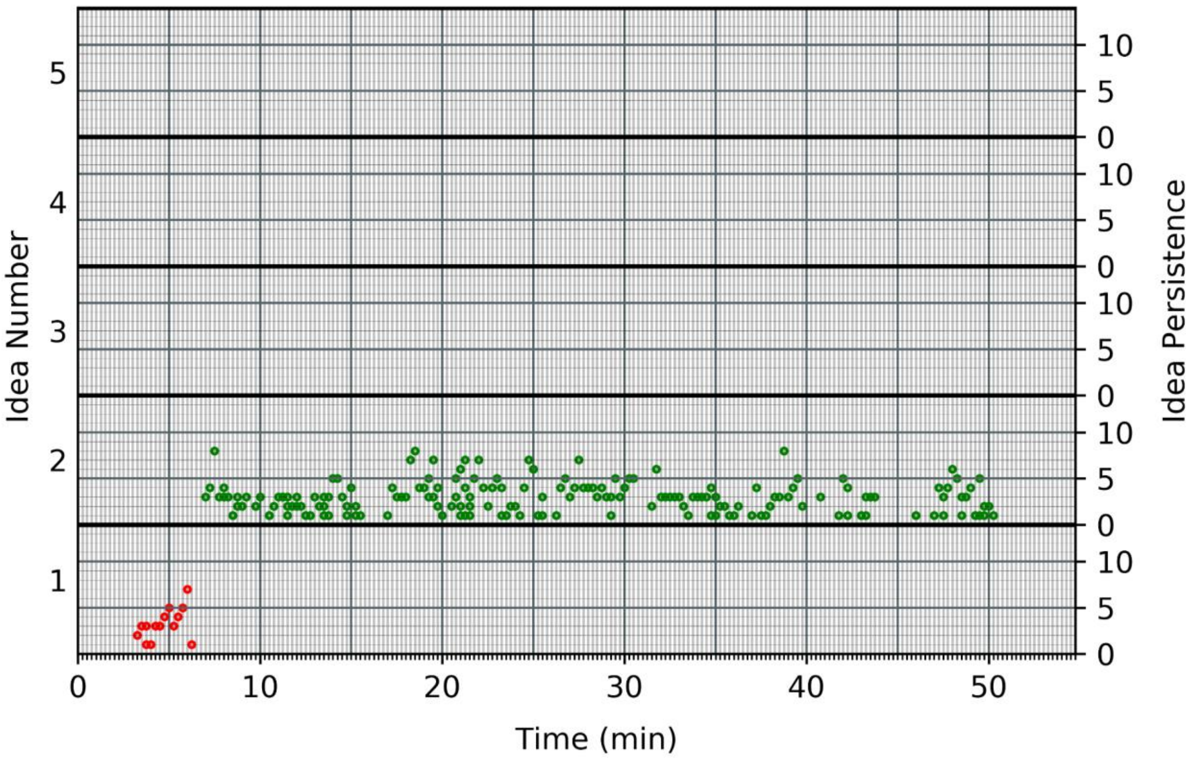
Figure 10: 0:00-16:59 of the Japan lesson

We encountered challenges in using this tool. First, the tool was not clear due to the sheer density of the ideas; that is, as they emerged and underwent (re)iterations, elaborations, and moved across the levels. Second, while the nested model allowed for chronological sequencing, it did not allow for mapping along linear time which meant that specific moments in time within any one lesson could not be compared with other tools being developed.

FROM ONE TO TWO TOOLS

In designing the second iteration of the tool, we separated the two dynamics: the persistence (i.e., the (re)iterations and elaborations) of the ideas and the movement (or lack thereof) of the ideas across the Pirie-Kieren levels in order to address the first challenge. For each of the dynamics, we then mapped them in 15-second increments to address the second challenge. The addition of the 15-second increments as a standard timeline allowed for identifying a specific moment in time and as well, comparison of any moment across tools.

PERSISTENCE OF IDEAS TOOL

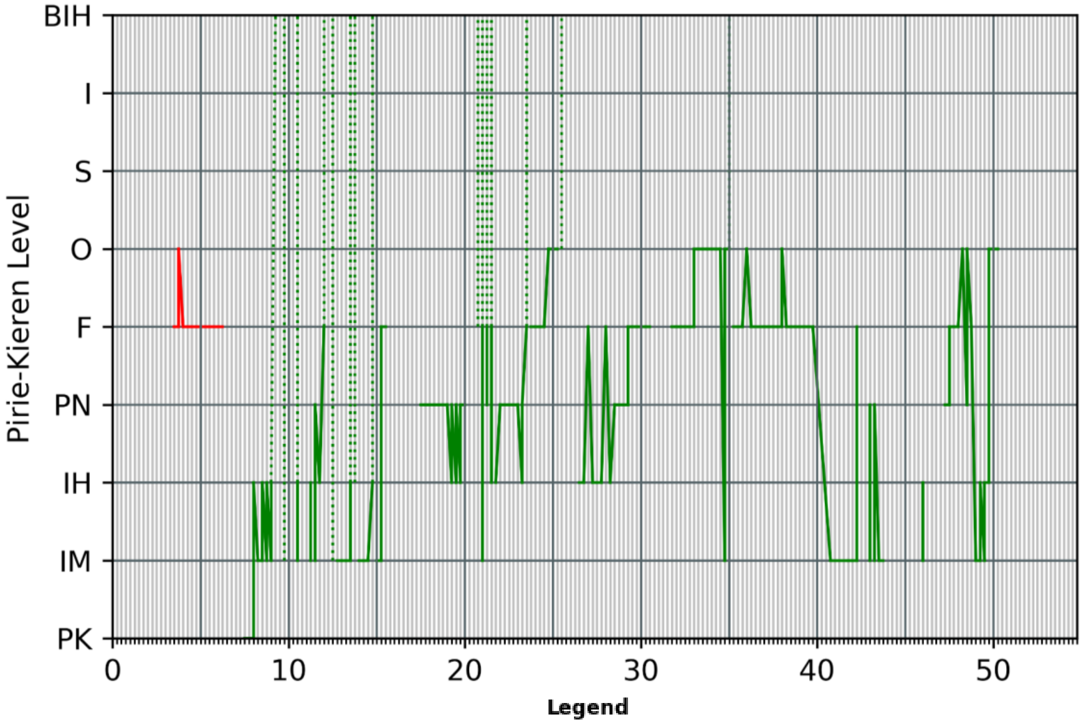


Legend of Ideas	
Idea 2	related to how an inequality expression could be used to model a specific context
Idea 1	involved the procedure(s) used to solve an inequality

Figure 11: Persistence of Ideas for JP 4

Graphically, we can clearly see the persistence of Idea 1 and Idea 2 as well the difference of persistence between the two ideas across the whole period of time (see Figure 11). The ways in which the ideas persisted across the whole lesson could not be seen in the Dynamics of Ideas Tool as mapped on the Pirie-Kieren model. This new tool monitors the observed ideas as they emerge, are elaborate upon, and reiterated within the classroom as a collective.

MOVEMENT OF IDEAS TOOL



BIH	Beyond Image Having
Pirie Kieren Levels	
I	Inventising
S	Structuring
O	Observing
F	Formalising
PN	Property Noticing
IH	Image Having
IM	Image Making
PK	Primitive Knowing

Figure 12: Movement of Ideas for JP4

The movement (or lack thereof) of ideas can be observed across the Pirie-Kieren levels and across the whole period of lesson time (see Figure 12). This could not be seen in the Dynamics of Ideas Tool as mapped on the Pirie-Kieren model. Neither were instances observed as not mathematical located on the model. The breaks in the graph are periods of time that could not be coded for a variety of reasons (e.g., no mathematical ideas were expressed for approximately the first 3 minutes of the lesson, and no mathematical ideas emerged at approximately the 7 minute and 15-17 minutes marks of the lesson). I1[JP] emerged, and for the most part, stayed at the Formalising level. Interestingly, I2[JP] arose in manners that were not specific to any one level in the model but indeed, clearly beyond Image Having. To distinguish these events, we mapped the moments in which I2[JP] occurred Beyond Image Having using dotted lines. Unlike I1[JP], I2[JP] moved across levels, moved back and forth, from Primitive Knowing through to Formalising and Observing, and Beyond Image Having throughout the lesson. This tool monitors how the ideas moved within the Pirie-Kieren levels as they are taken up within the classroom as a collective.

VERTICALLY ALIGNING THE TWO TOOLS

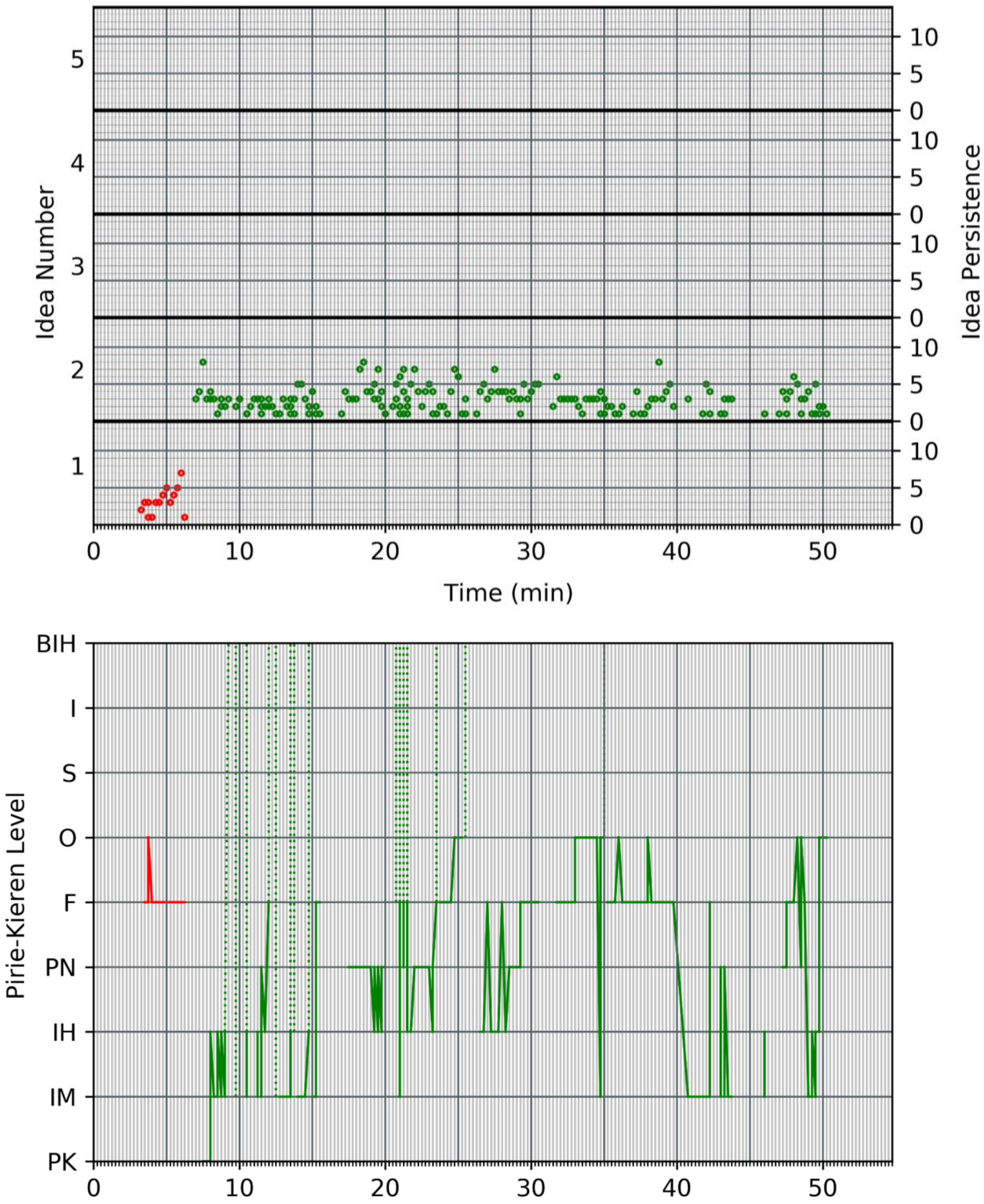


Figure 13: Aligning the Two Graphs

When the two tools are aligned vertically, we can see at any moment in time, which idea is being taken up in the collective, the persistence of that idea, and the Pirie-Kieren level at which the ‘taken up’ occurs (see Figure 13). So, for example, between the 17 and 20-minute period of the lesson, Idea 2 was elaborated upon or reiterated 1 to 8 times at any given moment within the collective. Within this time period, the idea moved back and forth between Property Noticing and Image Having. In contrast, during the 40- to 45-minute time period in the lesson, Idea 2 can be observed as persisting between 1 to 5 times while moving back and forth from Formalising to Image Making to Property Noticing then back to Image Making. These two examples could not be seen in the Dynamics of Ideas Tool. Further still, at approximately the 13 to 15-minute time period, the persistence of Idea 2 also occurs 1 to 5 times, however, the idea moves between Property Noticing, Image Having, and Beyond Image Having.

CONCLUDING THOUGHTS

The first iteration of the Dynamics of Ideas Tool attempted to observe two dynamics at the same time. The second iteration involved the separation of the two dynamics into two distinct Tools. The two tools offer a clearer way to monitor the persistence and movement of mathematical ideas within the classroom as a collective.

LAYERING THE TOOLS

We offer Figure 14 as an initial layering of three tools for the Japan lesson (JP4): Lesson Activity Mapping, Bodymarking, and Movement of Ideas. By aligning the tools vertically we may begin to see visual patterns across the three visualizations. As we might expect, the lesson pattern has many features in common with the Bodymarking. For example, boardwork and pointing occur predominantly in the public activities of presenting and sharing, while the writing occurs during private engagement times. These are also the time segments when we see more movement in ideas to the different levels of the Pirie-Kieren model.

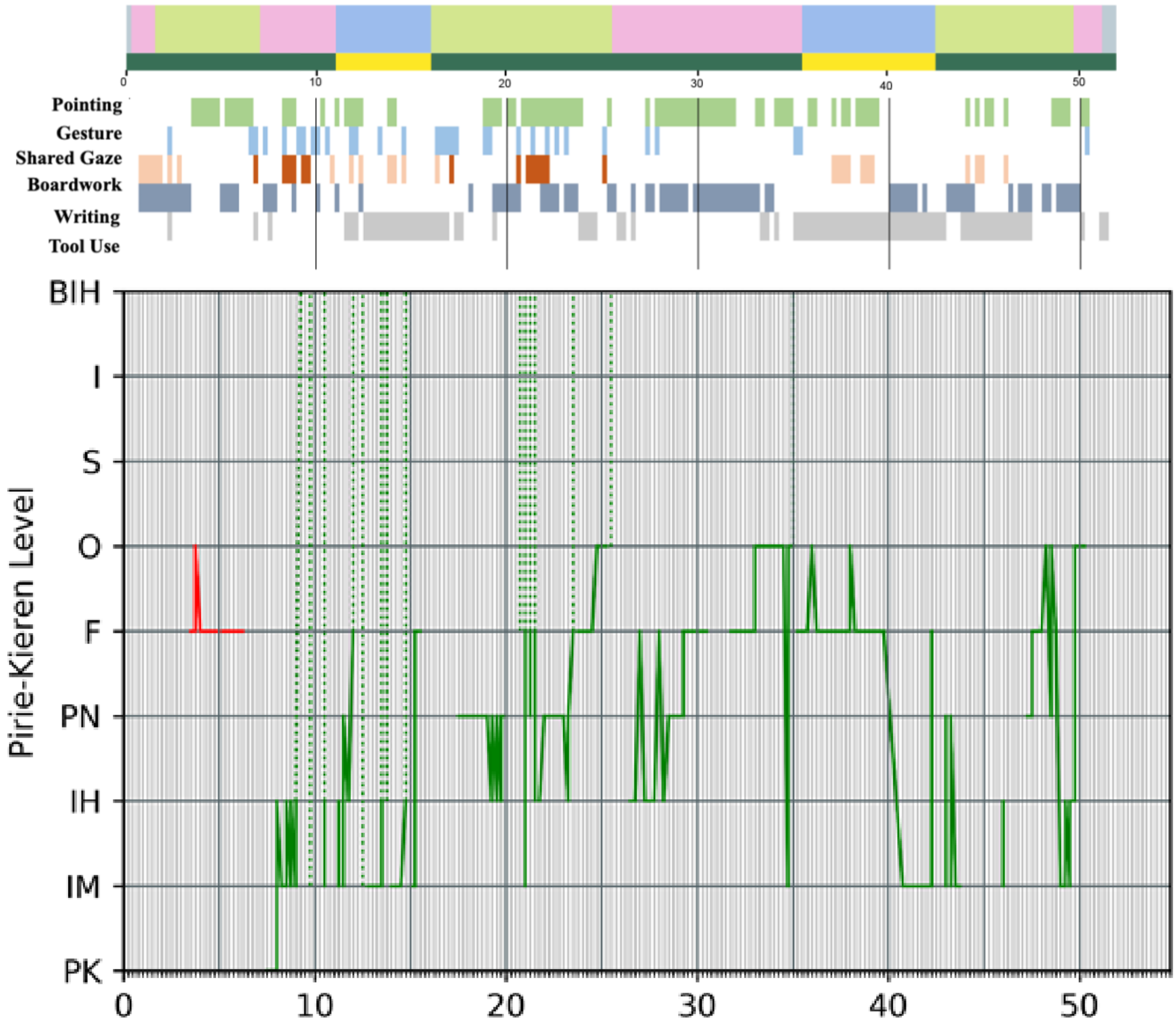


Figure 14: Layering Tools for Collective Activity

We believe that it is by layering multiple tools that we may be able to notice possible moments of interest, emergence, activity, and inactivity within a classroom. It is by exploring different modelling techniques of different aspects of collective activity that we can gain insight into global traits and group activity of collective systems.

Acknowledgement

Research was supported by the Social Sciences and Humanities Research Council of Canada.

References

Abrahamson, D., Shayan, S., Bakker, A., & van der Schaff, M. (2015). Eye-tracking Piaget: Capturing the emergence of attentional anchors in the coordination of proportional motor action. *Human Development*, *58*, 218–244.

Alibali, M. W., Nathan, M. J., Wolfgram, M. S., Church, R. B., Jacobs, S. A., Martinez, J. C., & Knuth, E. J. (2014). How teachers link ideas in mathematics instruction using speech and gesture: A corpus analysis. *Cognition & Instruction*, *32*, 65–100.

Anderrson, J., & Risberg, J. (2018). Embodying teaching: A body pedagogic study of a teacher's movement rhythm in the 'sloyd' classroom. *Interchange*, *49*, 179–204.

Anderrson, J., & Risberg, J. (2019). The walking rhythm of physical education teaching: An in-path analysis. *Physical Education and Sport Pedagogy*, 24(4), 402–420.

Clarke, D., Mesiti, C., Jablonka, E., & Shimizu, Y. (2006). Addressing the challenge of legitimate international comparisons: Lesson structure in Germany, Japan and the USA. In D. Clarke, J. Emanuelsson, E. Jablonka & I. Ah Chee Mok (Eds.), *Making connections: Comparing mathematics classrooms around the world*. Sense Publishers.

Clarke, D., Mesiti, C., O'Keefe, C., Xu, L.H., Jablonka, E., Mik, I.A.C., & Shimizu, Y. (2007). Addressing the challenge of legitimate international comparisons of classroom practice. *International Journal of Educational Research,* 46(5), 280-293.

Cooperrider, K. (2021). Fifteen ways of looking at a pointing gesture. https://psyarxiv.com/2vxft/

Davis, B., & Simmt, E. (2003). Understanding learning systems: Mathematics education and complexity science. *Journal for Research in Mathematics Education,* 34(2), 137–167.

Davis, B. & Simmt, E.S. (2016). Perspectives on complex systems in mathematics learning. In D. Kirshner & L.D. English (Eds.), *Handbook of international research in mathematics education, 3rd edition,* (pp. 416-432). Taylor & Francis.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher,* 25(4), 12-21.

Hiebert, J., Gallimore, R., Garnier, H., Givvin, K. B., Hollingsworth, H., Jacobs, J., et al. (2003). Teaching Mathematics in Seven Countries: Results From the TIMSS 1999 Video Study (NCES 2003–013). US Department of Education. National Center for Education Statistics.

Kääntä, L. (2012). Teachers' embodied allocations in instructional interaction. *Classroom Discourse,* 3(2), 166–186.

Kaur, B. (2021). Mathematics teacher practice and student perception of how they learn mathematics in the context of Singapore. *ZDM - Mathematics Education*. doi.org/10.1007/s11858-021-01318-2

Kieren, T., & Simmt, E. (2002). Fractal filaments: A simile for observing collective mathematical understanding. In D. Mewborn, P. Sztajn, D. White, H. Wiegel, R. Bryant, & K. Nooney (Eds.), *Proceedings of the 24th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, (Vol. 2, pp. 865-874). Eric Clearinghouse for Science, Mathematics, and Environmental Education.

Luff, P., Jirotka, M., Yamashita, N., Kuzuoka, H., Heath, C., & Eden, G. (2013). Embedding interaction: The accomplishment of actions in everyday and video-mediated environments. *ACM Transactions on Computer-Human Interaction,* 20(1), Article 6.

Martin, L. C., & Towers, J. (2003). Collective Mathematical Understanding as an Improvisational Process. International Group for the Psychology of Mathematics Education, 3, 245-252.

Martin, L. C., & Towers, J. (2015). Growing mathematical understanding through collective image making, collective image having, and collective property noticing. *Educational Studies in Mathematics*, 88(1), 3-18.

McGarvey, L.M., Davis, B., Glanfield, F., Martin, L., Mgombelo, J., Proulx, J., Simmt, E., Thom, J. & Towers, J. (2015). Collective learning: Conceptualizing the possibilities in the mathematics classroom. Bartell, T.G., Bieda, K.N., Putnam, R.T., Bradfield, K., & Dominguez, H. (Eds.), *Proceedings of the annual meeting of the North American Chapter of International Group for the Psychology of Mathematics Education* (pp. 1333-1342). Lansing, MI: Michigan State University.

McGarvey, L.M., Thom, J.S., Glanfield, F., Mgombelo, J., Simmt, E., Davis, B., Martin, L., Proulx, J., Towers, J., & Luo, L. (2017). Monitoring the vital signs of classroom life. Presentation at the 2017 NCTM Research Conference. San Antonio, Texas.

McGarvey, L.M., Thom, J.S., Mgombelo, J., Proulx, J., Simmt, E., Glanfield, F., Davis, B., Martin, L., Towers, J., Bertin, C., Champagne, K., L'Italien-Bruneau, R.-A., & Mégrourèche, C. (2018). Vital signs of collective life in the classroom. In E. Bergqvist, M. Österholm, C. Granbert, & L. Sumpter (Eds.), *Proceeding of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol I, pp. 155-184). Umeå, Sweden: PME.

McIntyre, N., A., Halszka, J., & Klassen, R. M. (2019). Capturing teacher priorities: Using real-world eye-tracking to investigate expert teacher priorities across two cultures. *Learning and Instruction, 60*, 215–224. https://doi.org/10.1016/j.learninstruc.2017.12.003

Mgombelo, J., Thom, J., Glanfield, F. & McGarvey, L. (2018). Vital sign 2: (Non)actions on the board. In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 165-169). Umeå, Sweden: PME.

Novack, M., & Goldin-Meadow, S. (2015). Learning from gesture: How our hands change our minds. *Educational Psychology Review, 27*, 405–412.

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterize it and how can we represent it? *Educational Studies in Mathematics, 26*, 165-190.

Russo, J., & Hopkins, S. (2017). How does lesson structure shape teacher perceptions of teaching with challenging tasks? *Mathematics Teacher Education and Development, 19*, 30–46.

Seidel, T., Schnitzler, K., Kosel, C., Stürmer, K., & Holzberger, D. (2021). Student characteristics in the eyes of teachers: Differences between novice and expert teachers in judgment accuracy, observed behavioral cues, and gaze. *Educational Psychology Review, 33*, 69–89.

Shilling, C. (2017). Body pedagogics: Embodiment, cognition and cultural transmission. *Sociology*, 51(6), 1205–1221.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning*, 10(4), 313-340.

Stigler, J. W., Gonzales, P., Kawanaka, T., Knoll, S., & Serrano, A. (1999). The TIMSS Videotape Classroom Study: Methods and findings from an exploratory research project on eighth-grade mathematics instruction in Germany, Japan, and the United States. Washington, DC: U.S. Government Printing Office.

Stigler, J. W. & Hiebert, J. (1999). The Teaching Gap. New York: Free Press.

Stodolsky, S. S. (1988). *The subject matters: Classroom activity in math and social studies*. Chicago: The University of Chicago Press.

Stromaier, A. R., Mackay, K. J., Obersteiner, A., & Reiss, K. M. (2020). Eye-tracking methodology in mathematics education research: A systematic literature review. *Educational Studies in Mathematics, 104*, 147–200.

Thom, J. S. & Glanfield, F. (2018). Live(d) topographies: The emergent and dynamical nature of ideas in secondary mathematics classes. In A. Kajander, J. Holm, and E. Chernoff (Eds.). *Teaching and learning secondary school mathematics: Canadian perspectives in an international context* (pp. 51-60). Springer.

Thom, J.S., Glanfield, F., Mgombelo, J. Proulx, J. McGarvey, L. & Towers, J. (2021). Research tools for collectivity: Tracking mathematics classes. In A.I. Sacristan, J.C Cortes-Zavala & P.M. Ruiz-Arias (Eds.), *Proceedings of the 2020 Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 159-161). Mazatlán: PME-NA.

Varela, F.J., Thompson, E. & Rosch, E. (2017). *The embodied mind, revised edition: Cognitive science and human experience*. The MIT Press.

Wilmes, S. E. D., & Siry, C. (2021). Multimodal interaction analysis: A powerful tool for examining plurilingual students' engagement in science practices. *Research in Science Education*, *51*, 71–91.