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Mechanical Models in Structural Engineering

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Laboratory dynamic structural testing. Methods and applications

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ABSTRACT

Regardless of the advances in simulation power of computers and material constitutive models, there is an agreement with the need of structural experimental testing. In particular, great effort has been made to understand the performance of structures under exceptional loads such as earthquakes or impacts.

Several methods have been used in structural testing: quasi-static testing, shake tables and hybrid simulation. In the latter, only some parts of the structure are experimentally tested, whereas the others are numerically simulated. Test progresses as the results of the physical test are fed into the model and its outputs are imposed on the physical substructure.

In this work, a review of these methods is presented comparing their strengths, weaknesses and areas of application. Emphasis is put on hybrid simulation and application possibilities to areas different than Seismic Engineering are suggested; for instance: testing of vibration damping devices under crowd induced forces or problems such as fluid or vehicle-structure interaction.

Keywords: Dynamic testing, Hybrid simulation, Substructuring, Pseudo-dynamic testing

1. INTRODUCTION

Thanks to the advances in simulation power of computers and in material constitutive models, there has been an undoubted improvement in the quality of numerical dynamic analysis of structures; however, there is still a strong need for experimental testing, particularly, when studying severe loading cases in which structures are likely to undergo non-linear deformations. A clear example of these scenarios are earthquakes, being Seismic Engineering the root discipline around which the testing methods discussed in this work originated.

Three techniques have been widely used to perform seismic testing of structures [1,2]; i.e.: quasi-static testing, shake table testing and hybrid simulation, with its several variants such as: conventional pseudo-dynamic testing [3], pseudo-dynamic testing with substructuring [4], real-time hybrid simulation [5] and the effective force method [6]. The first two are quite well stablished and mature

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approaches, whereas in hybrid simulation, despite the fact that it has been used for more than four decades now, there are continuous advances and developments; not only in Seismic Engineering, but also in other areas such as Mechanical, Aeronautical or Aerospace Engineering. Other structural testing techniques such as operational modal analysis (OMA), experimental modal analysis (EMA) or centrifuge machine tests will not be dealt with in this work.

The remainder of this paper is organized as follows. In section 2 a brief description of quasi-static method is given, section 3 deals with shake table testing, paying special attention to its critical subsystem; i.e.: the control system. Section 4 describes hybrid simulation in more detail, with its previously mentioned modalities, giving considerations on test set-ups, numerical time stepping algorithms, substructuring approaches and areas of application. In section 5, some classical applications of hybrid simulation are briefly reviewed, together with an outline of other potential ones. Finally, some conclusions are provided in section 5.

2. QUASI-STATIC TESTING

In quasi-static tests, the structure under test (SUT) is subjected to a predefined time history of displacements or forces (cyclic or monotonic) at given locations by means of hydraulic actuators commanded by servovalves. Displacements or loads on the SUT are imposed at low speed in comparison to those the specimen would experience in the real event [1,2].

Fig. 1 shows a schematic arrangement for quasi-static testing in a three-storey building. Typically, these set-ups consist on a strong floor where SUT is installed and a wall to react actuators forces. In order to minimize reaction wall deformation effects, displacement transducers are commonly installed in auxiliary support structures. Very often, actuators count with rod and body swivels to relieve them from side-loads and optimize load application onto the SUT.

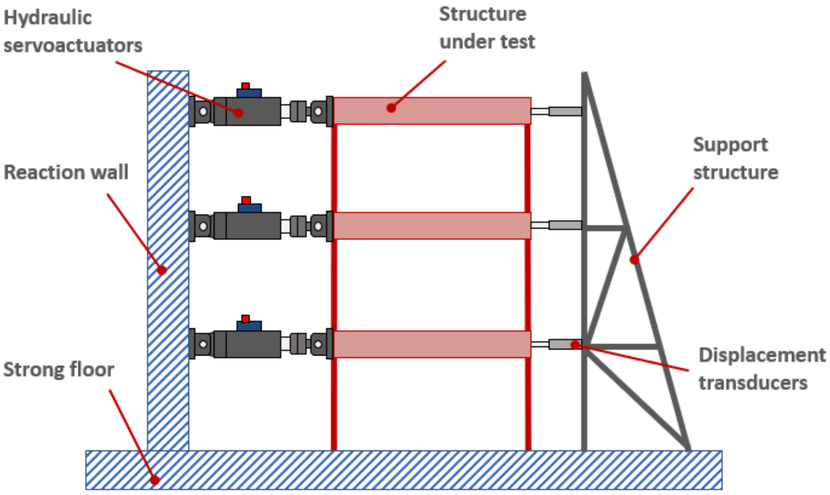


Figure 1. *Quasi-static test set-up.*

The fact that loads are slowly imposed on the SUT has several implications. Firstly, velocity and frequency range of operation demands for servoactuators are reduced; therefore, standard cylinders, with polymeric bearings are commonly employed. The reduced load application speed implies a reduction in flow rates demanded by the actuators; consequently, servovalves needed are simple with modest frequential performance requirements and hydraulic power unit (HPU) needed to provide the demanded flow rate can be of reduced size.

Control system required to perform this type of tests is also quite simple due to the low operation velocities and frequencies. Normally, standard proportional-integral-derivative (PID) controllers, using displacement or force transducers as feedback signals, are used with proportional and integral actions enabled. Since these tests are carried out at reduced speeds, the quality of time history tracking and recorded measurements is usually very good.

Quasi-static tests can be performed with full-scale specimens or single elements imposing unidirectional or multidirectional loads. Their aims are to characterize structural properties or performances of the SUT, for instance, under load reversals such as the ones taking place in earthquakes or low cycle fatigue as well as to obtain data that later can be used in numerical models. These tests do not capture dynamic behavior of the specimen and therefore cannot be used with SUTs which exhibit rate-dependent effects; nevertheless, they are widely used in many engineering fields because of the valuable information they provide.

3. SHAKE TABLE TESTING

In shake table testing, full-scale or scaled SUTs are installed on a rigid platform (shake table) which is able to move in one or several DoF (Degrees of Freedom) according to prescribed acceleration profiles such as accelerograms. These platforms are usually powered by hydraulic servoactuators set up in a configuration capable of reproducing the desired motion DoFs [7].

There are multiple morphologies of shake tables depending on their specific purposes. The simplest one corresponds to uniaxial motion and, excluding special applications such as multiple shake tables or table-on-table configurations [8], the most complex features six DoF (three translations and three rotations) usually employing more than six actuators; therefore leading to, difficult to control, over-constrained Multiple Input-Multiple Output (MIMO) set-ups [9]. Fig. 2 illustrates examples of one, two, three and six DoF shake table concepts.

Figure 2. *(a) one DoF, (b) two DoF, (c) three DoF and (d) four DoF shake table concepts. Courtesy of Vzero Engineering Solutions, S.L.*

Sizes of shake tables range, in their standard configuration, from one to five meters and payloads may be as high as 50 t. Due to the high forces developed by the servoactuators and their frequency content, an independent foundation with some kind of vibration isolation system (airmounts, metallic springs or polymeric layers), is usually required to isolate the source of unwanted vibration constituted by the testing system from the rest of the building [10].

A key factor in the success of this kind of testing facilities is the correct design of the shake table. On the one hand, the platform must be as rigid as possible, so that its dynamics do not interact with those of the specimen; that is: its natural modes do not fall within the operational frequency range of the system and the boundary conditions to which the specimen is intended to be subjected, are correctly reproduced. On the other, the table must be as light as possible to optimize servoactuators size while being able to withstand operational loads observing infinite-life criteria. Both circumstances lead to carry out exhaustive Finite Element Method (FEM) studies during the design process, in which representative SUTs must be incorporated. Fig. 3 illustrates typical outputs of these studies, showing first specimen mode shift.

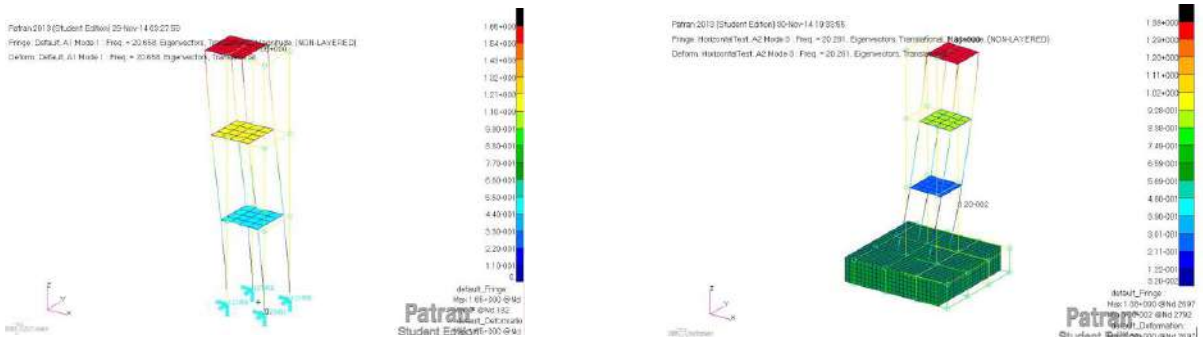


Figure 3. *Typical FEM studies carried out during shake table design.*

The actuation systems required for shake table systems are way more complex than those used for quasi-static tests, being the main reasons the velocity and frequency range requirements. These two facts imply increased demands for both actuators and servovalves. The former must allow for accurate dynamic operation while providing the required (high) loads. For this reason, they are usually equipped with advanced elements such as low friction hydrodynamic or hydrostatic rod bearings and adjustable backlash swivels, both of which minimize non-linearities, thus improving system controllability. The latter must deal with high flow rates at high frequencies; therefore three-stage servovalves are commonly used in these facilities. Due to the high loads and velocities demanded, it is clear that HPUs in these installations feature a high-power consumption.

The most challenging subsystem in a shake table is, however, the control system, especially in MIMO configurations. A commonly adopted approach for the control problem is the usage of two nested controllers; i.e.: the Inner Loop Controller (ILC) and the Outer Loop Controller (OLC).

OLC is in charge of overall system identification, adaptive DoF compensation and solution of inverse and direct kinematics relationships. Fig. 4 shows a block diagram illustrating the main components of a shake table control system using this approach.

The iterative algorithm shown in the block diagram takes the form in Eq. (1).

$$\boldsymbol{v}_{\mathbf{DoF}}(\omega)^{\mathrm{N}+1} = \boldsymbol{v}_{\mathbf{DoF}}(\omega)^{\mathrm{N}} + \mathbf{Z}\left[\boldsymbol{a}_{\boldsymbol{DoF,ref}}(\omega) - \boldsymbol{a}_{\mathrm{DoF}}(\omega)\right]\mathbf{K} \quad (1)$$

Where $\mathbf{Z}$ is the impedance matrix of the system, $\boldsymbol{K}$ is a matrix of correction gains, $\boldsymbol{v}_{\mathbf{DoF}}$ is the vector of DoF drives and $\boldsymbol{a}_{\mathbf{DoF,ref}}$ is the vector of desired DoF accelerations and $\boldsymbol{a}_{\mathbf{DoF}}$ is the vector of achieved accelerations, all expressed in frequency domain. The impedance matrix is identified at the beginning of each test by outputting low level, uncorrelated random stimuli for each DoF and recording accelerometers response, adequately combined by means of direct kinematics relationships and may be updated during the test to account for the changing behavior of SUT, especially when undergoes non-linear deformation [7].

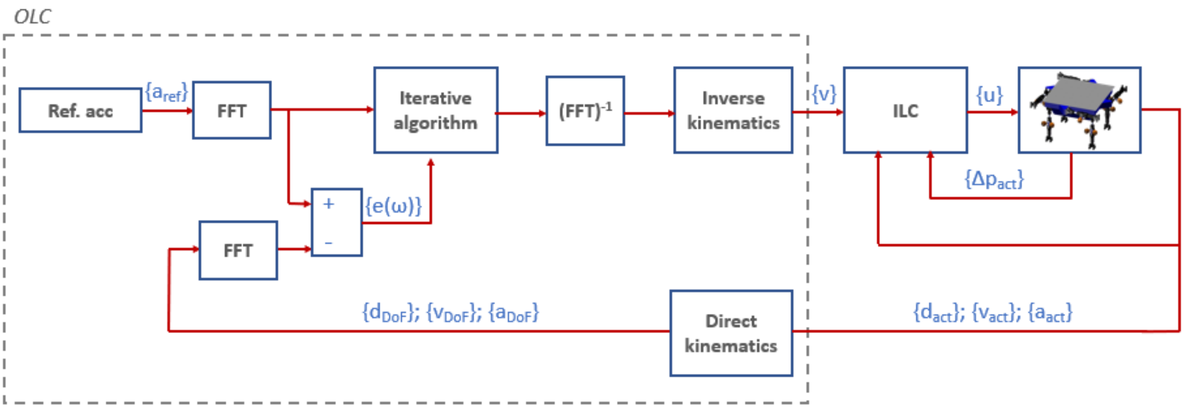


Figure 4. *Shake table control system block diagram*

ILC is in charge of single actuator control and receives the commands synthesized by the OLC. Several control strategies such as PID control, state space schemes or Three Variable Control (TVC) have been used for shake table control, being the main issues the high non-linearity of the hydraulic actuation system (Fig. 5), see Eq. (2), Eq. (3) and Eq. (4) describing time evolution of pressures in cylinder's chambers, servovalve flows and equation of motion for a single actuator respectively [11].

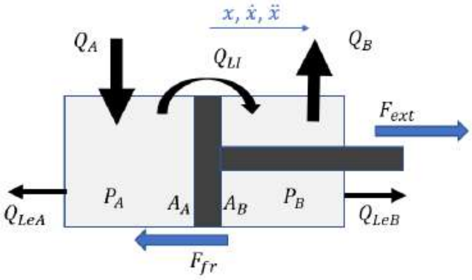


Figure 5. *Hydraulic actuator variables and sign criteria*

$$\frac{(V_{0A}+A_A x)}{\beta_A}\dot{P}_A + A_A\dot{x} = Q_A - Q_{LI} - Q_{LeA};\ \frac{(V_{0B}-A_B x)}{\beta_B}\dot{P}_B - A_B\dot{x} = -Q_B + Q_{LI} - Q_{LeB} \tag{2}$$

$$Q_A = \begin{cases} 0;\ if\ y_{sp} = 0 \\ f(y_{sp})sgn(P_s - P_A)\sqrt{|P_s - P_A|};if\ y_{sp} > 0 \\ f(y_{sp})sgn(P_A - P_R)\sqrt{|P_A - P_R|}\,;if\ y_{sp} < 0 \end{cases}$$

$$Q_B = \begin{cases} 0;\ if\ y_{sp} = 0 \\ f(y_{sp})sgn(P_B - P_R)\sqrt{|P_B - P_R|};if\ y_{sp} > 0 \\ f(y_{sp})sgn(P_S - P_B)\sqrt{|P_S - P_B|}\,;if\ y_{sp} < 0 \end{cases} \tag{3}$$

$$m_p\ddot{x}_p = P_A A_A - P_B A_B - F_f + F_{ext} \tag{4}$$

Where A and B denote actuator chambers (extension and retraction), A_A and A_B are wet areas, P_A and P_B pressures, V_{OA} and V_{OB} dead volumes, β_A and β_B effective Bulk moduli, Q_A and Q_B main flow rates, Q_{LeA} and Q_{LeB} external leakage flow rates, Q_{Li} internal leakage flow rate, y_{sp} servovalve spool position, P_S and P_R supply and return pressures respectively, m_p piston mass, F_f friction force and F_{ext} external force on piston rod. Displacement is represented by x and its time derivatives are denoted with successive upper dots.

Other approach for shake table control consists in using the Minimal Control Synthesis (MCS) algorithm [12] which tries to match the output of a non-linear system to that of a reference system and counts with the attractive advantage of not requiring a pre-test system identification. This approach can be implemented at each servoactuator control loop level, provided it is accessible, or at OLC level [2]. In both cases, MCS algorithm follows the structure shown in Fig. 6.

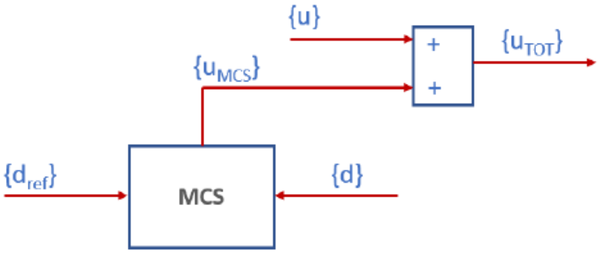


Figure 6. *MCS algorithm application block diagram*

Despite the fact that shake table testing yields the true dynamic response of SUTs, testing of full-scale structures is very often unpractical or impossible due to the prohibitive investment that such large tables would imply in terms of civil works, mechanical and hydraulic actuation systems; nevertheless, some large shake tables have been built in Japan and the U.S. [2,7]. Hence, shake table testing is usually reserved to scaled SUTs, with the difficulties associated to scale laws application [13], or real size components. Regarding component testing, a remarkable and relatively recent application of shake tables is RTHSTT (Real Time Hybrid Shake Table Testing) as it will be seen in next section.

4. HYBRID SIMULATION

In Hybrid Simulation, one or several parts of the SUT (usually the one(s) whose behavior is less understood and/or likely to undergo inelastic deformations) is experimentally tested, while the remainder of the structure is numerically modelled. In a typical test-even though there are different formulations-the displacements at the interface of both domains are calculated by means of a numerical time stepping algorithm and are applied to the experimental structure through hydraulic actuators; restoring forces are then measured and fed back to the numerical model, which calculates displacements to impose on SUT at next time step to proceed with the test. One key point of Hybrid Simulation is that the division of total system into both numerical and experimental subsystems does not need to be performed only along geometrical boundaries, but can also be based on stiffness, damping and inertial properties [14].

If hybrid tests are performed in an expanded time scale; i.e.: at reduced load application speeds, inertial and rate-dependent dissipative properties of the experimental structure need to be accounted for numerically. On the other hand, if tests are carried out at real speeds, which usually happens with reduced size experimental substructures, inertial and rate-dependent dissipative features are identified in real time by force measurements. In the former group of tests, simple set-ups and actuators similar to those shown on Fig. 1 are used. In the latter, dynamic actuators and a complex control system must be used, but the required power is low, since only small components need to be subjected to dynamic loading.

Consequently, Hybrid Simulation can count, in principle, with the advantages of both quasi-static and shake table; that is, dynamic behavior of full-scale SUTs can be reasonably well captured (when certain structural and material conditions are met) while keeping investment in testing equipment and infrastructure within reasonable limits. Different variants of Hybrid Testing are reviewed in next sections.

4.1. Pseudo-dynamic testing

Pseudo-dynamic testing was first introduced by Hakuno [15] and first implementation was carried out by Takanashi [3]. This approach can provide dynamic structural response if the following conditions are met: (a) SUT can be accurately modelled by a set of lumped masses and (b) structure does not exhibit rate-dependent effects.

The idea behind of this approach is to subject certain points of the SUT (nodes) at each time, in a quasi-static manner, to displacements calculated by a numerical time marching algorithm. The numerical scheme makes use of the known external forces value and values of structural restoring force measured by transducers at that time step, to solve equations of motion, Eq. (5) yielding displacement vector to impose to the structure at the next time step.

$$\boldsymbol{M}\ddot{\boldsymbol{x}}_{n+1} + \boldsymbol{C}\dot{\boldsymbol{x}}_{n+1} + \boldsymbol{r}_{n+1}(\dot{\boldsymbol{x}}_{n+1}, \boldsymbol{x}_{n+1}) = \boldsymbol{f}_{n+1} \tag{5}$$

Where $\boldsymbol{M}$ and $\boldsymbol{C}$ represent mass and viscous damping matrices respectively, $\boldsymbol{r}_{n+1}$ is the vector of restoring forces developed by the structure, $\boldsymbol{f}_{n+1}$ is the vector of external forces acting on the structure and $\boldsymbol{x}_{n+1}$ is the vector of nodal displacements evaluated at instant t_{n+1}. Differentiation with time is denoted by successive upper dots. A diagram describing this testing method is shown on Fig. 7.

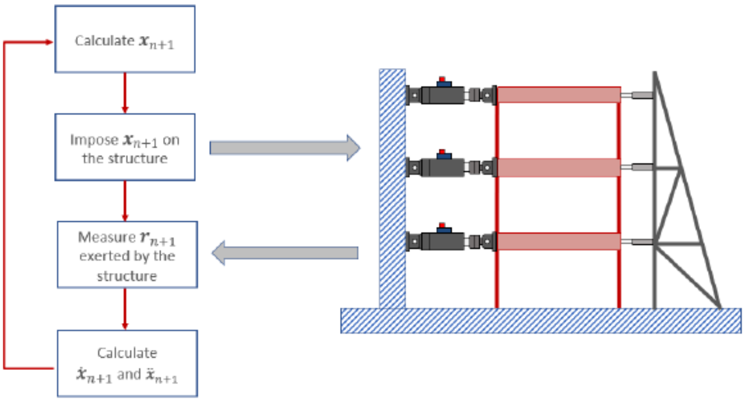


Figure 7. *Pseudo-dynamic testing method generic schematic diagram*

The numerical scheme used for the solution of the equations of motion is of capital importance, both in terms of accuracy and stability, for the successful performance of pseudo-dynamic tests. Newmark based integration algorithms have been widely used [16], see Eq. (6) and Eq. (7) which give expressions of velocities and displacements according to this method.

$$\dot{\boldsymbol{x}}_{i+1} = \dot{\boldsymbol{x}}_i + \Delta t[(1-\gamma)\ddot{\boldsymbol{x}}_i + \gamma\ddot{\boldsymbol{x}}_{i+1}] \tag{6}$$

$$\boldsymbol{x}_{i+1} = \boldsymbol{x}_i + \mathit{\Delta} t\dot{\boldsymbol{x}}_i + \mathit{\Delta} t^2\left[\left(\frac{1}{2}-\beta\right)\ddot{\boldsymbol{x}}_i + \beta\ddot{\boldsymbol{x}}_{i+1}\right] \tag{7}$$

Integration schemes defined by the family of methods defined in Eq. (6) and Eq. (7) can be explicit or implicit depending on the values of parameters γ and β. Explicit methods have the advantage that displacements at a certain time step can be calculated as a function of variables in previous time step while with implicit methods iteration of some kind must be performed, which may have negative effects on the structure such as overshooting and non-smooth trajectories until final desired position is achieved [17]. On the other hand, explicit methods are only stable for values of $\mathit{\Delta} t$ less than a critical time-step T_{min}/π where T_{min} is the minimum period of the structure.

In order to damp out higher frequency modes, which are prone to rapid error accumulation, while retaining lower modes contribution, a modified version of the Newmark method; i.e.: the Hilber Alpha method [18] has been used to solve a shifted version of the equations of motion, see Eq. (8) and Eq. (9), using the expressions for displacements and velocities given by Eq. (6) and Eq. (7).

$$\boldsymbol{M}\ddot{\boldsymbol{x}}_{n+1} + (1+\alpha)\boldsymbol{C}\dot{\boldsymbol{x}}_{n+1} + \alpha\boldsymbol{C}\dot{\boldsymbol{x}}_n + (1+\alpha)\boldsymbol{r}_{n+1} + \alpha\boldsymbol{r}_n = (1+\alpha)\boldsymbol{f}_{n+1} + \alpha\boldsymbol{f}_{n+1} \tag{8}$$

$$-\tfrac{1}{3} \leq \alpha \leq 0,\ \ \beta = \tfrac{1}{4}(1-\alpha^2),\ \ \gamma = \tfrac{1}{2}(1-2\alpha^2) \tag{9}$$

The dissipative features of the scheme increase when α decreases and classical Newmark method is recovered when α equals 0.

Nakashima [19] developed the Operator Splitting (OS) method which aims at combining the advantages of explicit and implicit methods. For that purpose, Eq. (6) and Eq. (7) can be recast using a predictor and a corrector term, see Eq. (8) and Eq. (9).

$$\dot{\boldsymbol{x}}_{i+1} = \dot{\boldsymbol{x}}^*_{i+1} + \Delta t\gamma\ddot{\boldsymbol{x}}_{i+1}\ ;\ \ \dot{\boldsymbol{x}}^*_{i+1} = \dot{\boldsymbol{x}}_i + \Delta t(1-\gamma)\ddot{\boldsymbol{x}}_i \tag{8}$$

$$\boldsymbol{x}_{i+1} = \boldsymbol{x}^*_{i+1} + \mathit{\Delta} t^2\beta\ddot{\boldsymbol{x}}_{i+1}\ ;\ \ \boldsymbol{x}^*_{i+1} = \boldsymbol{x}_i + \mathit{\Delta} t\dot{\boldsymbol{x}}_i + \mathit{\Delta} t^2\left(\frac{1}{2}-\beta\right)\ddot{\boldsymbol{x}}_i \tag{9}$$

And restoring forces estimated by the linear expansion in Eq. (10).

$$\boldsymbol{r}_{\boldsymbol{n+1}} = \boldsymbol{r}^*_{\boldsymbol{n+1}} + \widehat{\boldsymbol{K}}[\boldsymbol{x}_{i+1} - \boldsymbol{x}^*_{i+1}] \tag{10}$$

Where $\boldsymbol{r}_{n+1}^{*}$ is the vector of restoring forces corresponding to $\dot{\boldsymbol{x}}_{i+1}^{*}$ and $\widehat{\boldsymbol{K}}$ is an estimate of the tangent stiffness of the structure which can be taken as the initial structure stiffness. By substituting Eq. (8) and Eq. (9) in Eq. (10) a linear system of equations is obtained which allows for $\ddot{\boldsymbol{x}}_{i+1}$ calculation, see Eq. (11).

$$\overline{\boldsymbol{M}}\ddot{\boldsymbol{x}}_{n+1} = \overline{\boldsymbol{f}}_{\boldsymbol{n+1}} \tag{11}$$

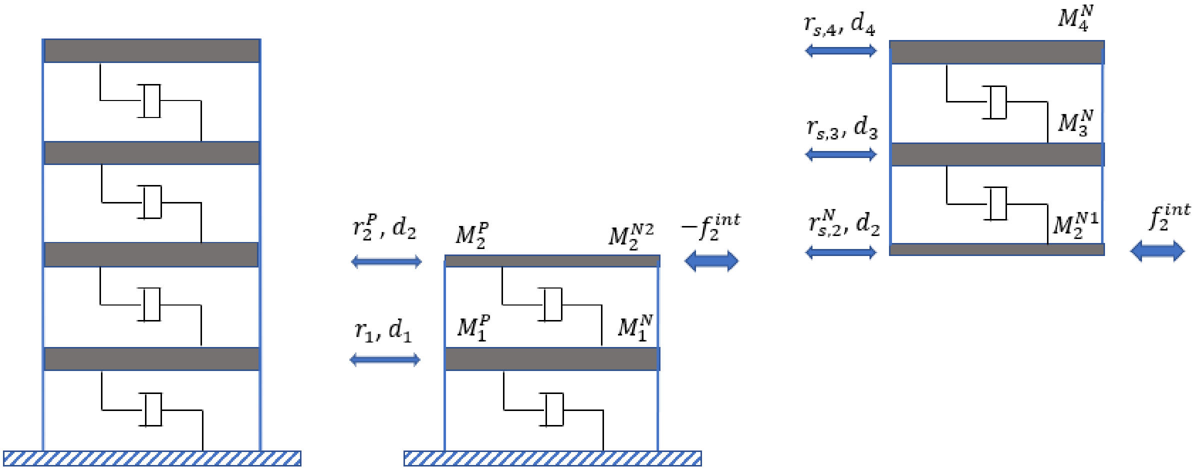
The test proceeds as follows: at the beginning of time step t_{n+1} predictors $\boldsymbol{x}_{i+1}^{*}$ and $\dot{\boldsymbol{x}}_{i+1}^{*}$ are calculated making use of variable values at previous time step. Then, displacements $\boldsymbol{x}_{i+1}^{*}$ are imposed on the structure and $\boldsymbol{r}_{\boldsymbol{n+1}}^{*}$ are measured. Finally, accelerations $\ddot{\boldsymbol{x}}_{n+1}$ are calculated by means of Eq. 11 and displacements $\boldsymbol{x}_{i+1}$ and velocities $\dot{\boldsymbol{x}}_{i+1}$ are calculated via Eq. (8) and Eq. (9). Versions of this algorithm with a fixed number of iterations can also be implemented [20].

Many researchers have studied the origin and effect of the unavoidable errors present in Pseudo-dynamic testing [21]. Besides the intrinsic errors due to structural discretization and numerical scheme, which should be assessed with numerical analysis techniques, those arising from electrical noise, AD and DA conversion, resolution and accuracy of measurement and control instruments, load relaxation in wait periods, support motion and friction among others, enter the numerical solution process and may be carried over subsequent solution steps. These errors can lead to incorrect simulation results or even instabilities during a test; in particular, due to their systematic components; i.e. systematic undershooting is equivalent to adding energy to the higher modes of the structure [22]

While most of these errors can usually be minimized by correct instrumentation, hardware and control strategy selection, others like load relaxation could also be considered intrinsic to the way tests are carried out. This is the reason why Continuous PsD testing was developed. In this modality of Pseudo-dynamic testing, the hold and stabilization periods used in its conventional counterpart for displacement imposition are eliminated, thus allowing for continuous actuator motion reducing relaxation issues and increasing signal to noise ratio [17].

4.2. Pseudo-dynamic testing with substructuring

Full advantage of the hybrid nature of Pseudo-dynamic testing can be attained when performing substructured tests. In this technique, one or more parts of the structure (usually the ones not sufficiently well understood) are physically tested while the remainder of the structure is numerically modelled. Since these tests are not performed at real event speed, the integration of a numerical model accounting for the inertial and dissipative properties of the experimental substructure is also required for the correct execution of the test and actual restoring forces of the physical specimen are read and incorporated in the loop. Consequently, numerical algorithms that best fit both domains characteristics can be selected. An example of a four storey shear building substructured Pseudo-dynamic test set up is shown in Fig. 8.



Total structure Physical substructure Analytical substructure

Figure 8. *Pseudo-dynamic testing with substructuring on a four storey shear building*

In what follows, the Coupled Subdomain Approach [14], will be described in more detail. First, it is assumed that the behavior of the structure can be appropriately described by the following set of spatially discretized equations of motion, see Eq. (12).

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{r}_s = \boldsymbol{f} \tag{12}$$

In which the meaning of the terms is the same as those in Eq. (5). These equations can be written in the form shown in Eq. (13).

$$(\boldsymbol{M}^N + \boldsymbol{M}^P)\ddot{\boldsymbol{x}} + (\boldsymbol{C}^N + \boldsymbol{C}^P)\dot{\boldsymbol{x}} + \boldsymbol{r}_s^N + \boldsymbol{r}_s^P = \boldsymbol{f} \tag{13}$$

Where superscripts N and P make reference to the numerical and physical subsystems. Matrices and vectors in Eq. (13), can be cast in a partitioned manner as shown in Eq. (14) and Eq. (15).

$$\ddot{\boldsymbol{x}} = \begin{Bmatrix} \ddot{\boldsymbol{x}}_N \\ \ddot{\boldsymbol{x}}_I \\ \ddot{\boldsymbol{x}}_P \end{Bmatrix}; \;\; \dot{\boldsymbol{x}} = \begin{Bmatrix} \dot{\boldsymbol{x}}_N \\ \dot{\boldsymbol{x}}_I \\ \dot{\boldsymbol{x}}_P \end{Bmatrix}; \;\; \boldsymbol{r}_s^N = \begin{Bmatrix} \boldsymbol{r}_{s,N} \\ \boldsymbol{r}_{s,I}^N \\ \boldsymbol{0} \end{Bmatrix}; \;\; \boldsymbol{r}_s^P = \begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{r}_{s,I}^P \\ \boldsymbol{r}_{s,P} \end{Bmatrix} \tag{14}$$

$$\boldsymbol{M}^N = \begin{bmatrix} \boldsymbol{M}_{NN}^N & \boldsymbol{M}_{NI}^N & \boldsymbol{0} \\ \boldsymbol{M}_{IN}^N & \boldsymbol{M}_{II}^N & \boldsymbol{M}_{IP}^N \\ \boldsymbol{0} & \boldsymbol{M}_{PI}^N & \boldsymbol{M}_{PP}^N \end{bmatrix}; \boldsymbol{M}^P = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M}_{II}^P & \boldsymbol{M}_{IP}^P \\ \boldsymbol{0} & \boldsymbol{M}_{PI}^P & \boldsymbol{M}_{PP}^P \end{bmatrix} \tag{15}$$

In which subscripts refer to the DoFs belonging to the numerical (N), physical (P) and interface (I) of both domains and superscripts to the subdomain in which quantities are accounted for. Now, proceeding according to the domain decomposition method, both subsystems are dealt with as separate domains coupled by the interaction forces present at their interface DoFs, see Eq. (16) and Eq. (17).

$$\begin{bmatrix} \boldsymbol{M}_{NN}^{N} & \boldsymbol{M}_{NI}^{N} \\ \boldsymbol{M}_{IN}^{N} & \boldsymbol{M}_{II}^{N1} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}}_{N} \\ \ddot{\boldsymbol{x}}_{I} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_{NN}^{N} & \boldsymbol{C}_{NI}^{N} \\ \boldsymbol{C}_{IN}^{N} & \boldsymbol{C}_{II}^{N1} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{x}}_{N} \\ \dot{\boldsymbol{x}}_{I} \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{r}_{s,N} \\ \boldsymbol{r}_{s,I}^{N} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f}_{N} \\ \boldsymbol{f}_{I}^{N} \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{f}_{I}^{int} \end{Bmatrix} \tag{16}$$

$$\begin{bmatrix} \boldsymbol{M}_{II}^{N} & \boldsymbol{M}_{IP}^{N} \\ \boldsymbol{M}_{PI}^{N} & \boldsymbol{M}_{PP}^{N2} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{x}}_{I} \\ \ddot{\boldsymbol{x}}_{E} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{C}_{II}^{N2} & \boldsymbol{C}_{IP}^{N} \\ \boldsymbol{C}_{PI}^{N} & \boldsymbol{C}_{PP}^{N} \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{x}}_{N} \\ \dot{\boldsymbol{x}}_{I} \end{Bmatrix} + \begin{Bmatrix} \boldsymbol{r}_{I}^{P} \\ \boldsymbol{r}_{P} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{r}_{I}^{P} \\ \boldsymbol{r}_{E} \end{Bmatrix} + \begin{Bmatrix} -\boldsymbol{f}_{I}^{int} \\ \boldsymbol{0} \end{Bmatrix} \tag{17}$$

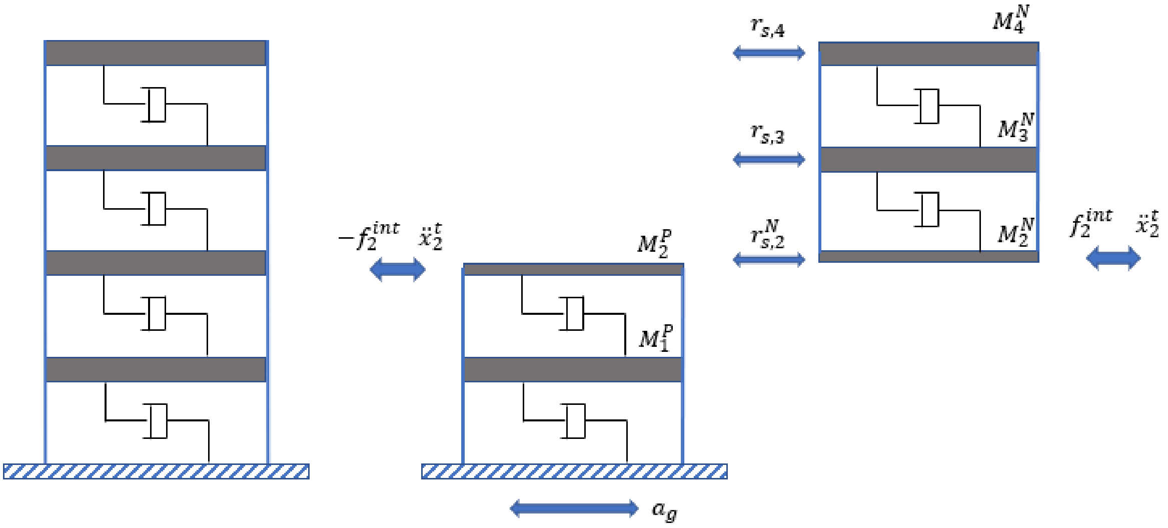
These systems of equations must be solved satisfying compatibility and equilibrium conditions, which leads to Lagrange multipliers problems with relatively complex domain interaction solution algorithms in which generally, interface velocity continuity is imposed. See [17] for more details.

4.3. Real-time hybrid simulation

One major drawback of Pseudo-dynamic testing with substructuring is its inability of capturing dynamic behavior of the SUT; i.e.: rate-dependent effects such as those manifested by secondary structures (fluid, magneto-rheological and pendulum dampers, etc.) or by the SUT itself. To be able to cope with velocity sensitive substructures, Real-time hybrid simulation was developed [5].

When employing this testing technique, only numerical substructure is solved by means of a time marching algorithm, see Eq. (16). Restoring forces developed by the numerical structure are directly measured by load transducers and include static, dissipative and inertial components, as these tests are carried out at actual deformation velocities. Therefore, numerical solution of Eq. (17) is not required, and structural response is obtained by pure dynamic testing without modelling assumptions [14]

An example of Real time hybrid simulation test set up for the same four storey building discussed previously is shown in Fig. 9.



Total structure

Physical substructure

Analytical substructure

Figure 9. *Real time hybrid testing on a four storey shear building*

In this case, the set of ordinary differential equations that needs to be solved numerically are shown in Eq. (18).

$$\begin{bmatrix} M_2^N & 0 & 0 \\ 0 & M_3^N & 0 \\ 0 & 0 & M_4^N \end{bmatrix} \begin{Bmatrix} \ddot{x}_2^t \\ \ddot{\hat{x}}_3 \\ \ddot{\hat{x}}_4 \end{Bmatrix} + \begin{bmatrix} 0 & -C_3^N & 0 \\ 0 & C_3^N + C_4^N & -C_4^N \\ 0 & -C_4^N & C_4^N \end{bmatrix} \begin{Bmatrix} \dot{x}_2^t \\ \dot{\hat{x}}_3 \\ \dot{\hat{x}}_4 \end{Bmatrix} + \begin{Bmatrix} r_{s,2}^N \\ r_{s,3} \\ r_{s,4} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -M_3^N \ddot{x}_2^t \\ -M_4^N \ddot{x}_2^t \end{Bmatrix} + \begin{Bmatrix} f_2^{int} \\ 0 \\ 0 \end{Bmatrix} \tag{18}$$

Where superscript t and hats denote absolute and relative to the second storey motions respectively. This example corresponds to a test set up in which ground acceleration is imposed to the physical substructure by means of a shake table. Taking into account that $r_{s,2}^N = r_{s,3} + r_{s,4}$, the last two equations can be integrated measuring $\ddot{x}_2^t$ from the physical structure and f_2^{int} to be imposed by means of an actuator can be factored out from the first equation. Tests of this nature have been extensively performed [23].

A remarkable difficulty found in Real-time Hybrid Simulation is that of delay of servoactuators, or more generally, transfer systems, in imposing the required reference profiles on the SUT. It has been shown that this delay is equivalent to introducing negative damping in the system even leading to instabilities in the test [24]. To overcome delays, several techniques have been used. As significative approaches it is worth noting polynomial extrapolation, phase lead compensators or model-based delay cancellation techniques. See [25] for more details.

4.4. Effective Force testing method

The Effective Force testing method was first devised by Dimig [6]. The basic idea of this approach is to exert on each of the (lumped) masses of the SUT the inertial forces they would experience in the real loading event, that is, the product of the mass times the ground acceleration. Fixed base test set-ups similar to those discussed in section 2 and subsection 4.1 of this work are commonly used for multi-storey mock-ups testing when using a total approach; i.e.: without substructuring.

By imposing the actual forces (known beforehand) on the masses, the need of solving equations of motion disappears and the real dynamic response of the SUT is retrieved directly. However, this sort of test requires advanced control systems able to impose loads accurately, especially at frequencies close to SUT resonant frequencies [6] and cope with servoactuator interactions in MIMO systems.

Substructuring approaches can also be employed in Effective Force testing method, making use of servoactuators and shake tables, as discussed above.

4.5. Applications of Hybrid Simulation in structural systems

In this subsection, classical applications of Hybrid Simulation in several branches of engineering as well as some possible applications are outlined. It is noted here that applications in Electronic Engineering in which Hybrid Simulation is commonly known as Hardware in the Loop (HIL) simulation are excluded from this work, since no transfer systems-that is, systems of mechanical actuation-are involved.

4.5.1. Civil Engineering

Pseudo-dynamic testing method originated within the field of Earthquake Engineering. A typical application in this field is the testing of multi-storey buildings subjected to uniaxial or multiaxial ground excitation, with or without substructuring [1]. In the latter case, non-linear geometric transformations required to translate displacements from actuators to SUT space have been used [21]. Other expanded time-scale applications, which make use of substructuring, include physical testing of bridge isolators with coupled numerical models of the remainder of the bridge [17], reinforced concrete frames or columns of various shapes. Non-planar extensions to these applications have also been implemented [26].

In order to adequately predict structural dynamic response in SUTs with rate dependent behavior caused by secondary systems mounted on structures [14], real time substructured tests of MR dampers, sloshing dampers and tuned mass dampers have been carried out. Some of these involve the joint use of dynamic servoactuators and shake tables. Other dynamic substructuring seismic applications related to soil-pier-structure interaction involving shake tables and actuators have been proposed.

The particular features of real time hybrid simulation make it attractive to explore its application in testing active vibration control devices such as those in study for footbridges under a wide variety of crowd induced vibration scenarios. Control laws employed in approaches described in [27] could be evaluated and pre-tuned in a hybrid setup consisting in two serial actuators in which the former simulates the motion of the vibration control device (VCD) installation point and the latter, the VCD itself. Forces exerted by the VCD on the SUT, sensed by a load cell, are transferred to the numerical model residing in a real-time computer which calculates installation point displacement, by means of a time stepping integration algorithm, accounting for prescribed external forces as well. Calculated displacement reference is then sent to a real-time controller in charge of ensuring accurate kinematic reference tracking of the actuator simulating footbridge motion. A concept of the Single Input-Single Output (SISO) version of the proposed test setup is shown in Fig. 10.

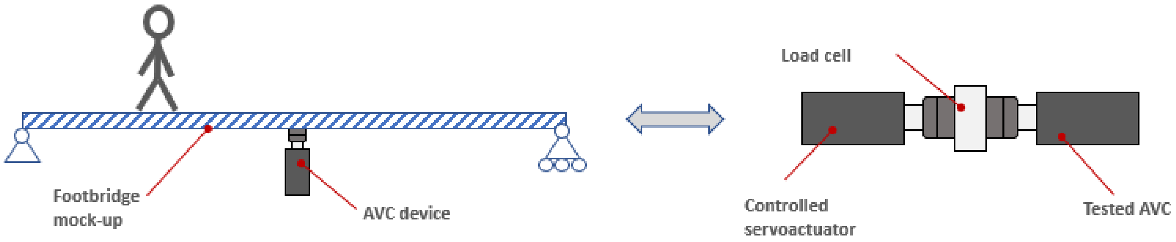


Figure 10. *Suggested concept for Active VCD testing*

Another appealing application of hybrid simulation in Civil Engineering structures is the evaluation of long span bridges behavior under the action of aerodynamic and traffic loads of variable intensity, nature and speed. Two approaches can be considered here: firstly, substructured tests of critical components and secondly, tests of scaled models; since full-scale testing would be possible in very few occasions. In both approaches, one of the key aspects of successful execution is the accuracy of the aerodynamic model. An additional difficulty in the second approach is the need to find an optimal number and location of actuators able to reproduce with sufficient accuracy overall structural response, due to the inherent distributed mass nature of this type of structures. Some applications of these tests would be, aside of assessment of component or structural performance under exceptional loads, studies of allowance of high-speed train traffic pass.

4.5.2. Automotive Engineering

In Automotive Engineering Hybrid Simulation has found application in testing components such as tires, suspensions, transmissions, or agricultural drives coupled to a chassis numerical model and subjected to ground, driver and environmental actions. This modality is sometimes known as Component in the Loop (CIL) testing [25]. In formula one industry, a tire coupled simulator, colloquially known as four poster test rig, exerts road actions on the complete vehicle by means of four vertical hydraulic actuators, while a system of three actuators simulates aerodynamic downforce and moments output by a real time mathematical model running in parallel [28].

Other applications of hybrid simulation have been implemented in the field of passive safety and were aimed at estimating seat belt anchorages resistance in the event of a crash using existing test rigs used to perform static tests prescribed by usual regulations [29]. This approach could also be used for other normative tests such as those specified for seat backs and head restraints, roof crush or side intrusion.

4.5.3. Aerospace and Aeronautical Engineering

In Aerospace Engineering, Hybrid Simulation has been used to perform Coupled Load Analysis (CLA) of spacecrafts which are vibrated by shake tables interacting in real time with a numerical simulation of the launcher [30].

Some examples of applications in Aeronautical Engineering such as testing of lag dampers used in helicopter rotor instabilities have been implemented [25]. Other applications oriented to aeroelastic behavior prediction or maneuver assessment under a wide range of operational and environmental conditions could be considered. These implementations would be based in software developments taking advantage of the already existing static and dynamic test rigs.

5. CONCLUSIONS

In this work, a brief description of the three approaches employed for dynamic assessment of structural systems; i.e.: quasi-static testing, shake table testing and hybrid simulation, has been given. Even though these testing methods originated in the field of Seismic Engineering, their application has been extended to other areas such as Automotive, Aeronautical and Aerospace Engineering.

In quasi-static testing, the structure under test is subjected to predefined time stories of load or displacement at some of its points by means of hydraulic servoactuators. Equipment required to perform these tests is inexpensive yet offering the capacity to test full-scale specimens, good quality measurements and control performances. Although this method does not capture dynamic behavior, it is still widely used to characterize behavior of structures in load reversals or under low cycle fatigue.

Shake tables are rigid platforms, powered by actuators, able to move in one or more degrees or freedom on to which structures under test are installed. Even though shake tables are able to reproduce true structural dynamic response, limitations in their size lead very frequently to tests of scaled specimens with the associated difficulties. Equipment required is complex and expensive due to the accurate dynamic performances demanded to shake tables. A key component of these testing systems is the control system which must deal with strong non-linearities inherent to hydraulic actuation systems and those related to specimen behavior. Adaptive control features are also required to cope with variations of mechanical properties of structures as the test is executed.

In hybrid simulation, the structure whose dynamic behavior needs to be evaluated is divided into a numerical part (with a well understood behavior) and physical substructures (difficult to model numerically). This separation does not need to be only along geometrical boundaries, but can also be performed based on inertial, dissipative and stiffness properties. In order to emulate total dynamic structural response, numeric and experimental domains are coupled. The test proceeds as data from physical testing is fed into the numerical model and the outputs of the latter are imposed on the physical system. The different variants of hybrid testing according to criteria of system substructuring, rate of imposed loads and controlled variables, have been succinctly discussed in this paper: pseudo-dynamic tests without and with substructuring, real time hybrid tests and the effective force method. Thanks to the hybrid approach, and as long as certain requirements are met, good estimates of complete structural dynamic behavior can be obtained without the need of large testing facilities.

Finally, some common applications of hybrid simulation in Civil, Automotive, Aerospace and Aeronautical Engineering fields have been outlined and some potential ones related to active vibration control devices and fluid-structure and vehicle-structure interaction have been proposed.

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