



# Determination of dynamic elastic modulus of materials under a state of simple stresses by using electrodynamic actuators in beam-type mechanical elements

J. Carbajo<sup>a,\*</sup>, P. Poveda<sup>a</sup>, E. Segovia<sup>b</sup>, E. Rincón<sup>a</sup>, J. Ramis<sup>a</sup>

<sup>a</sup> University of Alicante, Department of Physics, Systems Engineering and Signal Theory, San Vicente del Raspeig, Spain

<sup>b</sup> University of Alicante, Department of Civil Engineering, San Vicente del Raspeig, Spain

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## ABSTRACT

The dynamic elastic modulus is an important parameter for the study of solid mechanical elements. Even though there exist several well-established standardized methods that allow determining this parameter under a state of simple stresses, some of these may require sophisticated laboratory equipment or involve the use of multiple devices (impulser, sensor transducer...). The main purpose of this work is to propose an alternative approach that uses a single electrodynamic actuator both as an impulse exciter and vibration sensor simultaneously. By generating a random signal excitation and measuring the electrical input impedance of the actuator when coupled to a beam-type cylindrical element, the fundamental flexural mode of vibration of the latter can be identified and the dynamic elastic modulus of its material determined in a simple and straightforward manner. Preliminary experiments were performed over different size beams with rectangular cross-sections made of natural stone, marble, wood, and aluminium. Results were compared to those obtained using the standardized procedure described in the ASTM E1876 for the same specimens, these showing a good agreement in terms of dynamic elastic modulus.

## 1. Introduction

Obtaining the dynamic elastic properties of solid materials is of great interest in many scientific disciplines both from the practical point of view and for design purposes [1]. Most prediction methods involving solid media usually require prior knowledge of these properties to perform the corresponding simulations that let estimate the mechanical behaviour of the system under analysis. In this context, it turns out important the development of experimental characterization methods that let determine these properties over specimens of different composition so that this data can be used in the design stage of structures.

There exist several standardized methods that let determine the dynamic elastic properties of solid materials under a state of simple stresses. The ASTM E1876 [2] implements the Impulse Excitation Technique (IET) to determine the dynamic elastic modulus (or Young's modulus) of materials from the resonant frequencies of rectangular or cylindrical specimens, provided their geometry and mass are known beforehand. In a recent work by Torres-Romero et al. [3], it was shown

that the resonant modes of beam-type elements may be also analysed by exciting these using an electrodynamic actuator, results showing a good agreement when compared with well-known analytical solutions. However, to the author's knowledge, there is a lack of works dealing with the application of these devices for the characterization of the dynamic elastic properties of solid materials.

This work proposes a simple procedure that relies on the use of electrodynamic actuators to determine the elastic modulus of natural stone, marble, wood, and aluminium by using different size beam-type elements. It is shown that by measuring the electrical input impedance of such transducers when coupled to a simply supported beam, the flexural modes of vibration of the beam can be identified and the elastic modulus of its material retrieved.

\* Corresponding author.

E-mail addresses: [jesus.carbajo@ua.es](mailto:jesus.carbajo@ua.es) (J. Carbajo), [pedro.poveda@ua.es](mailto:pedro.poveda@ua.es) (P. Poveda), [enrique.segovia@ua.es](mailto:enrique.segovia@ua.es) (E. Segovia), [erc48@alu.ua.es](mailto:erc48@alu.ua.es) (E. Rincón), [jramis@ua.es](mailto:jramis@ua.es) (J. Ramis).

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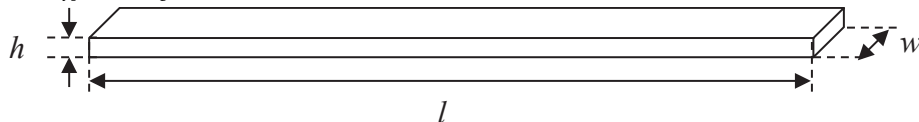
**Table 1**

Characteristics, flexural frequencies, and elastic modulus (obtained using the different methods) of the beam-type elements under study.

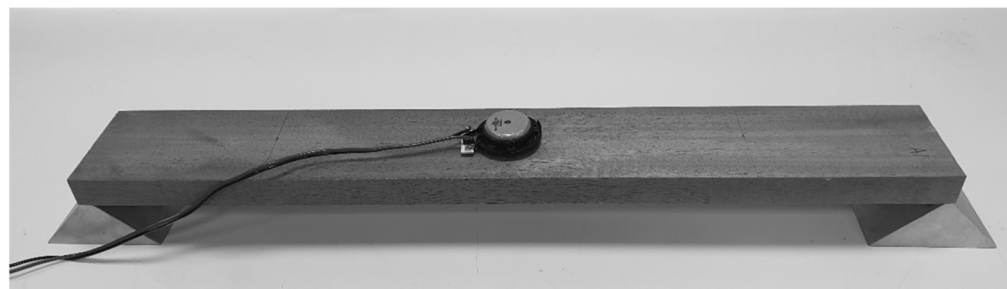
Sample	$h$ (m)	$l$ (m)	$w$ (m)	$\rho$ (kg/m <sup>3</sup> )	$f_1$ (Hz) <sup>1</sup>	$f_f$ (Hz) <sup>2</sup>	$E_{approach}$ (GPa)	$E_{ASTM\ E1876}$ (GPa)	Relative error (%)
BS#1 (natural stone)	0.03	1	0.07	2038	44.03	100.30	21.69	21.35	1.56
BS#2 (marble)	0.02	1	0.07	2562	40.19	92.86	52.42	50.32	4.00
BS#3 (wood)	0.025	0.635	0.09	629	112.20	258.20	9.98	10.42	4.30
BS#4 (aluminium)	0.02	0.705	0.05	2654	84.14	199.86	56.43	62.38	9.41

<sup>1</sup>: Proposed approach (use Eq. (1)). <sup>2</sup>: ASTM E1876 (use Eq. (2)).

Beam-type element general sketch



(a)



(b)

**Fig. 1.** General view of the experimental setups used: (a) ASTM E1876; (b) Proposed approach.

**2. Background theory**

*2.1. Flexural modes of a simply supported beam*

Let us consider a uniform beam of length  $l$  with a rectangular cross-section of width  $w$  and depth  $h$ . The natural frequencies of flexural vibration of the beam under simply supported boundary conditions can be obtained from [4].

$$\omega_n = \frac{n^2 \pi^2}{l^2} \sqrt{\frac{EI}{\rho wh}} \tag{1}$$

where  $n$  is a positive integer,  $\rho$  the mass density of the beam,  $E$  is Young's modulus of the beam material, and  $I = wh^3/12$  is the second moment of area.

**3. Material**

*3.1. Beam-type elements*

Four different beam-type elements of rectangular cross-section were

used for the experiments, their corresponding geometrical characteristics and mass density being summarized in Table 1. The materials chosen were natural stone, marble, wood, and aluminium. Besides, two metallic wedges were used to support the specimens and isolate these from undesired vibrations both in the standardized method and in the proposed approach.

*3.2. Measurement system*

The acquisition platform used to perform the electrical input impedance measurements consisted of the TEAX25C10-8/HS electrodynamic actuator and the CLIOwin 7 measurement system. As for the Impulse Excitation Technique (IET) test, a B&K accelerometer type 4534-B-002 connected to a B&K signal conditioner 1704-A-002 was used, the impact force being produced with a light self-made impulser following the recommendations in the ASTM E1876.

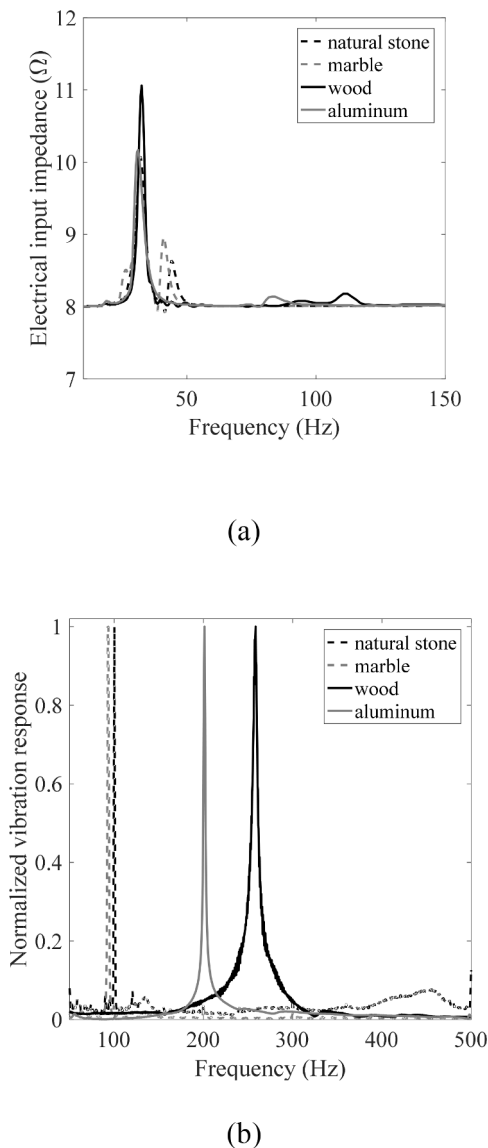


Fig. 2. Comparison of the different methods: (a) Proposed approach (electrical input impedance); (b) ASTM E1876 (normalized vibration response).

## 4. Methods

### 4.1. ASTM E1876

In the current work, each specimen was placed on the supports located at the fundamental nodal points (0.224 l from each end of the beam), whereas the contact transducer (i. e. the accelerometer) was placed only as far from the nodal points as necessary to obtain a reading while minimizing the damping effect resulting from the mass load. As for the impuler, it consisted of a metallic cylinder 0.7 cm in diameter attached to the end of a 10 cm long polymer rod (see Fig. 1a), as recommended in section 7.2 in the standard, its mass being sufficient to induce a measurable mechanical vibration. For this purpose, the beam must be excited at its centre using the impuler and the frequency spectrum of the signal received by the accelerometer used to identify the fundamental flexural mode. This frequency value is then substituted in the following equation to obtain Young’s modulus.

$$E = 0.9465 \left( m f_j^2 / w \right) (l^3 / h^3) T_1 \quad (2)$$

where  $m$  is the mass of the bar,  $f_j$  is the measured fundamental frequency

of bar in flexure, and  $T_1 = 1 + 6.585(h/b)^2$  a correction factor for slender bars ( $l/h \geq 20$ ). Fig. 1a shows a detailed view of the experimental setup used to reproduce the ASTM E1876.

### 4.2. Proposed approach

The proposed experimental approach consists of a simply supported beam-type element mechanically excited by an electrodynamic actuator located in the centre of the specimen (see Fig. 1b). Even though positioning the transducer at an antinode (location of maximum displacement) may slightly modify the natural vibration of the specimen due to the mass-load effect [2], given that the weight of the actuator was far below (between 2 and 9%) that of the beam, this had a minimal influence on the vibrational behaviour of the whole element. On the other hand, to minimize the excitation of additional modes of vibration (e. g. torsion modes), the actuator location was centred as much as possible. Unlike the standardized procedure, the support wedges were placed at the edges of the beam to guarantee simply supported boundary conditions that let attain lower fundamental resonance frequencies and ease the identification of the fundamental resonance frequency on the measured electrical input impedance spectrum.

In brief, when an electrical voltage is applied to the actuator it causes a movement of its mass and, hence, of the beam to which it is attached. Given that the actuator is placed over the top surface of the beam, this transverse displacement is responsible for the generation of flexural modes in the beam whose load effect on the transducer can be captured by measuring the electrical input impedance [5]. Therefore, once the fundamental flexural frequency of the beam is identified in the measured electrical input impedance, it is straightforward to determine the elastic modulus of its material by using Eq. (1).

## 5. Results and discussion

### 5.1. Proposed approach vs ASTM E1876

The proposed approach was compared to the method described in the ASTM E1876 in terms of the calculated dynamic Young’s modulus for the specimens under test. Following the procedures described in the previous section, both the frequency spectrums of the electrical input impedance and the vibration response were obtained. Experiments were repeated five times in each case to minimize measurement errors, the average results being used for the subsequent calculations. Fig. 2 shows the resulting spectrums for all the analysed specimens.

In Fig. 2a, the peak with a higher amplitude around 32 Hz corresponds to the mechanical resonance of the actuator mass-spring system, whereas the other peaks are the fundamental flexural mode of the corresponding beam under analysis. As expected, these latter frequency values differ from the resonance peaks obtained using the ASTM E1876 (Fig. 2b) because the boundary conditions were different in each case. Nevertheless, the dynamic elastic modulus obtained using Eq. (1) in the proposed approach and Eq. (2) in the standardized method yield very similar results as summarized in Table 1 (relative errors were calculated using as reference the standard).

### 5.2. Remarks on the applicability of the proposed approach

There are some important remarks that are worth discussing. On the one hand, the correct coupling of the electrodynamic actuator to the beam is critical to achieving a proper electrical input impedance response avoiding extra damping mechanisms, the use of self-adhesive mounting being recommended. On the other hand, and as stated in Section 4.2, the mass-load effect of the actuator may imply a source of error in those specimens whose mass density is too low, the use of alternative analytical expressions that account for this effect [6] being more appropriate to derive the fundamental resonance frequency in such cases. The simply supported boundary conditions used in the

proposed approach were chosen so that the fundamental resonance frequency of the beam lies below the coil inductance region of the electrical input impedance spectrum to minimize the masking effect and ease its identification. All the same, results showed the proposed approach to be a valid methodology to obtain the elastic modulus of materials by using beam-type elements with cost-effective laboratory equipment. Further research is yet encouraged to assess the applicability of the proposed approach on small-sized specimens or alternative materials to those analysed in the current work.

## 6. Conclusions

An experimental approach for measuring the dynamic elastic modulus of materials by using beam-type mechanical elements was proposed and tested for several beams made of materials commonly used in many applications such as natural stone, marble, wood, and aluminium. A good accuracy (error below 10%) was obtained when compared to the well-established standardized procedure described in the ASTM E1876, the approach being considered a reliable alternative for the specimens under study. Preliminary results encourage the use of the proposed procedure as a complementary approach for the determination of dynamic elastic modulus of solid materials under a state of simple stresses from beam-type mechanical elements.

*CRediT authorship contribution statement*

**J. Carbajo:** Conceptualization, Methodology, Writing – original

draft. **P. Poveda:** Investigation, Writing – review & editing. **E. Segovia:** Writing – review & editing. **E. Rincón:** Investigation. **J. Ramis:** Writing – review & editing, Supervision.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## References

- [1] A. Wolfenden. Dynamic elastic modulus. Measurements in materials, ASTM International, 1990.
- [2] ASTM E1876-15. Standard test method for dynamic Young's modulus, shear modulus, and Poisson's ratio by impulse excitation of vibration, ASTM International, 2015.
- [3] J. Torres-Romero, W. Cardenas, J. Carbajo, E. Segovia-Eulogio, J. Ramis-Soriano, An experimental approach to vibroacoustic study of beam-type structures, Arch. Acoust. 43 (2018) 283–295, <https://doi.org/10.24425/122376>.
- [4] F. Fahy, P. Gardonio, Sound and structural vibration: radiation, transmission and response, Noise Control Eng. J. 55 (3) (2007) 373.
- [5] L.L. Beranek, T.J. Mellow, Acoustics: sound fields and transducers, Academic Press, 2012.
- [6] G.-R. Gillich, Z.-I. Praisach, M. Abdel Wahab, N. Gillich, I.C. Mituletu, C. Nitescu, Free vibration of a perfectly clamped-free beam with stepwise eccentric distributed masses, Shock Vib. 2016 (2016) 1–10.