

# Optimal load scheduling for off-grid photovoltaic installations with fixed energy requirements and intrinsic constraints

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## **ABSTRACT**

*Solar energy is one of the most promising green energy sources. On-grid photovoltaic installations supply energy to consumers as a support energy source, but in isolated areas, it comes as the unique source. The decision-maker must dimension the installation, maintaining system performance with reasonable investments. In some scenarios, the utility manager can handle the energy delivered to consumers as every subsystem can be independently connected. A strategy for scheduling the energy consumption to decrease the number of photovoltaic modules required in a standalone system is proposed here. The problem formulation corresponds to generalising a more specific problem before published. We presented a real case study being the groups of hydrants that provide water to crops in a pressurized irrigation system for energy consumption to schedule.*

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33 *Keywords: energy efficiency; photovoltaic energy; solar plant size optimization*

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## 1. INTRODUCTION

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Photovoltaic (PV) systems have increased in the last times, covering fields with limited applications. But, in productive activities, the PV panels installation is large capital expenditures that can reduce the incomes and increase the payback period making it unaffordable. Then, procedures for the optimal design of photovoltaic plants are necessary. Typical approaches to the design problem are based on the guarantee of energy under the worst case. Special development of research has emerged around hybrid installations as PV-diesel (Xue, 2017) or PV-ES (energy storage) plants (Dufo-López and Bernal-Agustín, 2005; Kaldellis et al., 2010; Perez, E.; Beltran, H.; Aparicio, N.; Rodriguez, 2013). Design of big plants are defined by their problems and has also been an object of study in (Beltran et al., 2012).

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Solar energy production becomes one of the hottest topics in the water industry as irrigation occurs in isolated areas (no grid connection) and pressurized irrigation networks (PINs) are very energy hungry. For the last 70 years, irrigation networks shifted from gravity-fed irrigation into pressurized irrigation (PIN). The irrigated area grew 2.5 times, water consumption doubled and energy expenditure multiplied by 19 in Spain (Corominas, 2010). These enormous figures show efficient management of PIN has become of paramount importance (Moradi-Jalal and Karney, 2008; Pardo et al., 2018).

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Given that the anthropic pressure influences the environment, the photovoltaic (PV) technology incorporates in the agriculture industry, like green (no emissions) and

54 profitable choice to conventional power sources (Mérida García et al., 2019). Solar power  
55 management comes from one of the trendiest questions in the water industry (Aliyu et  
56 al., 2018; Chandel et al., 2015). Energy production in PV systems expanded overall energy  
57 production from 29.5 (2012) to 107 (2018) GW (Jäger-Waldau, 2019). This momentum is  
58 because of a shift to better large-scale service systems, shifts in legislation, and a universal  
59 devaluation of PV modules prices (Goodrich et al., 2013) (30–60% in 10 years; (Closas and  
60 Rap, 2017)).

61         Many researchers analysed how to convert direct-drive pumping devices fed by  
62 electricity grids into a standalone (off-grid) direct solar waterpower system (SPWS) (Betka  
63 and Attali, 2010; Elkholy and Fathy, 2016; Mohanty et al., 2018; Pardo et al., 2020a; Ru et  
64 al., 2013). Converting water pressurized network (WPN) into a standalone full PV supplied  
65 system involves accumulating energy in head tanks or batteries. But, in irrigation, the  
66 utility manager may regulate water (not in urban WPNs) and energy expenditure (Pardo  
67 et al., 2018). Standalone systems are the most widespread solution around the world  
68 because of simplicity, great efficiencies and adaptable to all sizes and irrigation methods  
69 (Hartung and Pluschke, 2018). Other optimisation problems have significant interest and  
70 an inherent difficulty, as determining the best tilt angle on locations under high diffusive  
71 and reflective factors (Gökmen et al., 2016; Kerekes et al., 2012). Design problems in  
72 extreme conditions involve considering other restrictions as cooling needs (Karira et al.,  
73 2004).

74         Scheduling problems to minimise (or maximise) an objective function by satisfying  
75 some set of constraints is frequently studied. To name a few, there are scheduling jobs in

76 machines (Liu and Yang, 2011; Luo and Chu, 2006; Pham et al., 2007), transportation  
77 (Forouharfard and Zandieh, 2010; Shrivastava et al., 2002; Thengvall et al., 2003),  
78 computer task scheduling (Chrétienne, 1989; Feitelson et al., 1996; Moore, 2004, 2003)  
79 and energy optimising (Ahmed et al., 2017). These optimisation problems are hard to  
80 solve because of the size of the configuration space. By example, the job shop problem  
81 (JSP) (Fang et al., 1996; Gholami and Zandieh, 2009; Hefetz and Adiri, 1982; Koonce and  
82 Tsai, 2000; Nakano and Yamada, 1991) is known as an example of NP-complete problems  
83 (nondeterministic polynomial time).

84         The proposed algorithms to solve scheduling problems are non-deterministic  
85 (Forouharfard and Zandieh, 2010; Liu and Yang, 2011; Luo and Chu, 2006; Pham et al.,  
86 2007; Shrivastava et al., 2002; Thengvall et al., 2003), (Fang et al., 1996; Gholami and  
87 Zandieh, 2009; Hefetz and Adiri, 1982; Koonce and Tsai, 2000; Nakano and Yamada,  
88 1991). For instance, genetic algorithms (Fang et al., 1996; Gholami and Zandieh, 2009;  
89 Nakano and Yamada, 1991; Shrivastava et al., 2002) are suitable for optimisation  
90 problems where the space of configuration is too big to be explored using a deterministic  
91 algorithm in workable computing time, or where the objective function presents local  
92 extremes. But the algorithm does not guarantee to get the global best solution to the  
93 problem (a limiting characteristic).

94         The present paper considers a case of the scheduling problem. It corresponds to  
95 the design of a photovoltaic (PV) installation where PV panels supply electricity to feed a  
96 pumping device in a pressurised irrigation network. In this approach, the irrigation

97 network runs as a standalone SPWS and we intend to quantify the effect of selected  
98 schedule in a rigid rotation predetermined scheduled PIN (Replöge and Kruse, 2007).

99         The irrigation network comprises a set of devices (hydrants, units and subunits)  
100 constrained to have a fixed time of operation. Two more requirements apply. The first is  
101 fixed operation time (water delivered to crops in every consumption node must be  
102 similar) but we can divide this period into intervals. The second is that the power  
103 consumption of the set of devices can depend on the connected appliances (nodes with  
104 high elevation far from the pumping system need more energy supplied by the pumping  
105 system).

106         For a set of devices with given shared consumption profiles, we parametrised the  
107 problem with the working times (hydrants opened supplying water to crops) and the  
108 number of intervals in which the day is divided. We have presented a restricted version  
109 of this problem (Pardo et al., 2018) applied to water pumping systems. We determine the  
110 optimal job scheduling to decrease the number of required solar panels for the power  
111 supply installation by the number of simultaneous devices ( $Q$ ) and the number of time-  
112 slices ( $N$ ). In (Pardo et al., 2018), we solve the problem by exploring the space of allowed  
113 combinations, composed by  $Q \cdot 2^N$  configurations. But this method is not useful for big  
114 cases because of its dependence on the parameters  $Q$  and  $N$ . We develop a genetic  
115 algorithm to find the schedule minimise differences between the aim and the real injected  
116 flows (Pardo et al., 2020b). This earlier approach shows that it is impossible to calculate  
117 the  $Q \cdot 2^N$  combinations. The present paper exploits a property of the feasible solutions  
118 that reduce the subset of the configuration space that is admissible as a solution. The fast-

119 numerical algorithm developed in this investigation guarantees the solution with a  
120 computational complexity of  $O(Q)$ .

121 The rest of the paper is organised as follows. Section 2.1 exposes the problem and  
122 section 2.2 presents the basis of solar irradiance and clear sky models. Section 2.3 shows  
123 the objective function to minimise and 2.4 presents the formulation of the problem;  
124 Section 3 shows the case study where to solve the problem and Section 4 presents the  
125 solution for the optimal size design problem. and the discussion about the research.  
126 Finally, Section 5 highlights the conclusion.

127

## 128 2. MATERIALS AND METHODS

129

### 130 2.1. Problem exposition

131

132 Let us consider the problem of designing a PV plant for supply  $d$  devices in several  
133 states (connected or powered off would be the simplest case), numbered from zero  
134 (powered off) to  $S$ :

$$135 \quad s \in \{0,1,\dots,S\} \quad (1)$$

136 Let call each one of the allowed combinations a system configuration, represented  
137 by the  $d$ -dimensional vector  $\zeta$ :

$$138 \quad \zeta = \{(s_1, \dots, s_d) \mid s_i \in \{0,1,\dots,S\}\} \subset N^d. \quad (2)$$

139 Where  $S^d$  combinations may appear. Each with a power consumption function given by  
140  $c(\zeta)$ .

141 Let us consider a work schedule for the devices of a determined system  
142 corresponding to a day. With different needs on different days, we would realise the

143 overall process for each day. If we divide the day into  $N$  slices,  $\{t_1, t_2, \dots, t_N\}$ , with size  
 144  $\Delta=24/N$ , there will be  $S^{d \cdot N}$  possible daily combinations for the system. We show a working  
 145 plan example in Table 1.

		WORKING PLAN FOR 6 DEVICES																		
		8:00	8:30	9:00	9:30	10:00	10:30	11:00	11:30	12:00	12:30	13:00	13:30	14:00	14:30	15:00	15:30	16:00	16:30	17:00
DEVICE 1		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
DEVICE 2		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
DEVICE 3		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
DEVICE 4		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
DEVICE 5		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
DEVICE 6		■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
Power demand		$P_1$	$P_1$	$P_{12}$	$P_{12}$	$P_{125}$	$P_{1235}$	$P_{2345}$	$P_{2345}$	$P_{345}$	$P_{345}$	$P_{345}$	$P_{345}$	$P_4$	$P_4$	$P_{24}$	$P_{234}$	$P_{236}$	$P_{36}$	$P_{36}$

Connected device     
  Disconnected device

Table 1: Working plan for 6 devices with slices of 30 minutes.

146 A working plan is then, a set of states associated with the group of slices,  
 147  $\Pi = \{\zeta(t_k)\}_{k=1}^N$ . Given a working plan, it is possible to calculate the total power  
 148 consumption and time of operation:

149 
$$P = \sum_k c(\zeta(t_k)). \tag{3}$$

150 
$$T = \Delta \cdot \sum_{i,k} Ind(\xi^i(t_k)), \tag{4}$$

151 where  $Ind(x)$  is the indicator function defined as:

152 
$$Ind(x) = \begin{cases} 1 & x \neq 0 \\ 0 & x = 0 \end{cases}. \tag{5}$$

153 Also, representing by  $E(t_k)$  the maximum power for generation by one PV panel  
 154 at a time  $t_k$ , the number of panels needed at this time is:

155 
$$n(t_k) = \frac{c(\zeta(t_k))}{E(t_k)}. \tag{6}$$

156 And the number of panels needed by a working plan is:

157 
$$\eta = \max_{k \in [1, N]} (n(t_k)). \tag{7}$$

158 A power-time representation of a working plan is a plot of the function  $c(\zeta(t))$  as

159 Figure 1:

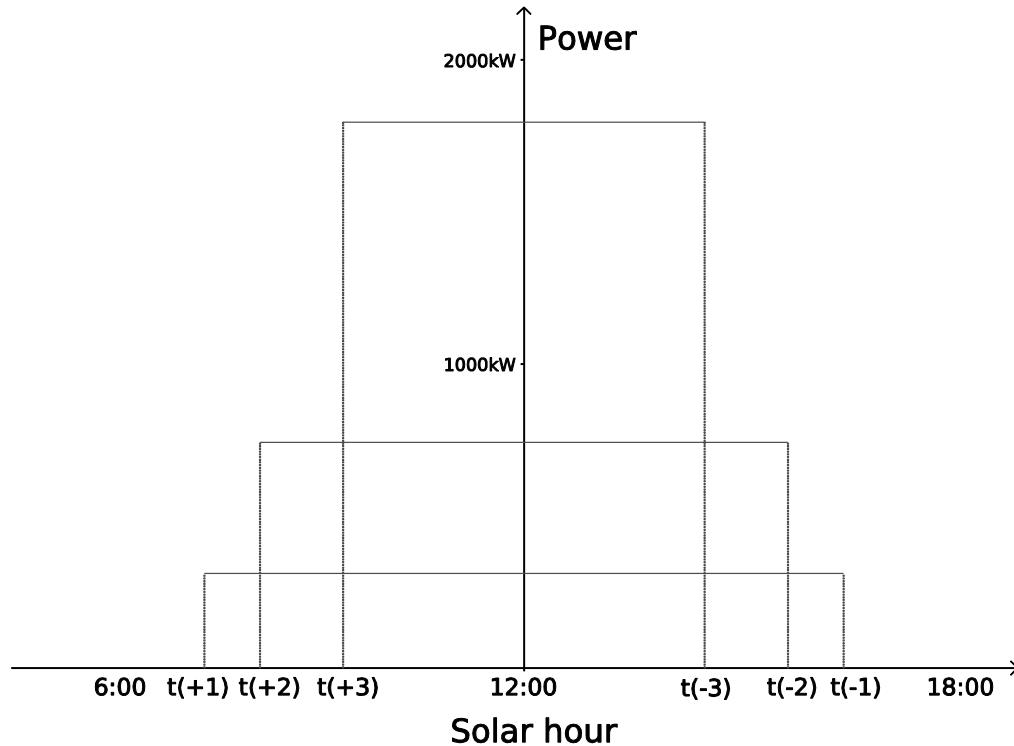


Figure 1. Power-time representation of a working plan

160 **2.2. Model of maximum hourly solar irradiance**

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162 **2.2.1. Basis of hourly solar irradiance**

163

164 Let us introduce briefly some concepts and terminology that will be needed in the  
 165 rest of the paper (for a more in-depth discussion see (Badescu, 2014; Cao and Lin, 2008;  
 166 Page, 2012; Wald, 2018)).

167 The irradiance  $E$  is the power received per area (Watt per square meter). To  
 168 calculate solar irradiance at ground level the first step is to consider its value on a normal



169 horizontal surface at the top of the atmosphere ( $E_{0N}$ ), and the value corresponding to a  
 170 horizontal surface situated in the same point  $E_0$ , given by:

$$171 \quad E_0 = E_{0N} \cdot \cos(\theta_s), \quad (8)$$

172 where  $\theta_s$  is the solar zenithal angle, related to the latitude of the point ( $\Phi$ ) and the  
 173 declination of the sun ( $\delta$ ) through the equation:

$$174 \quad \cos(\theta_s) = \sin(\Phi) \cdot \sin(\delta) + \cos(\Phi) \cdot \cos(\delta) \cdot \cos(\omega), \quad (9)$$

175 where  $\omega$  is the hour angle measured from the local meridian to the position of the sun at  
 176 each instant. Introducing the reduced time coordinate  $t$ , with  $t=0$  at solar noon, the hour  
 177 angle can be written as:

$$178 \quad \omega = \frac{\pi}{12} \cdot t. \quad (10)$$

179 Let consider a panel facing south with an inclination given by the angle  $\beta$  (tilt)  
 180 concerning the horizontal. The angle of incidence of the solar rays can be obtained as a  
 181 modification of the expression (9):

$$182 \quad \cos(\hat{r}) = \sin(\Phi - \beta) \cdot \sin(\delta) + \cos(\Phi - \beta) \cdot \cos(\delta) \cdot \cos(\omega). \quad (11)$$

183 From equation (11) the energy of the incident direction will have an expression:

$$184 \quad E_{lin}(t) = A + B \cdot \cos(\omega(t)). \quad (12)$$

185 The atmosphere affects the values calculated before because of light scattering  
 186 causes the dispersion of light around the incident direction (depending on the  
 187 wavelength). Then, the effective energy received at ground depends on the amount of  
 188 atmosphere that must be crossed.

189 Also, the radiation that is reflected on the ground can impact in the solar panel  
 190 contributing to the generated power. At ground level, the total irradiance is the sum of

191 radiation that comes directly from the direction of the sun (direct or beam radiation),  
192 radiation originated by scattering processes in the atmosphere (diffuse radiation) and the  
193 radiation coming from the ground (albedo), a fraction of the total radiation that falls upon  
194 it (reflected radiation).

195 But even if we perform the calculations using all the precision, some factors scape  
196 from the equations and must be determined in situ (for instance, influence from the local  
197 topography in the values of the albedo). Moreover, the randomness of nature appears in  
198 the form of clouds or other meteorological elements that break the validity of the  
199 forecast.

200 We calculate here the least number of PV panels required to feed the energy  
201 demand using a curve of hourly power generation representative of the set of panels used  
202 in the installation calculated with the Duffie and Beckman equations (Duffie and  
203 Beckman, 2013).

#### 204 **2.2.2. Clear sky models**

205

206 Equation (12) only considers the effect of direct radiation, but as we introduced  
207 previously, several other contributions determine the total power. The light beam is  
208 dispersed following a three-dimensional angular distribution around the direction given  
209 by the direct beam, and part of the energy arrives at the panel in angles different from  
210 that given by (12). Inclusion and calculation of diffuse irradiance is not a simple problem  
211 (see (Behar et al., 2015) for a comparison of the models).

212 The problem of determining the maximum irradiance available at any determined  
213 instant is studied using a class of models called clear sky models. Clear sky models

214 complement equation (12) considering the contribution of incident beams with direction  
215 different from the direct one.

216         There exist a great variety of clear sky models, as can be seen in [59, 60]. Each  
217 model considers different dispersion terms, but a common characteristic of these terms  
218 is that they depend on the amount of atmosphere traversed by the light. A measure of  
219 this distance is the parameter called air mass, which depends on the zenithal (or  
220 equivalently the solar elevation) angle, related through equation (9) with the hour angle.  
221 For elevation angles greater than 20-30 degrees, the value of air mass can be considered  
222 constant. It is for smaller angles that the different models predict different values.  
223 However, the final behaviour is similar in the most of models, that is a reduction  
224 concerning considering only the direct beam given by (12) of irradiance at solar noon and  
225 an increment of the generated power for earliest and latest moments of the day. As a  
226 result, the power generated by the panel changes respect to the direct component as it  
227 is shown in Figure 2.



237 where  $(n_1, \dots, n_N)$  are the number of panels used in each interval and  $\eta$  is  
 238 optimum.

239 Then, the optimal condition over is given by the null partial derivatives:

$$240 \quad \partial_i f = 2 \cdot \sum_{k=1}^N (\eta - n_k) \cdot \delta_{ki} = 0, \quad (15)$$

241 for every index  $i$ , so the solution is:

$$242 \quad n_1 = n_2 = \dots = \eta. \quad (16)$$

243 That is, the optimum is got when the number of required panels is the same at  
 244 every time, so the problem is to schedule a balanced load on the system, modifying the  
 245 working plan to obtain a constant ratio between the load and the available power at each  
 246 instant under the time operation constraint  $T$ .

247

#### 248 **2.4. Optimization Algorithm**

249

250 At any time, the number of active devices varies from 0 to  $Q$  (if there is no  
 251 constraint over the simultaneity in device working, this number would be the number of  
 252 devices  $d$ ). Depending on the vector  $\zeta$  the consumption can vary, so let introduce

$$253 \quad \pi_{(q)} = \min c(\zeta), \text{ where } \sum_i \text{Ind}(\xi^i(t)) = q. \quad (17)$$

254 Additionally, the problem is characterized by two different regimes of power  
 255 demand. At the first and last hours, the demand determines the need for power, but in  
 256 the central hours, the power generated exceeds the system needs. So, the optimization  
 257 problem is determining the connection  $t_{(+q)}$  and disconnection time  $t_{(-q)}$  with  $q$  from 1  
 258 to the maximum allowed number of simultaneous devices  $Q$  (Figure 3).

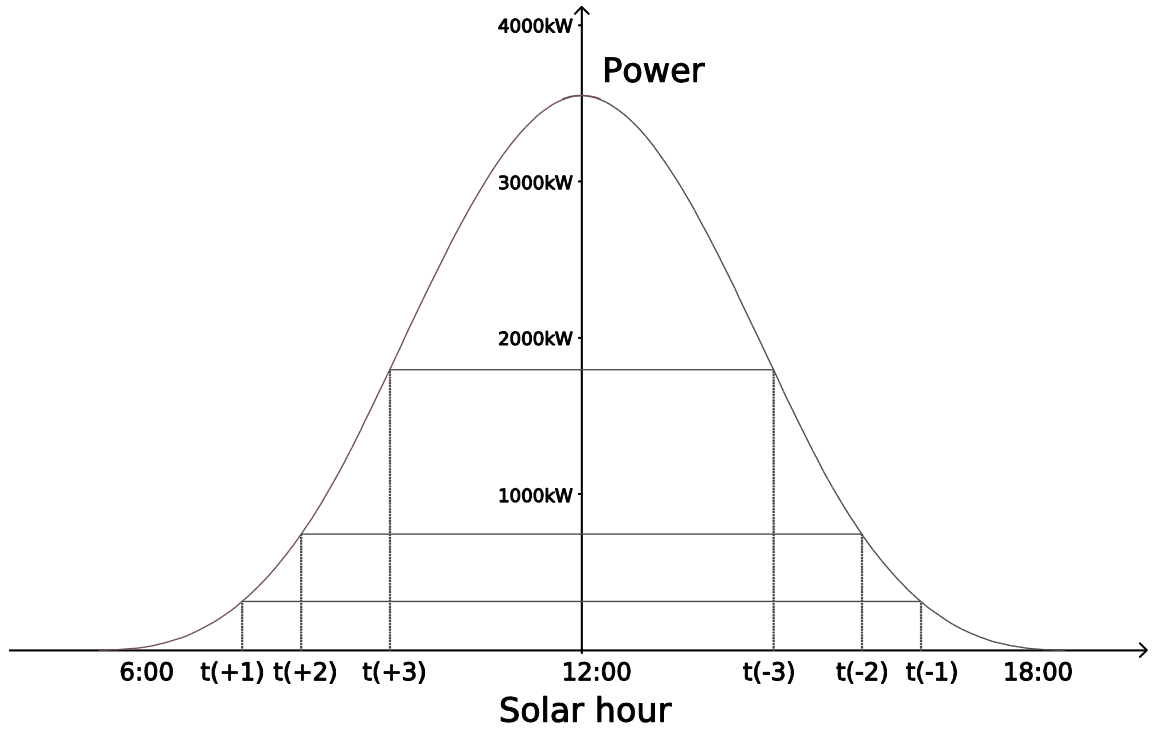


Figure 3. Example of power demand for  $Q=3$  and curve of maximum power supply.

259 From equations (6) and (16):

$$260 \quad \frac{\pi_{(1)}}{E(t_{(\pm 1)})} = \frac{\pi_{(2)}}{E(t_{(\pm 2)})} = \dots = \frac{\pi_{(Q)}}{E(t_{(\pm Q)})} = \eta. \quad (18)$$

261 This implies that:

$$262 \quad t_{(\pm q)} = E^{-1} \left( \frac{\pi_{(q)}}{\pi_{(Q)}} \cdot E(t_{(\pm Q)}) \right). \quad (19)$$

263 As the total time of working for each device is fixed, any modification of its  
 264 connection time is got diminishing the number of connected devices from  $q$  to  $q-1$  at this  
 265 moment. Considering the symmetry of the power curve,  $t_{(+q)} = -t_{(-q)}$ , and expression  
 266 (4) takes the form:

$$267 \quad T = \sum_q (t_{(-q)} - t_{(+q)}) = 2 \cdot \sum_q t_{(-q)}. \quad (20)$$

268 Now, the problem is equivalent to obtaining the solution of the following  
 269 equation, valid from noon to sunset.

$$270 \quad E^{-1}\left(\frac{\pi(1)}{\pi(Q)} \cdot E(t_{(-Q)})\right) + \dots + E^{-1}\left(\frac{\pi(Q-1)}{\pi(Q)} \cdot E(t_{(-Q)})\right) + t_{(-Q)} = \frac{T}{2} \quad (21)$$

271 For the clear sky model derived from equation (13), the solution can be obtained  
 272 through an analytic expression given by:

$$273 \quad \arccos\left(\frac{\frac{\pi(1)}{\pi(Q)} \cdot E(t_{(-Q)}) - B}{A}\right) + \dots + \arccos\left(\frac{\frac{\pi(Q-1)}{\pi(Q)} \cdot E(t_{(-Q)}) - B}{A}\right) + t_{(-Q)} = \frac{T}{2}. \quad (22)$$

274 We calculate the parameters A and B from equation (13).

275 In the general case, the equation (21) can be solved using by example a  
 276 Newton-Raphson approach. This solution can be applied to solve the problem of  
 277 scheduling the pump connection time presented in [26] in a numerical, faster and exact  
 278 form.

279

## 280 **2.5. Pseudocode**

281 The algorithm to calculate the optimum size for the PV installation is based on the  
 282 developments and equations presented before. In a short exposition:

283 Equation (16) states for the main result of the algorithm. The relation between the  
 284 available and required power must be constant in all the critical times, corresponding  
 285 with the connection and disconnection moments as Figure 3 illustrates.

286 Equation (4) represents the constraint corresponding with the total working time  
 287 of the installation.

288 Equation (6) introduces the expression for calculating the number of required  
 289 panels at any time. Using it in equation (16) derives in equations (18) and (19).

290 Finally, equation (21) is obtained from the equations (4) and (19) and represents  
 291 the expression that resumes the optimization problem as a non-linear equation that can  
 292 be solved for different irradiance models, as it is done, by example, in equation (22).

293 The pseudocode corresponding to the algorithm is the following:

---

**Algorithm 1** Optimize installation scheduling and size

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**Input:**

$Q$ : Number of maximum simultaneous devices,  
 $\pi_q(i)$ : Power consumption of each allowed configuration  $i$  of  $q$  devices,  
 $T$ : Total work time of the installation

**Output:**

$t_{(-1)}, \dots, t_{(-Q)}$ : Disconnection times,  
 $N_{opt}$ : Number of pannels

**procedure** OPTIMIZEINSTALLATION

**for**  $q$  from 1 to  $Q$  **do**

$\pi_{(q)} \leftarrow \min(\pi_q(i))$  ▷ Minimum power of  $q$  devices

  /\*Solve nonlinear equation\*/

$t_{(-Q)} \leftarrow \text{Root}(\sum_{k=1}^{Q-1} E^{-1}(\pi_{(k)}/\pi_{(Q)}) * E(x) + x - T/2 = 0)$

$N_{opt} \leftarrow E(t_{(-Q)})/\pi_{(Q)}$  ▷ Calculate  $N_{opt}$

**for**  $i$  from 1 to  $Q - 1$  **do**

$t_{(-i)} \leftarrow E^{-1}(\pi_i/N_{opt})$  ▷ Calculate disconnection time  $t_i$

**return**  $t_{(-1)}, \dots, t_{(-Q)}, N_{opt}$

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Figure 4. Flowchart Optimization process.

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296 **3. CASE STUDY**

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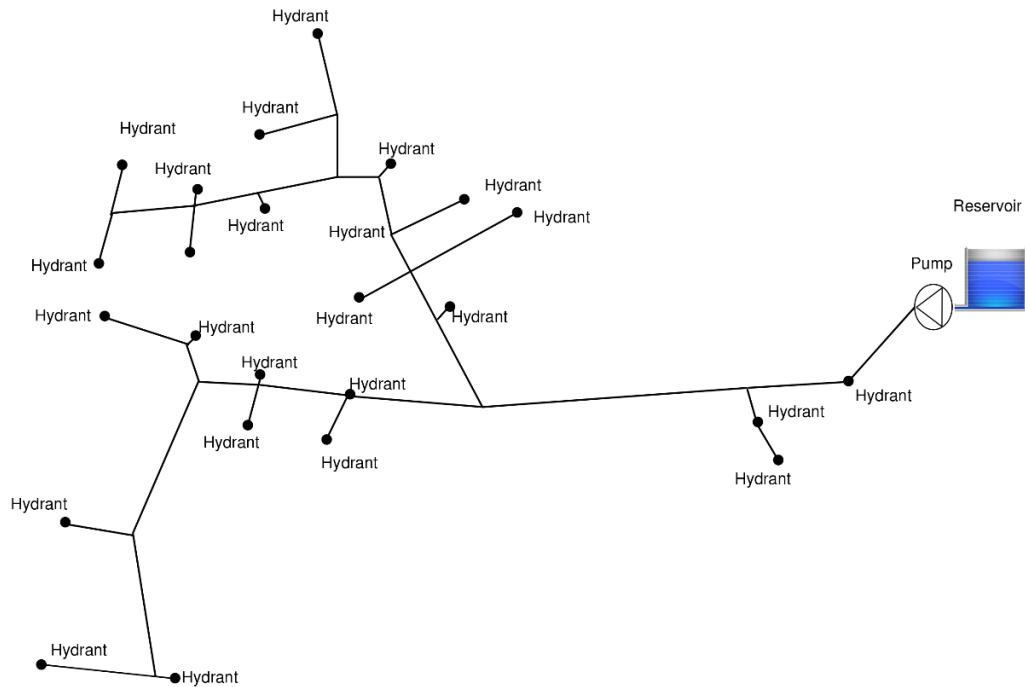
298 Figure 5 shows the Albamix network layout, located in Comunidad Valenciana.

299 This PIN comprises 132 nodes and 4.05 km of PVC pipes. It supplies water to 167.7 ha to

300 irrigate orchards containing different varieties of citrus fruit. The minimum service



301 pressure required is  $\left(\frac{P}{\gamma}\right)_{threshold} = 25$  meters of the water column. We grouped the  
 302 irrigation networks into five segments. The water demands for each segment were  
 303 around 80-85 L/s. The irrigation management system is a central system scheduled  
 304 delivery and the total irrigation time presents monthly variation. We recover  
 305 meteorological information to calculate water demands. We compute the reference crop  
 306 evapotranspiration using the Penman-Monteith method. The irrigation time values are  
 307 shown in Table 2.



308

Figure 5. General Layout of the PIN.

Month	January	February	March	April	May	June
Irr. time (h)	0.52	0.69	1.12	1.29	1.93	2.92
Month	July	August	September	October	November	December
Irr. time (h)	3.25	2.70	1.62	0.75	0.43	0.30

Table2. Monthly irrigation time in Albamix network.

309 In July (the month with the highest water requirements), every segment should  
310 be irrigated for 3.25 h/day (Table 2). The network features are incorporated into a  
311 hydraulic simulation software such as EPANet (Rossman, 2000) which represents reality.  
312 The energy consumption in pumps and nodes (Pardo et al., 2013) are calculated using  
313 UAEnergy (M.A. Pardo et al., 2019).

314

### 315 **3.1. Practical restrictions**

316 The utility manager operates a PIN dimensioned for delivering water during nights  
317 to exploit low electricity tariffs. Solar irradiance produces energy, but the irrigation time  
318 decreases. In local conditions, the number of hours in which it produces photovoltaic  
319 energy can be 9h and according to the values showed in Table 2 and the number of  
320 segments, the irrigation time is  $3.25 * 5 = 16.25h$ .

321 .Given that irrigation time is lower, higher flow rates and higher headlosses owing  
322 to friction in the pipes are likely. We defined the upper and lower network flowrate  
323 threshold ( $Q_{up,th}=194.9$  l/s,  $Q_{low,th}=152.5$  l/s) as the highest flowrate injected  
324 maintaining the pressure above the standards and the least injected flow not meeting  
325 pressure requirements (M A Pardo et al., 2019). The flowrate availability is another  
326 constraint. It is the maximum flow rate that can be delivered. If the injected flow is lower  
327 than the flowrate availability, no limitation arises in our optimisation problem.

328 The working time is 9 hours in the month with higher irradiance (July) in these  
329 latitudes as a consequence of the technical characteristics of the inverter.

330

331 **3.2. Seasonal variation**

332

333 Many considerations regulate power management in PV modules (tilt angle,  
334 latitude, azimuth, temperature, etc.). Seasonal variation is a well-known fact, the smallest  
335 solar irradiation level occurs in winter while the highest in summer (Bou-Rabee and  
336 Sulaiman, 2015; Osinowo et al., 2015; Pardo et al., 2020a) in these latitudes.

337 On the other hand, water and energy demands are higher during summer and this  
338 period coincides with greater energy production. The utility manager must conduct a  
339 hydraulic test (considering the network details) to choose the most adverse month. The  
340 PV modules must be sized for the worst month (the month selected to size the system).  
341 In this month, the pump can operate enough hours to guarantee crops irrigation.

342

343 **4. RESULTS AND DISCUSSION**

344

345 **4.1. Energy production variation**

346

347 We figured out the monthly energy production and consumption to establish July  
348 as the most unfavourable month (M A Pardo et al., 2019). July has the smallest rate  
349 between solar radiation and water needs, which does not coincide with works in urban  
350 WPN (Pardo et al., 2020a) in which low values of irradiance in December are more  
351 important than water consumption.

352

353 **4.2. Energy consumption by segment**

354

355 As an example of the application of equations (21) and (22), let us consider the  
 356 data corresponding to the problem presented in (Pardo et al., 2018), where the working  
 357 power for each combination of devices is represented in Table 3:

358

<b>Combination</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>1+2</b>	<b>1+3</b>	<b>1+4</b>
<b>Power</b>	5.24	5.20	5.26	5.25	5.18	8.41	8.42	8.42
<b>Combination</b>	<b>1+5</b>	<b>2+3</b>	<b>2+4</b>	<b>2+5</b>	<b>3+4</b>	<b>3+5</b>	<b>4+5</b>	
<b>Power</b>	8.40	8.41	8.41	8.39	8.43	8.41	8.40	

359 Table 3: Required power for the combination of each device.

360 As every segment injects approximately around 80-85 L/s, and the higher and the  
 361 upper and lower flow rates are ( $Q_{up,th}=194.9$  l/s,  $Q_{low,th}=152.5$  l/s) (as stated above), we  
 362 know that as the number of maximum simultaneous devices  $Q = 2$  (Figure 5). In short,  
 363  $Q_{low,th} = 152.5 < 2 * 80 < 194.9 = Q_{up,th}$ , and it is not possible to supply three  
 364 segments as we do not meet the pressure standards.

365 Dealing with the figures shown in Table 3, segment 5 and the combination 2-5 are  
 366 the least energy requirements when we feed one or two groups of segments. In short,  
 367  $\pi_{(1)} = 5.18$  and  $\pi_{(Q)} = 8.39$ .

368

### 369 4.3. Scheduling and number of PV modules calculation

370

371 To calculate the scheduling that minimises the number of photovoltaic panels, we  
 372 used two irradiance models. The first is a Kasten–Czeplak model where we get A and B

373 from the irradiance data presented in (Pardo et al., 2018). Under the conditions of equal  
 374 sunset hour angle ( $\omega_s$ ) and total day-generated energy, the irradiance equation takes the  
 375 form:

$$376 \quad E(t) = 0.0158997233 \cdot \cos(\pi \cdot (t/12 - 1)) + 0.0051107985. \quad (23)$$

377 This is a direct-beam method and will serve as a first approach to the result. Then,  
 378 the scheduling will start and finish with the device 5 and then pass to the combination 2-  
 379 5. We can fulfil other slides with any of the possibilities offered that the required power  
 380 will be much fewer than the power provided by the PV panels.

381 The question now is to establish the instant of connection and disconnection of  
 382 these two 'extreme solutions. We solve equation (22) to calculate variable  $t_2$ :

$$383 \quad \frac{12}{\pi} \cdot \arccos\left(\frac{\frac{5.18}{8.39} \cdot (0.01590 \cdot \cos(\pi \cdot (\frac{t_2}{12} - 1)) + 0.00511) - 0.00511}{0.01590}\right) + t_2 - 12 = \frac{16.25}{2} \quad (24)$$

384 The solution is  $t_2 = 15.23 \text{ h}$  (15:14), when the power generated by the PV  
 385 panels is 0.01565 W. We calculate the disconnection time for the single device  
 386 combination using equation (6) and (16):

$$387 \quad t_1 = 12 + \frac{12}{\pi} \cdot \arccos\left(\frac{\frac{5.18}{8.39} \cdot (0.01590 \cdot \cos(\pi \cdot (\frac{15.23}{12} - 1)) + 0.00511) - 0.00511}{0.01590}\right) = 16.89 \quad (25)$$

388 Using equation (6) and the condition about the number of panels in each period,  
 389 the number of panels are:

$$390 \quad \eta = n(t_2) = \frac{8.39}{0.01565} \approx 537 \quad (26)$$

391 We can consider this result as a first approach because of the simplicity and  
 392 inaccuracy of the selected irradiance model. We will calculate the second result using an  
 393 irradiance function obtained through a polynomial regression over the numeric data for

394 the panel generated power. We got these numbers with Duffie and Beckman equations  
 395 (Duffie and Beckman, 2013; Pardo et al., 2018).

$$396 \quad E(t) = -1.7 \cdot 10^{-19} \cdot t^5 + 7.52 \cdot 10^{-6} \cdot t^4 - 3.6 \cdot 10^{-4} \cdot t^3 + 5.64 \cdot 10^{-3} \cdot t^2 -$$

$$397 \quad -0.32 \cdot t + 0.058 \quad (27)$$

398 Now, the equation to solve is:

$$399 \quad E^{-1} \left( \frac{5.18}{8.39} \cdot E(t_2) \right) + t_2 = \frac{16.25}{2} \quad (28)$$

400 And the solution is also  $t_2 = 15:29$  (15.48h) and the available power is now 0.0146.

401 Now, we compute the disconnection time for segment 5.

$$402 \quad E^{-1} \left( \frac{5.18}{8.39} \cdot E(15.23) \right) = 16:38 \text{ (16.63h)} \quad (29)$$

403 And the optimum number of panels is:

$$404 \quad \eta = n(t_2) = \frac{8.39}{0.0146} \approx 577 \quad (30)$$

405 Optimum scheduling for the polynomial irradiance is displayed in Table 4. We may  
 406 find that we deliver the same water to plots for every segment (irrigation time for every  
 407 segment is 3:15 (3.25h)).

Devices	Connection	Disconnection	Devices	Connection	Disconnection
5	7:21	7:57	1+4	12:00	12:50
2	7:57	8:31	3+4	12:50	13:39
2+5	8:31	9:34	1+3	13:39	14:28
1+4	9:34	10:23	2+5	14:28	15:31
3+4	10:23	11:12	2	15:31	16:05
1+3	11:12	12:00	5	16:05	16:39

408 Table 4: Required power for the combination of each device.

409

#### 410 4.4. Discussion

411 Converting direct-drive pumping systems (on-grid) into standalone direct  
 412 pumping photovoltaic system without dealing with the seasonal fluctuation of energy  
 413 production may contribute to inaccuracies and operation issues. In this analytical  
 414 procedure, the large-water use in summer is more energy-hungry than the increase in  
 415 energy production. In other words, this PIN is more sensitive to water consumption than  
 416 by energy production. We must calculate this for every future case.

417 The scheduling problem produces better results than those obtained before as we  
 418 cut down the amount of PV modules (537 and 577) for two different irradiation models.  
 419 But the key advantage here is not that one, but the quick response of the tool developed  
 420 to solve this new non-linear equation problem. Authors are aware of the practical  
 421 restraints indicated here (hydraulics and resource availability) as well as the future  
 422 functional conditions that may come out at the installation stage. Authors would like to  
 423 use this tool to consider segments change (in number and water consumption) for some  
 424 other rigid rotation predetermined scheduled.

425 To sum up, Table 5 shows the results got solving the scheduling problem.

<b>Irradiance</b>	<b>Trigonometric</b>	<b>Polynomic</b>
$t_{(-1)}$	16:53	16:38
$t_{(-2)}$	15:14	15:29
$\eta$	537	577

426 Table 5: Required power for the combination of each device.

427

428 **5. CONCLUSION**

429 The sizing of photovoltaic supplier systems is a problem of paramount importance,  
430 and if PV is the primary energy source, this point becomes crucial. When the devices  
431 (hydrants or units) allow flexibility for connection and disconnection time but maintaining  
432 the total consumed energy, we can plan the problem as a scheduling problem. We reduce  
433 the problem to a frontier problem where the solution corresponds to the lower power  
434 supply curve that includes all the consumption vertexes on the energy-time  
435 representation of the best working plan.

436 The presented method proposes a numeric algorithm solving of a non-linear  
437 equation. It is an advance as the genetic algorithm does not guarantee to have the real  
438 smallest size for the system.

439 This issue bears a direct use in pressurised irrigation networks as utility managers  
440 can adjust water and energy needs. They can deliver water to crops at every moment of  
441 the day. We develop a tool to solve this scheduling problem (Pardo et al., 2020b) using a  
442 genetic algorithm, and the result was very time-consuming. Here, we converted this  
443 problem into a non-linear equation problem.

444 But we must recognize several aspects to apply the method to an actual case. First,  
445 the accuracy of the results at earlier and later hours depends on the adopted clear sky  
446 models for low solar elevation angles. The effect of clouds on the effective irradiance is a  
447 complex question because of its stochastic nature. A question opens at this point:  
448 Including confidence intervals on irradiance curves, and then in the design of PV



449 installation. The results correspond to the most optimistic configuration to the power  
450 supply, an object that can be unreal, a probabilistic analysis to examine the confidence  
451 interval of the result might be regarded.

452

#### 453 **AUTHOR'S CONTRIBUTIONS**

454

455 All authors contributed equally to this work.

456

#### 457 **ACKNOWLEDGMENTS**

458

459 The authors have no conflicts of interest to declare. This research did not receive any  
460 specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

#### 461 **DATA AVAILABILITY STATEMENT**

462

463 AIP Publishing believes that all datasets underlying the conclusions of the paper should  
464 be available to readers. We encourage authors to deposit their datasets in publicly  
465 available repositories (where available and appropriate) or present them in the main  
466 manuscript. All research articles must include a data availability statement informing  
467 where the data can be found. By data, we mean the minimal dataset that would be  
468 necessary to interpret, replicate and build upon the findings reported in the article. The  
469 data that support the findings of this study are available from the corresponding author  
470 upon reasonable request.

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